Journal of Pedagogical Research
Volume 9, Issue 1, 2025
https://doi.org/10.33902/JPR.202527952

Research Article

Visualizing relative position of two straight lines in space: An exploratory study of the anaglyph in GeoGebra

Tang Minh Dung[1], Le Thai Bao Thien Trung[2] and Nguyen Thi Nga[3]

[1]*Ho Chi Minh City University of Education, Vietnam (ORCID: 0000-0001-5401-1395)*
[2]*Ho Chi Minh City University of Education, Vietnam (ORCID: 0000-0001-7269-9461)*
[3]*Ho Chi Minh City University of Education, Vietnam (ORCID: 0000-0002-0689-2295)*

This study investigates whether Vietnamese students face specific challenges when visualizing skew versus intersecting lines and whether using the anaglyph feature of GeoGebra can help overcome these difficulties. Employing a design-based research approach over four phases with 12th-grade students, the study involved tasks in traditional paper-pencil and GeoGebra's anaglyph environment, which offers 3D depth perception through color separation. The findings showed an improvement in students' accuracy in spatial visualization on 2D surfaces after using anaglyphs, as they were better able to interpret the relative positions of lines in space. This research highlights the potential of low-cost, accessible digital tools like the anaglyph feature of GeoGebra in enhancing spatial understanding in educational systems with limited resources. The insights gained from this study contribute to geometry education by demonstrating how anaglyph representations can aid in 3D visualization.

Keywords: Anaglyph; Relative position of two straight lines; Space; Spatial visualization

Article History: Submitted 26 April 2024; Revised 6 November 2024; Published online 6 March 2025

1. Introduction

1.1. Difficulties of Teaching Spatial Geometry

Spatial geometry is a vital component of the mathematics curriculum, enhancing students' ability to visualize and understand complex mathematical concepts and their applications in real-world situations (Lowrie et al., 2019). There is a strong correlation between spatial thinking abilities and mathematics performance. Students with developed spatial skills tend to perform better in mathematics, as these skills aid in understanding and solving mathematical problems (Atit et al., 2022; Cheng & Mix, 2014). Spatial thinking is not only crucial for mathematics but also for broader STEM fields. Well-developed spatial skills are linked to success in science, technology, engineering, and mathematics, highlighting the importance of integrating spatial skills training into STEM education (Newcombe & Shipley, 2015; Uttal et al., 2013). Beyond academic achievements, spatial skills have practical applications in everyday tasks and professional

Address of Corresponding Author

Dung Minh Tang, PhD, Ho Chi Minh City University of Education Address: 280 An Duong Vuong Street, District 5, Ho Chi Minh City, Vietnam.

dungtm@hcmue.edu.vn

How to cite: Dung, T. M., Trung, L. T. B. T., & Nga, N. T. (2025). Visualizing relative position of two straight lines in space: An exploratory study of the anaglyph in GeoGebra. *Journal of Pedagogical Research, 9*(1), 203-219. https://doi.org/10.33902/JPR.202527952

activities. From navigation to architecture and engineering, spatial skills enable individuals to interact effectively with their physical environment (Gilligan et al., 2017).

Geometry requires the ability to transform and manipulate spatial properties, which is a crucial component of spatial visualization (Baki et al., 2011; Lowrie et al., 2019). Spatial visualization refers to the ability to mentally represent, manipulate mentally, and understand spatial information of three-dimensional objects (Jones, 2001; Leopold, 2015). It involves understanding and interpreting two-dimensional representations of three-dimensional objects, such as drawings, and visualizing their spatial relationships and configurations in one's mind (Seah, 2015). It plays a crucial role in understanding and solving geometry problems, enabling individuals to comprehend the spatial aspects of geometric concepts and manipulate them mentally (Weckbacher, 2007; Yurmalia & Herman, 2021). Thus, spatial visualization ability is a crucial competency for students engaged in geometry, as it facilitates enhanced comprehension and effective communication of spatial objects and their properties. Lowrie et al. (2019) also suggest that the benefits of spatial visualization extend beyond solving geometry problems to include other types of mathematical issues, such as word problems and those requiring the interpretation of graphics (non-geometry).

Improving spatial visualization skills can be achieved through various methods. One approach is implementing academic interventions such as specific courses or instruction focused on enhancing spatial visualization abilities. These interventions can include Computer-Assistant Design courses (Rodriguez & Rodriguez-Velazquez, 2020), technical drawing training (Prieto & Velasco, 2010), and standard materials and games (Berciano & Gutiérrez, 2015) to engage students in spatial visualization tasks. Nevertheless, research findings have revealed a broad spectrum of spatial visualization capabilities among students, with some students having a better ability than others (Lyon et al., 2019). However, students frequently encounter substantial challenges in visualizing and comprehending objects in the three-dimensional space when relying on traditional two-dimensional instructional materials (Bakó, 2003; Gal & Linchevski, 2010; Gutiérrez, 1996; Susilawti et al., 2017). This difficulty is exacerbated by the limitations of physical models and drawings in accurately representing 3D objects, leading to misconceptions and a lack of intuitive understanding of geometric concepts (Accascina & Rogora, 2006; Clements & Battista, 1992). This issue highlights the inherent challenge in teaching spatial geometry and underscores the importance of innovative methods to enhance visualization skills in educational settings.

1.2. Anaglyph as a Potential Tool in Teaching Spatial Geometry

In mathematics instruction, visualization in spatial geometry can be achieved through three kinds of activities: (1) physical manipulative activities on items, physical models, origami, etc.; (2) computer-based activities, especially dynamic geometry software such as Cabri 3D, GeoGebra (see Kösa, 2016); (3) combination of the two kinds of activities above, for instance, Lieban (2019) developed activities exploring geometric modeling while combining physical and digital resources. In the first one, the real objects/physical models give the most realistic feel to 3D objects. However, they are pretty inconvenient due to their bulkiness, which takes up much space (so it is difficult to move and store). They require regular maintenance and much work to manufacture, and they are significantly often only suitable for representing simple 3D objects (without too many components). Hence, these characteristics lead to certain anxiety about its use in teaching practice.

Meanwhile, with the advent of the Internet and tech boom, the use of ICT as a tool for visual teaching has been of interest to many researchers (Lagrange et al., 2003). 3D objects can be visualized through rotation and interaction on plan representations of dynamic geometry software (Christou et al., 2006; Mithalal & Balacheff, 2019). Therefore, one successful method used to accomplish this and widely adopted today in the classroom is allowing students to manipulate 3D objects on a computer monitor. Advancing the study one step further, we ask if there is any other method to develop visualization in the classroom regardless of dynamic representations. This is an essential question because students will only sometimes benefit from utilizing a computer to manipulate geometric objects, including while taking an exam. We are interested in the anaglyph perspective as a strong candidate in this context.

Anaglyph is a stereoscopic representation method invented by mathematician Wilhelm Rollmann and is even more relevant in the complex modern world (Jančařík, 2016). Two half-images make it in different colors (often two color systems: red and blue/green/cyan) created from two central projection images projected from the centers approximately 6.5 centimeters apart (generally corresponding to the center distance between a set of human eyes). By using 3D glasses (in which each eye can only see a half-image of one color), the observer's brain synthesizes two half-images of each eye into a three-dimensional object (Ferdiánová, 2017; Guedes et al., 2012; Kmetova, 2015). Due to the biological mechanism of the brain's vision, though anaglyph representation is static, it can still highlight the third dimension (depth perception) of representation through the senses and interpret by mental reflection (Hemmerling & Lemberski, 2008; Kmetova, 2015). This representation is an intermediate element between 3D objects and their static 2D model.

While studies on visualization in the dynamic geometry environment are numerous, the existing studies on the impact of anaglyph (static status) on visualization in mathematics education still need to be more extensive and mainly deal with generic 3D objects. In particular, no study mentions using anaglyph to build teaching situations using an approach that meets the institutional constraints and difficulties specific to an education system. For example, learning spatial geometry in Vietnam requires a high spatial ability to understand geometric shapes with complex configurations. Not all students can satisfy that demand if they only work on flat representation. This perspective interests us in the question: How can the anaglyph help Vietnamese students overcome the institutional difficulties of learning spatial geometry? Answering this question allows the sustainable embedding of the anaglyph into the Vietnamese education system and enhances its viability (Lagrange et al., 2003). It leads to the benefit of developing the spatial visualization of Vietnamese students.

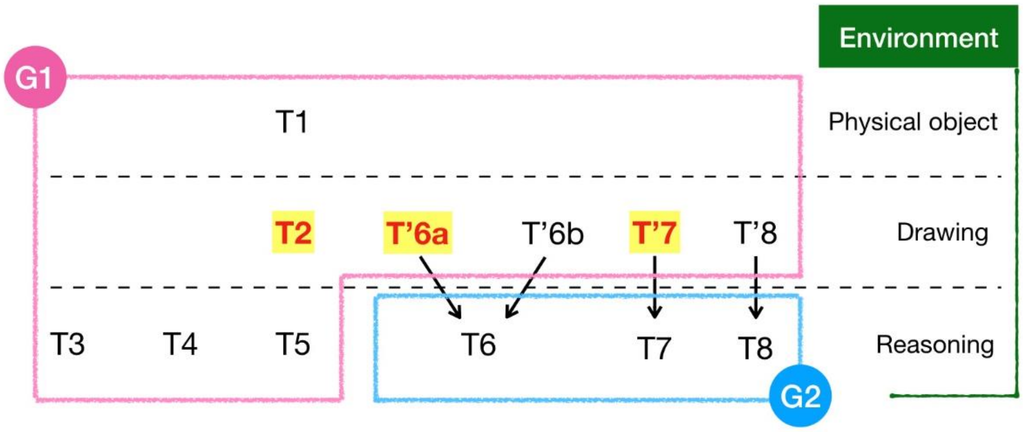
In this study, there are many spatial objects and relationships, but we limit them to the relative positions of two straight lines in space. Studies have yet to be conducted on this content, even though this subject is taught in all education geometry programs and has been reviewed in spatial geometry reasoning. Moreover, a two-dimensional drawing of two straight lines can hardly distinguish their relative position in space due to the loss of information during the projection.

Technically, the anaglyphs in this study were generated by the feature "Anaglyph" of GeoGebra, version 5.0 (Kmetova, 2015). It is a free software widely used in mathematics education in many countries. In particular, it has also been included in the Vietnamese Mathematics textbook of the new General Education Curriculum (2018). Hence, the students in this study's experiment worked with GeoGebra's interface. However, this is optional in teaching practice (in schools where computer equipment is lacking) because the anaglyph representations can be printed in color and distributed to students.

1.3. Case of Teaching Relative Position of Two Straight Lines in Space in Vietnam

According to the Anthropological Theory of the Didactic (Chevallard, 1992), research on the spatial visualization of an individual cannot be separated from the system of education where it is geographically located. The analysis of the types of tasks proposed in the textbook is considered an official document for teachers and the basis for constructing their lesson plans. The textbook contains the exercises that all Vietnamese students must complete. Assude et al. (1996) state that the textbook is a sound representative of a "weighted average of several constraints" of institutional relation to the knowledge object. Consequently, Chaachoua and Comiti (2010) supposed that textbook research is an indispensable entrance to understanding or characterizing an education system's state at a specific moment. In Vietnam, only the Ministry of Education and Training has the authority to organize the compilation, evaluation, and approval of school textbooks. Therefore, the influence of textbooks on teaching is enormous. We classified the proposed exercises in 11th-grade textbooks (Geometry 11) into types of tasks to identify types of tasks that can pose difficulties in visualization. The activities and exercises in the Vietnamese textbook Geometry 11 related to the relative positions of two straight lines were separated into two main groups. The first group (G1) consisted of the types of tasks directly related to the relative positions of the two straight lines (T1, T2, T3, T4, T5). The second group (G2) includes the indirect ones (T6, T7, T8). A part of their technique contains sub-task types directly related to the relative positions of the two straight lines (T'6a, T'6b, T'7, T'8).

Figure 1
Types of tasks related to the relative positions of the two straight lines



Note. T1: Find two skew lines on a physical object; T2: Find two skew lines on the drawing; T3: Prove that two straight lines are skew; T4: Prove that two straight lines do not intersect; T5: Prove that two straight lines are parallel; T6: Determine the intersection of two planes; T7: Determine the intersection point of a straight line and a plane; T8: Prove that a line is parallel to a plane; T'6a: Find two intersecting lines in two given non-parallel distinct planes on the drawing; T'6b: Find two parallel lines in two given planes on the drawing; T'7: Find a straight line in the given plane intersecting a given straight line on the drawing; T'8: Find a straight line in the given plane parallel to a given straight line on the drawing.

Suppose the techniques for reasoning-based types of tasks (T3, T4, T5, T6, T7, T8) can be found in the examples, and the technology that explains them is described explicitly in theorems and consequences in the textbook. In that case, techniques for the remaining types of tasks (T1, T2, T'6a, T'6b, T'7, T'8) have yet to be introduced. Among them, T1, working on the physical object, is required in only a single exercise; all the others (T2, T'6a, T'6b, T'7, T'8) require students to study relative positions of two straight lines on the drawing. According to Tang (2014), the drawing in teaching Spatial Geometry in Vietnam is based on parallel projection, which preserves the parallelism of two lines. Thus, in solving the types of tasks T'6b and T'8, students can find two parallel lines on drawing through their representations, which are now parallel lines.

Nevertheless, the types of tasks T2, T'6a, and T'7 require visualization to distinguish the intersection or skewing of two lines on the drawing. These three types of tasks can cause difficulties due to the loss of information in the transition from 3D objects to 2D objects by parallel projection. For example, a point on the drawing drawn by a parallel projection - which is a surjective mathematically - could be an image of more than one point in space. Thus, the intersection on the drawing may differ from the space intersection. If seen merely as a projection image, the drawing is not sufficient to replicate the 3D objects (Tang, 2014). From here, we are interested in how students used spatial visualization ability to expand the relative positions of two straight lines through drawing.

In this context, the study seeks to explore two primary research questions:

RQ 1) Do Vietnamese students encounter difficulties in distinguishing between two intersecting lines and two skew lines when interpreting planar representations?

RQ 2) Can anaglyphs help Vietnamese students overcome the difficulties of visualizing the relative positions of two straight lines in space?

2. Method

2.1. Research Design

To examine the two research questions above, this study used the design-based research approach with four phases summarized in Table 1.

Table 1
Four phases in the design research

Phase	*Environment*	*Representation of geometry objects*	*Type of work*	*Task*
1	Paper-Pencil	Drawing	Individual	Find the intersection of two given plans
2	GeoGebra	Anaglyph	Individual	Study relative positions of two straight lines
3	GeoGebra	Anaglyph	Group	Find two intersecting lines in two given planes
4	Paper-Pencil	Drawing	Individual	Draw the intersection of two planes

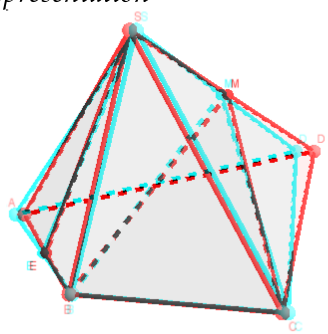
In the phase 1, we investigated the student's ability to solve the type of tasks T6 "Determine the intersection of two planes." We used a descriptive case study approach to consider students' answers. The survey included 126 students from three 12th-grade classes (encoded as 12A1, 12A15, 12B5) in Vietnam. The students completed 11th-grade geometry and were entering 12th-grade geometry. Set in a case study, this phase helps us confirm Vietnamese students' difficulties in distinguishing between two intersecting lines and two skew lines when interpreting planar representations.

If two-dimensional representation is not enough to support students in visualizing the relative positions of two straight lines in the type of tasks T6, we will conduct a didactic situation (phases 2-4) for students (classe 12A1) who failed the task of the phase 1, to enhance 3D visualization by using the feature of an anaglyph representation of GeoGebra.

In phase 2, wearing the provided 3D glasses (Figure 2), look at the anaglyph representation of the geometry object in the phase 1 (Figure 3), students worked individually to study relative positions of two straight lines. These couples of straight lines were skewed, but the students (in the phase 1) thought that they were intersecting. Our goal is to uncover which relative positions between two lines students see on the anaglyph, especially if the students repeat their previous mistakes.

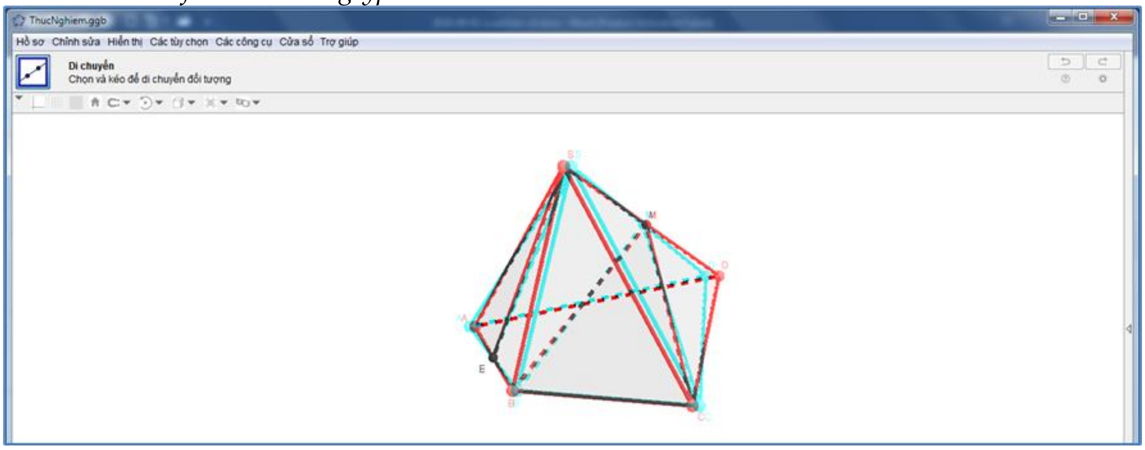
Figure 2
Anaglyph 3D glasses

Figure 3
Anaglyph representation



In phase 3, we introduced the type of task T*: "Find two intersecting lines in two given planes on the anaglyph of GeoGebra." Students were required to wear anaglyph 3D glasses while observing the pyramid drawing, while GeoGebra software was set in the companion mode "Projection for 3D Glasses". GeoGebra allows us to restrict the applications (buttons) accessible during testing from the default, and we made available only the option to "Draw a straight line through two points in space."

Figure 4
GeoGebra interface with anaglyph



In this activity, students entered a virtual environment (but were shut out of their real 3D environment). The anaglyph in the 3D mode of GeoGebra generated a perception of seeing objects in 3D (in this instance, a pyramid object). At the same time, we made only a single tool on GeoGebra available – "Draw a straight line through two points in space," which specifically blocked the ability to automatically generate the intersection of two planes ("Intersect two surfaces" button), a standard feature of GeoGebra. As a result, students were limited to utilizing the "Draw a straight line through two points in space" button as a ruler and pencil (the method students were already accustomed to) but also had another potential benefit explored later. Additionally, group collaboration created a dynamic environment where students could observe, learn, solve problems, and aid their peers with the process.

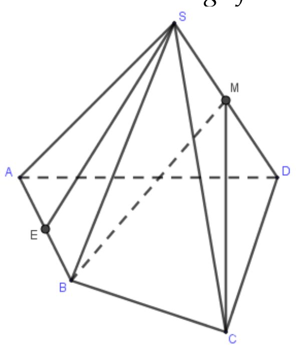
In phase 4, students were required to work individually to draw the intersection of two planes, but this time without the benefit of the GeoGebra software and instead with traditional rulers and paper.

2.2. Research Instruments

2.2.1. Phase 1

In the initial phase of this study, students were asked to solve the problem (with the available drawing, Figure 5) "Suppose that S.ABCD is a pyramid whose bottom ABCD is a convex quadrilateral with no parallel edges. M and E are, respectively, the midpoints of SD and AB. Find the intersection of plans (SAE) and (MBC)." This problem belongs to the type of task T6 "Determine the intersection of two planes." The supposition that "ABCD is a convex quadrilateral with no parallel edges" brings the solution to the type of task T'6a because, at this time, the absence of elements of parallelism forces students to determine the intersection through points of intersection.

Figure 5
Available drawing of the geometry object in the problem of the phase 1



Traditionally, the study of spatial geometry in Vietnam and many other countries frequently starts from solids. Therefore, students have plenty of time to understand the properties of solids (e.g., rectangular, prism, pyramid). In the requirement "Find the intersection of plans (SAE) and (MBC)," the properties of the two planes (SAE) and (MBC) are not the same. Indeed, the plane (SAE), coinciding with the plane (SAB), contains one face of the pyramid S.ABCD. Therefore, the links of the plane (SAE) to the elements (edges, faces) of the pyramid are evident to the students. Meanwhile, the plane (MBC) lies separately from all faces of the pyramid. This makes the plane (MBC) link to the pyramid loose. This choice challenges students to establish new connections, namely the relative position (intersect / skew) between the lines.

2.2.2. Phase 2

Students still worked on the setting of the pyramid S.ABCD in phase 1. They were asked to "mark the cross (x) in the relative positions (parallel, intersecting, skew) of the ten couples of straight lines: (1) SA and MC; (2) SE and MB; (3) AE and MC; (4) SE and MC; (5) SE and BC; (6) SB and MC; (7) SA and BC; (8) SA and MB; (9) AD and MB; (10) ME and SB."

2.2.3. Phase 3

Still with pyramid S.ABCD in phase 1, students were asked to "find two intersecting lines in two planes (SAE) and (MBC)" with the support of anaglyph representation in GeoGeora. Initially, the student can find the couple of straight lines AE and BC (intersecting at B) that satisfy the requirement. But, the second couple that did not pass through point B is a challenge.

2.2.4. Phase 4

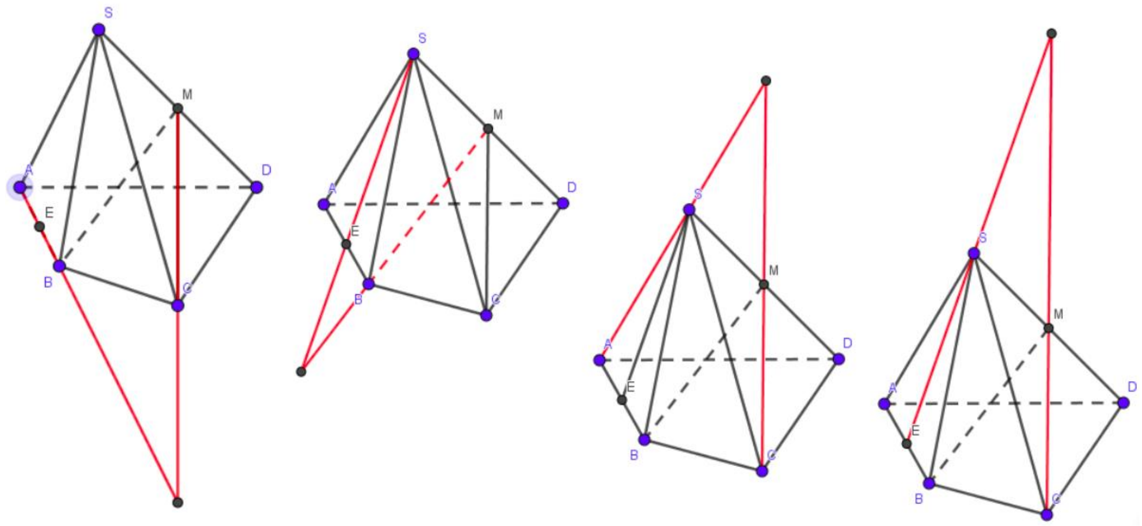
Returning to drawing (in paper-pencil environment), students were asked to "draw the intersection of two planes" with the setting of the pyramid S.ABCD in phase 1. The achievements in phase 3 is expected to help students solve this traditional requirement.

2.3. Data Analysis

2.3.1. Phase 1

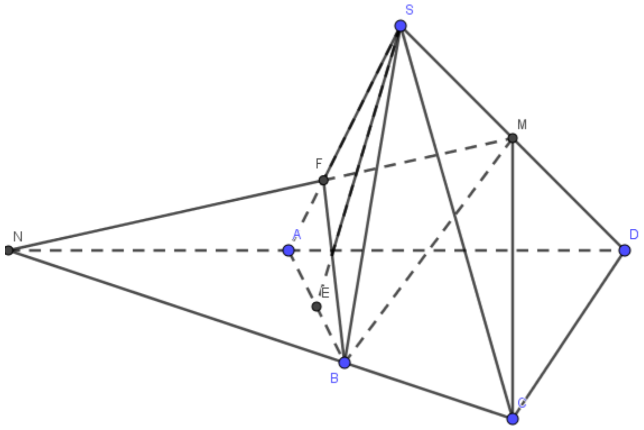
The drawing is made available to illustrate the most likely appearance of the intersecting lines. This provides an opportunity to develop strategy S1: Extending lines on the drawing and determining the intersection of straight lines in space through one of their representations (lines) on the drawing. We want to see if the test students make this mistake (Figure 6).

Figure 6
Drawings possible in the case of strategy S1



The correct solution to the problem comes from strategy S2: Determining the intersection point of straight lines in space through one of the coplanar lines (Figure 7). Students must determine a common plane containing both lines and find their intersection points. Especially in the case of the above problem, two straight lines, AD and BC, belong to the same bottom plane of the pyramid, and they intersect (by supposition). Thus, an intersection N exists for these straight lines. Then, because SA and MN are located in the face (SAD) of the pyramid, intersection point F of these lines exists. Because B and F are common points of two planes (SAE) and (MBC), BF is the intersection of these planes.

Figure 7
Drawing in the case of strategy S2



3.2.2. Phase 2

A descriptive statistic was used to determine the proportion of correct answers, and also to compare each student's answers in phase 1 to consider whether the analytical representation helped them correctly recognize the relative position of two straight lines.

3.2.3. Phase 3

In the drawing, two planes (SAE) and (MBC) were represented by triangles SAE and MBC, and these surfaces were small. To determine intersections, students were required to widen them. We predict three possible methods in Table 2.

Table 2
Illustrating the three anticipated student methods for the problem

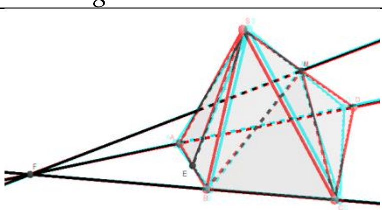
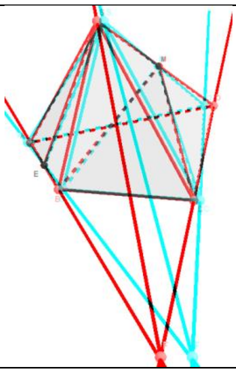
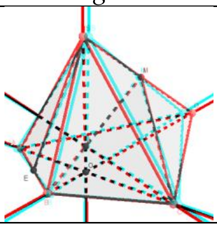
Method	*Drawing*	*Solution*
First method		Extend AD and BC. Suppose that F is the intersection point of AD and BC. Plane (MBC) is now represented by triangle MFC.
Second method		Extend AB and CD. Suppose that F is the intersection point of AB and CD. Triangle SAF now represents plan (SAE).

Table 2 continued

Method	*Drawing*	*Solution*
Third method		Suppose that O is the intersection point of AC and BD, I is the intersection point of SO and MB, and J is the intersection point of CI and SA. Quadrilateral MCBJ now represents plan (MBC).

These extensions allow the appearance of new intersection points. Thus, students can indicate the second point that is common to the two planes. We collected their answers when students completed the problem.

3.2.4. Phase 4

In this final phase, the study examined the success rate of students in drawing the intersection of two planes. It should be noted that these were students who had failed the task "Find the intersection of two given plans" in the previous phase 1. Therefore, this rate would reflect the need for an anaglyph addition in teaching the relative positions of two straight lines in space.

4. Findings

4.1. Phase 1

Table 3 classifies the solutions of 126 students. Accordingly, the ratio of students who correctly completed the problem was very low (3/126), and about half used strategy S1.

Table 3

Classification of 126 participants' solutions

Students' solution	*12B5*	*12A1*	*12A15*	*Amount*
S1: Extending the straight lines on the drawing	22	21	20	63
S2: Finding two coplanar straight lines	2	1	0	3
Other solutions				
Intersection line is SB	6	3	6	15
Intersection line is ME	2	1	1	4
Tracing parallel lines	3	1	2	6
Incomplete	8	10	12	30
Empty	0	4	1	5
Total	43	41	42	126

To find the intersection of two planes (SAE) and (MBC), it is necessary to determine their two common points. The first can be found easily (point B), but the second is more complicated. We extract some students' solutions based on strategy S1 in Table 4.

Table 4

Some of the solutions for the strategy S1 of students

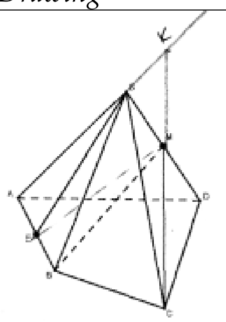
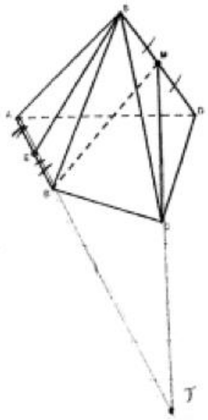
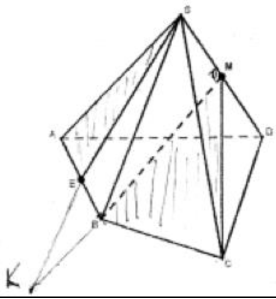
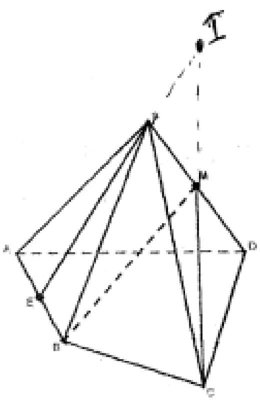
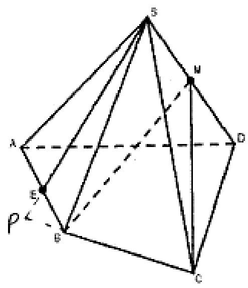
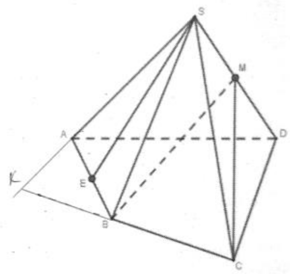
Student	*Drawing*	*Solution (translated into English)*
A15-04		We have: $\begin{cases} K \in SA \\ K \in CM \end{cases}$ $\Rightarrow K$ is a common point

Table 4 continued

Student	*Drawing*	*Solution (translated into English)*
A15-14		$AE \in (SAE)$ $MC \in (MBC)$ and AE and MC intersect in J $\Rightarrow J$ is the first common point $MD \subset (MBC$ MB cut AE in B We have $MB \in (MBC)$ $AE \in (SAE)$ $\Rightarrow B$ is the second common point $\Rightarrow (SAE)$ Intersection line is JB
B5-12		$(SAE) \cap (MBC) = ?$ We have $SE \cap MB = K$ $AB \cap BC = B$ and $AE \in AB$ $\Rightarrow AE \cap BC = B$ $\Rightarrow (SAE) \cap (MBC) = BK$
B5-14		$(SAE) = (SAB)$ $(SAB) \cap (MBC) = B$ Extend SB and MC, which intersect in I $\Rightarrow$ Int. is IB
A1-12		$\begin{cases}(SAE) \cap (MBC) = B \\ SE \cap BC = P\end{cases}$ $\Rightarrow (SAE) \cap (MBC) = BP$
A1-15		$AE \cap BC = B$ $AE \cap MC = K$ $\Rightarrow (SAB) \cap (MBC) = BK$

The student solutions shown in Table 4 demonstrate that the students attempted to exploit the available straight lines (in the name) on the planes, such as SA, SE, AE in the plane (SAE) and MB, MC, and BC in the plane (MBC). Students extended the lines, and if they intersected on the drawing, it allowed a "common point" to be determined. Experimental data shows the students' typical thinking pattern: intersecting on the drawing so intersecting in space. In other words, students do not consider the coplanar condition when considering two straight lines that intersect. Nevertheless, "coplanar" is a necessary property, allowing the review of the third dimension of space.

On the other hand, it is difficult for students to visualize the third dimension of space in a 2D drawing. Therefore, the pattern mentioned above may originate from the limited use of 3D representation models (skewing position of two straight lines). The question is in the classroom: Which other means do the teachers use to support the representation of 3D objects in general, and two skewing lines in particular, next to/in the paper-pencil environment?

4.2. Phase 2

For Phase 1, we contrast the students' answers on the relative positions of ten couples of straight lines in the second experiment with their erroneous answers from the first experiment (see Table 5).

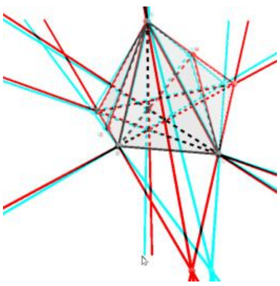
However, students made some mistakes. For example, students A1-03 and A1-31 made four mistakes, and A1-15 and A1-16 made three mistakes. Overall, the errors were clustered around just several questions: no5 (SE and BC): 5/40 students, question no7 (SA and BC): 6/40 students, and question no8 (SA and MB): 5/40 students. Re-examining the anaglyph, we found that brushstrokes red and cyan nearly overlap for the lines SA, SE, BC, and MB. This could reduce the 3D effect of anaglyph; then, students may go back to strategy S1 on classic drawing. For lines SA and MB, their representation was nearly parallel. These mistakes could have been prevented if the student had moved the position of the vertices of the pyramid, making the representation of these lines more explicit.

4.3. Phase 3

In phase 3, five groups of ten (groups 1, 4, 5, 7, 8) were successful in finding two intersecting lines without point B in two planes (SAE) and (MBC). We are interested in the answers of groups 1 and 7, whose 2 of 4 members still gave the wrong answers in phase 1, to consider the effect of teamwork on individual opinions.

Group 1, with four students, A1-10, A1-12, A1-13, and A1-31, answered the question as follows:

> Trace $AC \cap BD = O$. So SO cut BM at H. Join C and H. CH intersect SA at K. CK and SA intersect themself.



This answer corresponds to the third method in Table 2. Notably, student A1-31 has suggested that SA and BC intersect themselves in phase 1, but SA cut CH, not BC, in the group's solution.

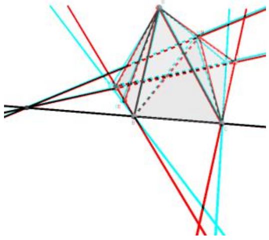
Table 5
Students' opinion on relative positions of two straight lines in the first experiment (with drawing) and the second experiment (with anaglyph)

Student	*Group*	*Phase 1 (with drawing)*										*Phase 2 (with anaglyph)*									
		1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10
A1-01	7		Intersect								Intersect	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-02	5					Intersect						Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-03	10			Intersect								Intersect	Skew	Skew	Intersect	Intersect	Skew	Skew	Skew	Skew	Intersect
A1-04	7					Intersect						Skew	Skew	Skew	Skew	Skew	Skew	Intersect	Parallel	Skew	Skew
A1-05	6					Intersect	Intersect					Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-06	5				Intersect							Skew	Skew	Skew	Intersect	Skew	Skew	Skew	Skew	Skew	Skew
A1-07	9				Intersect							Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-08	3			Intersect								Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-09	6	Intersect						Intersect				Skew	Skew	Skew	Skew	Skew	Skew	Intersect	Intersect	Skew	Skew
A1-10	1	Intersect	Intersect									Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-11	5	Intersect										Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-12	1					Intersect						Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-13	1		Intersect									Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Intersect
A1-14	4		Intersect									Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-15	2							Intersect				Skew	Skew	Skew	Skew	Skew	Skew	Intersect	Parallel	Intersect	Skew
A1-16	3							Intersect				Skew	Skew	Skew	Skew	Intersect	Skew	Skew	Parallel	Skew	Intersect
A1-17	2							Intersect				Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-18	9					Intersect						Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-19	3		Intersect									Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-20	4				Intersect							Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-21	8						Intersect					Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-22	9											Intersect	Skew	Skew	Skew	Skew	Skew	Intersect	Skew	Skew	Skew
A1-23	7										Intersect	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-24	8											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-25	4										Intersect	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-26	4											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-27	7											Skew	Skew	Skew	Skew	Intersect	Skew	Intersect	Skew	Skew	Skew
A1-28	8											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-29	10											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-30	5											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-31	1											Skew	Skew	Intersect	Skew	Intersect	Skew	Intersect	Parallel	Skew	Skew
A1-32	2											Skew	Skew	Parallel	Skew	Intersect	Skew	Skew	Skew	Skew	Skew
A1-33	9											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-34	10											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-35	2							Intersect				Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-37	6								Parallel			Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-38	8											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-39	10											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-40	3											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew
A1-41	6											Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew	Skew

Note: ■Two straight lines are skew. ■Two straight lines intersect themselves. ■Two straight lines are parallel. □No information.

Group 7, with four students, A1-01, A1-04, A1-23, and A1-27, answered as follows:

Extend AD and BC cutting at I.
So MI cut SA at K



This answer corresponds to the first method in Table 2. If, in phase 1, student A1-04 thought that SE and BC, SA, and BC were intersecting, now, in the group's solution, he has replaced SE and SA with AD.

This suggests that students could identify and eliminate mistakes by working in groups. Anaglyph created an environment where the students could confirm their remarks on the relative positions of two straight lines in space.

Two groups out of ten chose two skew lines: for group 2, they were SA and MC, and for group 10, SA and BC. These lines appear in the name of two planes. Also, students from these two groups did not create other lines on the drawing. Thus, these students limited themselves to a set of lines available in the name or on the representation of the planes (triangles, in this case).

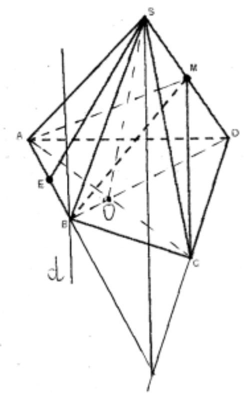
Although we had removed the function from GeoGebra that generates the parallel or perpendicular lines, three groups of ten proposed new lines based on these relations. Group 6 wanted to create the parallel line to MC, and for groups 3 and 9, the perpendicular lines to SA. They thought that these lines belonged to the plane (MBC). This is a residual result of previously working in two dimensions on parallel and perpendicular relations in space in 11th grade.

4.4. Phase 4

For Phase 4, analysis of students' answers for the problem of drawing the intersection of two planes (SAE) and (MBC) after working by group shows that 12 students of 20 in successful groups (1, 4, 5, 7, 8), correctly solved the problem.

For the failed groups (2, 3, 6, 9, 10), four students of 20 had the incorrect answer, and six of 20 submitted no response. At least in this instance, anaglyph did not assist them in solving the problem. Only one student, A1-35, who answered all the questions in Table 2 correctly, had the correct answer. Nevertheless, nine students of 20 could estimate the direction of the section, for instance, student A1-05's drawing in Figure 8.

Figure 8
Student A1-05's drawing



5. Discussion

Our research was propelled by how anaglyph technology could aid Vietnamese students in surmounting the educational challenges associated with learning the relative positions of two straight lines. This study was structured around a didactical situation designed to address this query effectively.

In contrasting our study with existing research on enhancing spatial visualization, it is evident that various approaches and tools have been explored, ranging from intensive courses specifically designed to bolster spatial visualization skills to the incorporation of spatial thinking within STEM education (Rodriguez & Rodriguez-Velazquez, 2020). While traditional methods, such as reliance on 2D drawings and physical models, have provided foundational support, the advent of modern technologies, including computer-aided design software (Hemmerling & Lemberski, 2008; Mjenda et al., 2023; Park et al., 2011; Yue, 2008) and immersive experiences through Virtual Reality (Kwon & Kim, 2002), Augmented Reality (Eh Phon et al., 2019), offer profound advancements in visualizing complex spatial relationships. Our study diverges notably from these methods by integrating the anaglyph feature of GeoGebra software, specifically targeting the educational context of Vietnam. In this country, low financial support significantly impacts educational resources. This approach underscores the feasibility and effectiveness of employing advanced digital technologies in contexts that might not afford the latest in Virtual Reality, Augmented Reality, and Mixed Reality technologies. Notably, our study validates using digital anaglyph features to improve spatial visualization and align with institutional constraints within a traditional paper-pencil environment. Indeed, differentiating the representation of 3D objects on the screen through dynamic geometry software—by rotation and change of point of view—has advantages. However, it fundamentally contrasts with anaglyph representation, where objects do not move. This distinction is crucial. Anaglyph technology offers a unique approach that closely mimics the static drawing of 3D objects. Nevertheless, it transcends the limitations of traditional drawings by providing a depth dimension that static images cannot.

The use of anaglyph technology in GeoGebra for visualizing these spatial relationships aligns with the pedagogical goals of enhancing visualization skills in a way that is accessible and feasible under the constraints of limited educational resources (Jančařík, 2016). By providing a low-cost, high-impact tool, educators in countries like Vietnam can effectively introduce complex concepts of geometry and spatial visualization without the need for expensive hardware or software. This approach democratizes access to advanced educational technologies, ensuring that students in resource-constrained settings are not left behind in the digital leap forward. The static anaglyph images bridge traditional paper-pencil methods and modern digital tools, offering a balanced solution that leverages the strengths of both environments.

This study ventures into relatively uncharted territory by focusing on the anaglyph feature of GeoGebra, a tool that has yet to be addressed in existing literature. Despite the widespread acknowledgment of GeoGebra as a powerful instrument for teaching and learning mathematics, the specific potential of its anaglyph capabilities to enhance spatial visualization has not been extensively explored (Méxas et al., 2015). By spotlighting this feature, our research contributes to a deeper understanding of how specific technological functionalities can be leveraged to overcome educational challenges, particularly in spatial geometry. This exploration broadens the scope of technology-enhanced learning research and underscores the need for continued innovation and experimentation with digital tools in educational settings.

While our study marks a significant step forward in understanding the impact of digital tools on spatial visualization, it also highlights areas for further investigation and acknowledges certain limitations. Primarily, the qualitative nature of this research and its reliance on a relatively small sample size underscores the necessity for subsequent quantitative studies. Future research could aim to quantitatively measure the extent of improvement in spatial visualization attributable to the use of the anaglyph feature in GeoGebra within geometry teaching. Such studies could employ larger sample sizes and diverse educational contexts to provide more generalized evidence of the effectiveness of this approach.

Additionally, integrating anaglyph technology in educational settings raises essential considerations regarding teacher education. There is a pressing need to incorporate training on using anaglyphs into pre-service and in-service teacher education programs. This training would familiarize educators with the technological aspects and address pedagogical strategies for effectively incorporating these tools into teaching practices. Understanding and mitigating potential barriers and challenges to adopting anaglyph technology in schools is crucial. On the other hand, teachers' acceptance of new technologies is influenced by various factors, including perceived usefulness, ease of use, and the availability of resources and support (Mazman Akar, 2019; Mirzajani et al., 2016). Future research should explore these dimensions to identify and address the obstacles educators face in implementing anaglyph technology, facilitating its broader acceptance and utilization in educational settings.

5. Conclusion

The investigation into the use of anaglyph technology within the GeoGebra software has illuminated its potential to enhance spatial visualization capabilities in students, a cornerstone skill in the study of spatial geometry and broader STEM fields. By embracing this tool, we can engage students more effectively, and ultimately elevate the quality of education in spatial geometry and beyond. Integrating anaglyph technology into schools represents a forward-looking step in preparing students for the complex spatial challenges of the future, aligning education with the rapid advancements in digital and technological landscapes.

Author contributions: All the authors contributed significantly to the conceptualization, analysis, and writing of this paper.

Declaration of interest: The authors declare that no competing interests exist.

Data availability: Data generated or analyzed during this study are available from the authors on request.

Ethical declaration: All participants provided informed consent prior to their involvement in the study. They were informed about the study's purpose, procedures, and their right to withdraw at any time without consequence.

Funding: This research is funded by Ho Chi Minh City University of Education Foundation for Science and Technology under grant number CS.2017.19.05.

References

Accascina, G., & Rogora, E. (2006). Using Cabri3D diagrams for teaching geometry. *International Journal for Technology in Mathematics Education, 13*(1), 11–22.

Assude, T., Capponi, B., Bertomeu, P., & Bonnet, J.-F. (1996). De l'économie et de l'écologie du travail avec le logiciel Cabri-Géomètre [The economics and ecology of work with Cabri-Géomètre software]. *Petit x, 44*, 53–79.

Atit, K., Power, J. R., Pigott, T., Lee, J., Geer, E. A., Uttal, D. H., Ganley, C. M., & Sorby, S. A. (2022). Examining the relations between spatial skills and mathematical performance: A meta-analysis. *Psychonomic Bulletin and Review, 29*(3), 699–720. https://doi.org/10.3758/s13423-021-02012-w

Baki, A., Kosa, T., & Guven, B. (2011). A comparative study of the effects of using dynamic geometry software and physical manipulatives on the spatial visualisation skills of pre-service mathematics teachers. *British Journal of Educational Technology, 42*(2), 291–310. https://doi.org/10.1111/j.1467-8535.2009.01012.x

Bakó, M. (2003). Different projecting methods in teaching spatial geometry. In M. A. Mariotti (Eds.), *Proceedings of the Third Conference of the European Society for Research in Mathematics Education* (pp. 1-9). University of Pisa and ERME.

Berciano, A., & Gutiérrez, G. (2015). How to improve spatial visualization ability of preservice teachers of childhood education: A teaching experiment. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 2755–2761). Charles University in

Prague.

Chaachoua, H., & Comiti, C. (2010). L'analyse du rôle des manuels dans l'approche anthropologique [Analysis of the role of textbooks in the anthropological approach]. In A. Bronner, M. Larguier, M. Artaud, M. Bosch, Y. Chevallard, G. Cirade & C. Ladage (Eds.), *Diffuser les Mathématiques (et les Autres Savoirs) Comme Outils de Connaissance et d'Action* [Disseminating Mathematics (and Other Knowledge) as a Tool for Knowledge and Action] (pp. 771–789). IUFM de l'Académie de Montpellier.

Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development, 15*(1), 2–11. https://doi.org/10.1080/15248372.2012.725186

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: perspectives apportées par une approche anthropologique [Fundamental concepts of didactics: perspectives from an anthropological approach]. *Recherche en Didactique des Mathématiques, 12*(1), 73–112.

Christou, C., Jones, K., Mousoulides, N., & Pittalis, M. (2006). Developing the 3Dmath dynamic geometry software: theoretical perspectives on design. *International Journal for Technology in Mathematics Education, 13*(4), 168–174. https://eprints.soton.ac.uk/42114/

Clements, D. H., & Battista, M. T. (1992). Geometry and Spatial Reasoning. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 420–464). Macmillan.

Eh Phon, D. N., Abdul Rahman, M. H., Utama, N. I., Bilal Ali, M., Dayana Abd Halim, N., & Kasim, S. (2019). The effect of augmented reality on spatial visualization ability of elementary school student. *International Journal on Advanced Science Engineering Information Technology, 9*(2), 624–629. https://doi.org/10.18517/ijaseit.8.5.4971

Ferdiánová, V. (2017). GeoGebra materials for LMS Moodle focused Monge on projection. *Electronic Journal of e-Learning, 15*(3), 259–268. https://files.eric.ed.gov/fulltext/EJ1146049.pdf

Gal, H., & Linchevski, L. (2010). To see or not to see: analyzing difficulties in geometry from the perspective of visual perception. *Educational Studies in Mathematics, 74*(2), 163–183. https://doi.org/10.1007/s10649-010-9232-y

Gilligan, K. A., Flouri, E., & Farran, E. K. (2017). The contribution of spatial ability to mathematics achievement in middle childhood. *Journal of Experimental Child Psychology, 163*, 107–125. https://doi.org/10.1016/j.jecp.2017.04.016

Guedes, K. B., Guimarães, M. de S., & Méxas, J. G. (2012). Virtual reality using stereoscopic vision for teaching/learning of descriptive geometry. In J. Valerdi, B. Kramer & S. White (Eds.), *Proceeding of The Fourth International Conference on Mobile, Hybrid, and On-Line Learning* (pp. 24–30). ThinkMind/IARIA.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th International Conference for the Psychology of Mathematics Education*, *vol.1* (pp. 3–19). Universitat de Valencia.

Hemmerling, M., & Lemberski, D. (2008). Anaglyph representation as medium for spatial design. In M. Muyelle (Ed.), *Architecture "in Computro": Integrating Methods and Techniques* (pp. 209–214). The Higher Institute of Architectural Sciences.

Jančařík, A. (2016). Dynamic models using 3D projection. In J. Novotná & A. Jančařík (Eds.), *Proceedings of the 15th European Conference on e-Learning* (pp. 296–304). Academic Conferences and Publishing International Limited.

Jones, K. (2001). Spatial thinking and visualisation. In B. Barton (Ed.), *Teaching and Learning Geometry 11-19* (pp. 55–56). Royal Society.

Kmetova, M. (2015). Rediscovered anaglyph in program GeoGebra. *Acta Mathematica Nitriensia, 1*(1), 86–91. https://doi.org/10.17846/amn.2015.1.1.86-91

Kösa, T. (2016). The effect of using dynamic mathematics software: Cross section and visualization. *International Journal for Technology in Mathematics Education, 23*(4), 121–128. https://doi.org/10.1564/tme_v23.4.01

Kwon, O. N., & Kim, S. H. (2002). Enhancing spatial visualization through virtual reality (VR) on the web: Software design and impact analysis. *Journal of Computers in Mathematics and Science Teaching, 21*(1), 17–31. https://www.learntechlib.org/p/10769/

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: A multidimensional study of the evolution of research and innovation. In A. Bishop, M. K. Clements, C. Keitel, J. Kilpatrick & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 237–269). Springer. https://doi.org/10.1007/978-94-010-0273-8_9

Leopold, C. (2015). Visual representations for spatial thinking. In L. Cocchiarella (Ed.), *The visual language of technique* (Vol. 3, pp. 37–56). Springer. https://doi.org/10.1007/978-3-319-05326-4_5

Lieban, D. (2019). *Exploring opportunities for connecting physical and digital resources for mathematics teaching and learning* [Doctoral dissertation]. Johannes Kepler University Linz, Linz.

Lowrie, T., Logan, T., & Hegarty, M. (2019). The influence of spatial visualization training on students' spatial reasoning and mathematics performance. *Journal of Cognition and Development, 20*(5), 729–751. https://doi.org/10.1080/15248372.2019.1653298

Lyon, D. R., Gunzelmann, G., & Krusmark, M. (2019). Path visualization: a method for objective measurement of spatial visualization. *Spatial Cognition and Computation, 19*(4), 309–333. https://doi.org/10.1080/13875868.2019.1643867

Mazman Akar, S. G. (2019). Does it matter being innovative: Teachers' technology acceptance. *Education and Information Technologies, 24*(6), 3415–3432. https://doi.org/10.1007/s10639-019-09933-z

Méxas, J. G. F., Guedes, K. B., & Tavares, R. da S. (2015). Stereo orthogonal axonometric perspective for the teaching of descriptive geometry. *Interactive Technology and Smart Education, 12*(3), 222–240. https://doi.org/10.1108/itse-09-2014-0027

Mirzajani, H., Mahmud, R., Fauzi Mohd Ayub, A., & Wong, S. L. (2016). Teachers' acceptance of ICT and its integration in the classroom. *Quality Assurance in Education, 24*(1), 26–40. https://doi.org/10.1108/qae-06-2014-0025

Mithalal, J., & Balacheff, N. (2019). The instrumental deconstruction as a link between drawing and geometrical figure. *Educational Studies in Mathematics, 100*(2), 161–176. https://doi.org/10.1007/s10649-018-9862-z

Mjenda, M., Mutarutinya, V., & Owiti, D. (2023). Assessing the effectiveness of computer-aided instructional techniques in enhancing students' 3D geometry spatial visualization skills among secondary school students in Tanzania. *International Journal of Learning, Teaching and Educational Research, 22*(6), 613–637. https://doi.org/10.26803/ijlter.22.6.31

Newcombe, N. S., & Shipley, T. F. (2015). Thinking about spatial thinking: New typology, new assessments. In J. Gero (Ed.) *Studying Visual and Spatial Reasoning for Design Creativity* (pp. 179–192). Springer. https://doi.org/10.1007/978-94-017-9297-4_10

Park, J., Kim, D. E., & Sohn, M. H. (2011). 3D simulation technology as an effective instructional tool for enhancing spatial visualization skills in apparel design. *International Journal of Technology and Design Education, 21*(4), 505–517. https://doi.org/10.1007/s10798-010-9127-3

Prieto, G., & Velasco, A. D. (2010). Does spatial visualization ability improve after studying technical drawing? *Quality and Quantity, 44*(5), 1015–1024. https://doi.org/10.1007/s11135-009-9252-9

Rodriguez, J., & Rodriguez-Velazquez, L. G. (2020). Predictive model for improvement of spatial visualization skills. In M. Auer, H. Hortsch, & P. Sethakul (Eds.), *The Impact of the 4th Industrial Revolution on Engineering Education* (Vol. 2, pp. 181–187). Springer. https://doi.org/10.1007/978-3-030-40271-6_19

Seah, R. (2015). Developing geometric reasoning abilities through visualisation. In J. Vincent, G. FitzSimons, & J. Steinle (Eds.), *Maths rocks* (pp. 136–143). Mathematical Asssociation of Victoria.

Susilawti, W., Suryadi, D., & Dahlan, J. A. (2017). The improvement of mathematical spatial visualization ability of student through cognitive conflict. *International Electronic Journal of Mathematics Education, 12*(2), 155–166. https://doi.org/10.29333/iejme/607

Tang, M. D. (2014). *Une étude didactique des praxéologies de la representation en perspective dans la géométrie de l'espace, en France et au Viêt-Nam* [A didactic study of the praxeologies of perspective representation in space geometry, in France and Vietnam] [Doctoral dissertation]. Université de Grenoble Alpes, Grenoble.

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352–402. https://doi.org/10.1037/a0028446

Weckbacher, L. M. (2007). *The role of visualization in geometric problem solving* (Publication No. 3285847) [Doctoral dissertation, University of California]. ProQuest Dissertations and Theses Global.

Yue, J. (2008). Spatial visualization by realistic 3D views. *Engineering Design Graphics Journal, 72*(1), 28–38. https://doi.org/10.18260/1-2--1619

Yurmalia, D., & Herman, T. (2021). Student visualization in solving geometry problem: The case of reflection. *Journal of Physics: Conference Series, 1806*(1), 012052. https://doi.org/10.1088/1742-6596/1806/1/012052