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Research Article

Understanding through explanation in AI-supported instructional design: Preservice mathematics teachers' pedagogical transformations

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This qualitative research investigates how preservice mathematics teachers structure their instructional design processes with artificial intelligence [AI] by establishing a connection between mathematical understanding and explanation. This study, which views explanation as both a knowing and a teaching action that supports understanding, investigates how AI-powered environments affect preservice mathematics teachers' transitions in explanation. Using a qualitative multiple-case design, data were collected from twelve preservice mathematics teachers through lesson plans, AI prompt logs, micro-teaching videos (with and without AI support), and pre- and post-interviews. Thematic analysis revealed five interrelated themes: pedagogical transformation through AI support; shifts in explanatory strategies; transformation of mathematical understanding; reflective and epistemological awareness; and constraints and tensions in AI integration. The findings suggest that AI functions not merely as a technical assistant providing ready-made content, but as a dialogical partner prompting a rethinking of how explanations are constructed, justified, and adapted. AI-powered instructional designs enhanced relational understanding by promoting visualization, contextualization, and multiple forms of representation. By demonstrating how this process and understanding can support epistemological growth, it may contribute to research on AI in teacher education. Also, based on the findings, some suggestions for further research and significant implications for teacher education are provided.

Keywords: Artificial intelligence-assisted teaching; Mathematical explanation and understanding; Preservice mathematics teacher education

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1. Introduction

AI has rapidly become a significant force in education, with growing implications for how teaching and learning are organized and supported (Holmes et al., 2022; Luckin, 2018). Within teacher education, recent research has increasingly begun to examine how preservice teachers understand and use AI in relation to pedagogical planning, professional learning, and classroom practice, including in mathematics-specific contexts. Earlier research on AI in education was more heavily concentrated on areas such as profiling and prediction, assessment and evaluation,

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adaptive systems, and intelligent tutoring systems (Zawacki-Richter et al., 2019). More recent work, however, has begun to examine teacher-facing uses of generative AI. This emerging line of research leaves an important question concerning how interaction with AI may mediate teachers' own pedagogical thinking, particularly how they interpret, justify, and explain mathematical ideas as they move from lesson planning to classroom enactment. Examining this relationship is especially relevant in mathematics education, where teaching depends on specialized mathematical knowledge and on creating learning environments that support disciplinary thinking, sense making, and robust understanding (Ball et al., 2008; Schoenfeld, 2018).

In mathematics education, explaining is not merely a communicative act but a cognitive process that reveals how teachers construct, justify, and convey mathematical meaning (Bergqvist, 2007; Stylianides et al., 2024). Teachers' explanations are intertwined with their epistemological views about mathematics—whether they perceive mathematics as a fixed body of truths or as a dynamic system of relationships (Ernest, 1991). Consequently, any shift in teachers' explanatory strategies can indicate a deeper transformation in how they understand and conceptualize mathematics itself. As AI systems such as ChatGPT, GeoGebra AI, or Mathigon increasingly assist teachers in generating examples, representations, or analogies, questions arise about how these tools reshape teachers' explanatory practices, their sense of mathematical meaning, and their reflective awareness during the instructional design process.

Recent studies have begun to examine both the pedagogical affordances and the limitations of AI in teacher education. Research with preservice teachers suggests that AI-related readiness and use are associated with lesson-design practices as well as self-awareness and self-regulatory processes relevant to professional learning (Asare & Boateng, 2025; Tusyanah et al., 2026). Qualitative evidence further indicates that AI-supported practicum experiences can contribute to reflective and metacognitive decision-making, instructional justification, and responsiveness to students' needs, although concerns regarding the accuracy, opacity, and appropriate pedagogical use of generative AI remain salient (Acuyo Cespedes, 2025). Thus, emerging research is beginning to move beyond viewing AI merely in terms of the instructional products it generates. Nevertheless, considerably less is known about how teachers' mathematical explanations themselves evolve through sustained and iterative interaction with AI—how alternative representations and lines of reasoning are evaluated, accepted, resisted, or revised, and how these emerging understandings are reconciled with prior experiences and pedagogical beliefs. This issue is particularly important because teachers enter AI-mediated practice with existing conceptions of AI, including misconceptions that may be associated with how they understand and intend to integrate the technology into their teaching (Antonenko & Abramowitz, 2023). A more fine-grained account is therefore needed of how these prior conceptions interact with AI-mediated reasoning over time and whether such interactions contribute to epistemic shifts in teachers' explanations and instructional judgments.

The present study addresses this gap by examining how preservice mathematics teachers [PMTs] design and explain mathematical concepts—specifically, the Pythagorean theorem and the summation formula from 1 to n—before and after receiving AI support. These two topics were chosen because they embody both procedural and conceptual dimensions of mathematical understanding. The Pythagorean theorem involves geometric reasoning and proof, whereas the summation formula engages numerical pattern generalization and symbolic representation. The reason for choosing these two topics is to analyze how PMTs' explanations transformed across these two mathematical domains and capture the nuances of AI's role in shaping both visual–spatial and algebraic reasoning. The data, drawn from lesson videos, AI-supported redesigns, and post-lesson interviews, reveal how teachers' explanatory discourse, use of representations, and reflective justifications evolved through interaction with AI.

This research is theoretically grounded in sociocultural and dialogic perspectives on teacher learning (Mercer, 2010; Vygotsky, 1978). From this viewpoint, teaching knowledge is not merely transmitted but co-constructed through mediated action and reflective dialogue. Here, AI is conceptualized as a cognitive partner—a semiotic tool that participates in teachers' meaning-making processes (Holmes et al., 2022; Pea, 1993). By engaging in conversational and design-based interactions with AI, teachers externalize their reasoning, negotiate meanings, and potentially reconstruct their epistemological stances toward mathematics. In this sense, AI does not replace pedagogical expertise but instead provides a new form of scaffolding that prompts teachers to articulate and refine their understanding.

Despite these promising potentials, integrating AI into teacher education also raises tensions and constraints (Crompton & Burke, 2024). Moreover, as teachers navigate between their professional judgment and AI suggestions, moments of dissonance or uncertainty may arise—challenging them to make epistemic decisions about what counts as a valid mathematical explanation. Investigating such tensions is crucial for understanding how teachers maintain agency and critical awareness in AI-supported contexts.

In light of these considerations, this research aims to explore how PMTs transform their explanations and understanding through AI mediation. Specifically, it seeks to uncover how AI tools shape teachers' conceptualizations of mathematical meaning, their design of explanatory discourse, and their reflections on their professional learning processes. By focusing on both the process and substance of these transformations, the study contributes to emerging discussions on AI-mediated teacher learning and epistemic change in mathematics education. The following research questions guide the study:

RQ 1) How do PMTs transform their explanatory practices when supported by AI tools in lesson design and implementation?

RQ 2) In what ways does AI mediation influence teachers' mathematical understanding and epistemic positioning during instructional design?

RQ 3) How do PMTs reflect on the role of AI in their professional learning and instructional reasoning processes?

Through addressing these questions, this research seeks to illuminate the transformative and dialogic potential of AI in shaping PMTs' pedagogical and epistemological development, offering insights into how AI can serve as a reflective companion in teacher education rather than a prescriptive tool.

2. Theoretical Framework

This study is based on an integrative theoretical framework that links three core constructs: mathematical explanation, mathematical understanding, and AI-enhanced teacher learning. It explores how PMTs restructure their pedagogical reasoning and epistemic orientations as they design and implement an AI-mediated course.

2.1. Mathematical Explanation as an Epistemic and Pedagogical Act

The term explanation is often self-evident and requires no further definition. However, the explanation is ambiguous and encompasses a range of meanings, from stating a concept to specifying a method of use. In essence, mathematical explanation is an epistemic activity that transforms procedural knowledge into conceptual understanding (Hanna, 1990; Hersh, 1993). It involves not only communicating results, but also articulating why a mathematical relationship holds. It is a statement that a mathematical concept, such as a theorem, diagram, or proof, explains a phenomenon (D'Alessandro, 2019).

Mathematicians and philosophers of mathematics consider explanations in mathematics. Some of the most prominent mathematical explanations are those of Steiner (1978), Kitcher (1981), and Lange (2014). Steiner emphasizes the characteristic features, Kitcher's unification, and Lange's exploitation of a distinct feature of the result. Furthermore, Gowers (2000) writes: "Other branches of mathematics derive their appeal from an abundance of mysterious phenomena that demand explanation." (p. 10). According to Wilkenfeld (2014), the operational definition of explanation is: An explanation is something that, under the right conditions and in the right way, leads to understanding. In mathematics education, explanation is both a tool for understanding and a teaching tool (Stylianides & Stylianides, 2009). When teachers explain mathematics, they articulate their conceptual structures and epistemological beliefs, revealing how they understand mathematical reality (Ernest, 1991).

While philosophical explanations focus on why a mathematical truth is valid, classroom explanations address pedagogical functions such as meaning-making, interaction, and epistemological positioning. Explanations can operate in multiple representational modes, including verbal, visual, symbolic, and dynamic (Duval, 2006). For example, when a teacher models the Pythagorean theorem through domain decomposition or number addition through pattern recognition, each explanatory action reveals an interaction between representation and reasoning. These multifaceted explanations are pedagogically important because they bridge the gap between the abstract, rigid structure of mathematics and the human mind in learning (Kaput, 1998).

In this study, mathematical explanation is conceptualized as a meaning-making process that unfolds through design, teaching, and thinking. The analysis comparing teachers' pre- and post-AI lessons reveals how AI engagement drives shifts in explanatory structure—from procedural expression to conceptual, inquiry-based, and explanatory reasoning.

2.2. Mathematical Understanding: From Instrumental to Relational Knowing

Understanding is one of the highest cognitive achievements humans can achieve, and it is a complex matter (Kelp, 2016). According to Kvanvig (2003), internal cognition is what makes understanding clear and distinctive. Crucial to understanding is the internal recognition or recognition of explanatory and other coherent relationships within a body of knowledge. Elgin (2009) considers understanding as a form of knowledge. Generally, he argues that understanding is a comprehensive body of shared knowledge that is grounded, responds appropriately to evidence, and enables non-trivial reasoning, argumentation, and perhaps action on the subject.

It is important to note that insights into why mathematical knowledge functions, rather than how to achieve it, will be grounded in the relationships among concepts. Therefore, in mathematics education, understanding was classically divided by Skemp (1976) into instrumental and relational understanding. This distinction remains fundamental for examining teachers' cognitive and pedagogical orientations. Relational understanding allows for flexible restructuring of knowledge and deeper transfer of concepts. However, because of the nature of mathematics, procedures at one level can be automated, enabling independent thinking at higher levels.

D'Alessandro and Lehet (2024) advocated for an understanding-based view of explanation in mathematics and offered a noetic account of it. According to this view, a piece of mathematics is explanatory if and only if it produces the appropriate type of understanding. According to this perspective, explanatory understanding emerges when certain explanatory goals are achieved. Inglis and Mejía Ramos (2021) argued that explanatory understanding is closely linked to the existence of well-structured cognitive schemas. This view posits that the defining feature of an explanation is its capacity to facilitate understanding. From a constructivist perspective, mathematical understanding involves coordinating conceptual schemas through reflection and explanation. Teachers who cultivate understanding emphasize reasoning, justification, and pattern recognition rather than purely procedural fluency. Furthermore, understanding is inherently social: it is developed, expressed, and negotiated through communicative acts such as explanation, questioning, and analogy (Sfard, 2008).

In the context of this study, understanding is not considered a static entity, but rather a dynamic epistemic stance that evolves as teachers interact with AI-generated cues. The data demonstrate that AI's dialogic feedback (through suggestions, analogies, or visual examples) encourages teachers to reframe problems, identify hidden structures, and recognize alternative explanations. Therefore, the framework interprets AI interaction as supporting teachers' transition from instrumental to relational forms of understanding.

2.3. AI-Supported Teacher Learning: Mediation and Co-Construction

Recent research has positioned AI as a mediating partner in teacher learning (Luckin, 2018; Zawacki-Richter et al., 2019). Instead of functioning as content providers, AI systems operate as dialogic agents that co-construct understanding through feedback loops and knowledge reorganization (Holmes et al., 2022). This theoretical orientation is consistent with Vygotsky's (1978) concept of mediation and extends human cognitive capacity. In this context, AI cues function as semiotic intermediaries (Wertsch, 1998), enabling teachers to externalize implicit reasoning and reorganize it through interaction. Wegerif's (2021) theory of dialogic education, which conceptualizes learning as a dynamic engagement in a dialogue with diverse perspectives, also supports this process. From this standpoint, AI has the potential to expand teachers' dialogical space and encourage reflection, reasoning, and epistemic negotiation.

From a co-constructivist perspective, AI-assisted teacher learning can be conceptualized not merely as the use of tools, but as a dialogical process in which pedagogical meaning is co-shaped through interaction (Guberina & Procházka, 2026; Katsenou et al., 2025; Kim, 2024). Instead of functioning as an external content provider, AI can serve as a dialogical partner, encouraging teachers to articulate, justify, and refine their instructional reasoning (Magne et al., 2025; Tang & Putra, 2026; Wan & Gu, 2025). In this sense, mediation extends beyond scaffolding to an epistemological co-construction in which ideas are iteratively negotiated between humans and machines. Teachers do not passively adopt the explanations, representations, or lesson structures generated by AI while critically evaluating them; instead, they participate in the process of co-authoring pedagogical meaning (Agarwal & Bhatnagar, 2026; Dadhich et al., 2025; Kasneci et al., 2023; Kim, 2024; Kim et al., 2022). This form of dialogical mediation is compatible with sociocultural accounts of learning as socially mediated through interaction and cultural tools (Vygotsky, 1978), while emerging human–AI collaborative pedagogies similarly emphasize feedback, reflection, iterative refinement, and human mediation rather than technological replacement (Okanya & Asogwa, 2026). AI can therefore be conceptualized as a dialogical and cognitive resource that may extend teachers’ professional reasoning without displacing their pedagogical agency.

2.4. The Interplay between Explanation, Understanding, and AI Mediation

The three theoretical perspectives outlined previously – mathematical explanation, mathematical understanding, and AI-enhanced teacher learning – are synthesized in the concept of AI-mediated explanatory development. This framework provides an analytical lens for the current study. While traditional mathematics teaching models rely heavily on teachers' existing conceptual schemes, AI offers an external, dialogic layer of mediation through which teachers develop their mathematical ideas (Li et al., 2025; Matteucci et al., 2024). From this perspective, explanations and understanding are not static entities, but dynamic, co-constructed processes that emerge through interaction with semiotic tools, including AI systems (Fleisher, 2022; Robbemond et al., 2022; Wang & Yin, 2023).

In traditional lesson design, teachers tend to rely on fixed cognitive resources, such as predetermined stored procedural plans, memory-based metaphors, or familiar representations, to structure their explanations (Bergqvist, 2010). These explanations often emerge from teachers' internalized mathematical meanings and pedagogical routines derived from their experiences. While internal resources are important, they can also constrain the range of explanations teachers can offer (Schoenfeld, 2018). Conversely, AI-mediated environments have been shown to encourage teachers to externalize their reasoning processes, encounter alternative representations, and reevaluate their explanatory decisions. In this sense, AI does not provide ready-made explanations, but rather serves as a provocative interlocutor, a dialogue partner who introduces conceptual variations and encourages reflective elaboration (Holmes et al., 2022; McDermott et al., 2025).

This mediation occurs through various mechanisms. First, AI can generate alternative narrative frameworks, requiring teachers to evaluate the conceptual accuracy and pedagogical appropriateness of different explanatory options (Watson & Shi, 2024). Such evaluation processes have been shown to deepen teachers' relational understanding by exposing them to content that they could not have produced independently (Skemp, 1976). Second, dialogic interaction with AI often elicits teachers' meta-explanatory reasoning; teachers explain why a particular representation works, how an evidence structure supports meaning, or which analogy best captures mathematical essence. This form of reflective explanation aligns with research on epistemic cognition, which emphasizes the importance of teachers' awareness of the justifications for and communication of mathematical knowledge (Hofer & Pintrich, 1997). Thirdly, AI-mediated learning restructures the temporal rhythm of explanation production. Instead of designing explanations individually and then testing them in practice, teachers engage in cycles of proposing, critiquing, and refining ideas with immediate feedback. This iterative development resembles design-based learning processes (Cobb et al., 2003; McKenney & Reeves, 2025). It positions AI as a co-constructor of pedagogical reasoning. In the course of these cycles, teachers' understanding evolves in alignment with their explanation strategies. Through the process of reworking explanations, teachers reorganize underlying conceptual connections, and new explanatory forms emerge as their understanding shifts (Dai et al., 2023; Yang, 2025).

Recent debates in the educational technology literature draw a sharp distinction between utilizing AI strictly as an efficiency-enhancing assistant versus engaging it as a co-designer (Kohnke et al., 2023). When AI is framed as an epistemic interlocutor rather than a ready-made content generator, it fosters productive struggle and epistemic engagement (Kafai & Proctor, 2022), creating a space where teachers must critically evaluate the epistemological adequacy of AI-generated explanations. Thus, the interaction between explanation, understanding, and AI-mediated learning is inherently reciprocal. Teachers' evolving understandings and explanations mutually shape and reveal each other. AI-mediated feedback reorganizes both. Teachers respond by establishing coherence between their mathematical structures of meaning and the possibilities AI creates. This AI-mediated interactive process becomes a site of epistemological transformation, where teachers not only learn new ways to explain mathematical ideas but also reconceptualize what it means to understand and teach mathematics in an age of intelligent tools.

3. Method

The present study employed a multiple case study design (Yin, 2018) to investigate how PMTs conceptualize and apply mathematical explanations and understanding with the support of AI. Each participant was a case in point, allowing for an in-depth examination of personal pedagogical and epistemological transformations across two mathematical subjects (the Sum Formula and the Pythagorean Theorem). The design allowed for within-case and across-case comparisons to track changes in participants' explanatory practices before and after integrating AI-based support into lesson planning and instruction. The study adopted iterative cycles of teaching, reflection, and interaction with AI tools to identify how these tools influence teachers' conceptualizations of mathematical explanations and understanding.

3.1. Participants and Context

The study group consisted of twelve PMTs enrolled in the Teaching Practice course. These PMTs were fourth-year students in the Mathematics Education department of a state university, selected through convenience and criterion sampling (Patton, 2014). Participants were selected based on the criteria of being PMTs who had completed mathematics content courses, pedagogy courses, mathematics content education pedagogy courses, and technology-related courses throughout their four-year internship program.

The study was conducted in a microteaching context, in which each PMT was tasked with designing and delivering two brief lessons (each spanning 6–10 minutes) on two distinct mathematics topics: the Pythagorean Theorem and the Addition Formula ($1 + 2 + ... + n$). For each subject, the initial lesson was prepared without AI support, while the subsequent lesson was designed with AI support. The experimental design employed a double-loop structure, enabling comparisons of pedagogical, epistemological, and explanatory changes across each participant's teaching process. Two mathematical topics were specifically selected to investigate the impact of AI support on different forms of mathematical explanation and understanding. In the literature, most mathematical philosophers offer explanations of these subjects (Lehet, 2021; Steiner, 1978). The Pythagorean Theorem is a geometric proof-based concept that relies on spatial reasoning and visual representation. The Addition Formula requires algebraic and pattern-based reasoning to generalize numerical relationships. The integration of these two subjects enabled the study to encompass the spectrum of mathematical thinking, ranging from visual-conceptual explanations to symbolic-analytical reasoning. The investigation further examined how AI tools influenced teachers' explanatory and interpretive practices across these diverse cognitive contexts. The dual-task design also provided a means to triangulate data across conceptual domains, thereby strengthening the validity and interpretive depth of the findings.

3.2. Data Sources and Data Collection

In the study, interviews, lesson plans, lesson videos, and AI prompt logs were examined. The dataset comprised multiple qualitative sources gathered across three phases. The data sources used in the stages of the research are defined in Table 1.

Table 1
Data collection in the research

Phase and Data Source	*Description*
Pre-AI Phase	
Initial interview	Explored PMTs’ conceptions of mathematical explanation, understanding, and views on AI use in education.
First lesson plans (without AI)	Lesson plans for both mathematical topics were prepared individually without digital assistance.
Video recordings of first lessons	6–10 min microteaching sessions capturing PMTs’ natural explanatory approaches.
AI-Supported Phase	
Prompt logs	All AI–teacher interactions during lesson planning (collected verbatim as prompt transcripts).
Second lesson plans (with AI)	AI-supported revisions of lesson plans on the same two topics.
Video recordings of the second lesson	Captured classroom enactments with AI-informed designs.
Post-AI Phase	
Final interviews	Conducted after both teaching cycles to explore perceived changes in understanding, explanation, and AI-related pedagogical reasoning.

The interview questions were developed by the researcher and reviewed by two experts in mathematics education to ensure content validity and consistency with the study's research objectives. The first interview aimed to determine PMTs' perspectives on understanding and explanation, and to explore whether AI could provide these. Questions such as "*What is understanding?*", "*How does understanding occur?*" "*Are there stages of understanding?*" "*What is an explanation?*" "*How would you know if an explanation is effective?*" "*What is the relationship between understanding and explanation?*" and "*Does artificial intelligence contribute to the processes of understanding and explanation?*" were asked. In these interviews, PMTs were given information about the topics they would teach, their lesson plans, and the AI-related processes. The choice of AI was left to the PMTs' discretion. Eleven PMTs used ChatGPT, while one used Copilot. Prior to implementation, an interactive lesson-plan development session was conducted to improve PMTs' ability to use AI effectively. In this session, PMTs were shown how to write prompts, manage AI dialogues, and critically evaluate AI-generated pedagogical outputs. Following the microteaching sessions, final interviews were conducted, marking the end of data collection.

In this study, the AI tool was used not as a passive information-gathering tool, but to provide pedagogical support to PMTs in their instructional design processes. To establish this pedagogical interaction, the dialogue with the AI was entirely left to the PMTs' initiative.

This study was conducted in accordance with the ethical guidelines of the Bartın University Ethics Committee, and formal approval was obtained. All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee. Informed consent was obtained from all individual participants included in the study. PMTs were comprehensively informed about the study's purpose, scope, and their roles, as well as their right to withdraw at any stage without any negative consequences. Their consent was documented through signed written consent forms. To ensure confidentiality, all data—including interview transcripts and lesson video recordings—were anonymized, and no personally identifiable information was used in the analysis or the findings. Anonymized data were stored securely in password-protected digital environments accessible only to the researcher.

3.3. Data Analysis

In case studies, research questions and relevant findings are formulated and reported for each study, and these are correlated with similar findings in the literature (Creswell, 2013). Data were analyzed through a four-phase qualitative analysis, combining open, axial, and selective coding (Strauss & Corbin, 1998) with cross-case thematic synthesis (Braun & Clarke, 2021).

Two authors of the research analyzed the initial interview transcripts, lesson plans, and lecture videos. Then, AI-generated lesson plans, lecture videos, and AI prompt logs were analyzed. An inter-coder coefficient was calculated to assess inter-coder reliability, yielding 0.81, indicating a high level of agreement among coders (Miles & Huberman, 1994). Any inconsistencies in the coding were resolved through discussion and, if necessary, by re-examining the raw data until complete consensus was reached.

Overall, this analytical triangulation, combining interviews, lesson plans, lecture videos, and AI prompt logs, contributed to a comprehensive and reliable understanding of participants' experiences with AI-generated lesson planning and mathematical explanation (Patton, 2014). The process from open source coding to thematic synthesis is outlined in the following subheadings.

3.3.1. Within-case open coding

Each PMT's dataset (interviews, prompt transcripts, lesson plans, and videos) was analyzed separately to identify: Explanatory approaches, forms of understanding, and AI interaction patterns. Codes such as analogical reasoning, AI-suggested visualization, teacher reinterpretation, and shift from rule to concept emerged during this phase.

3.3.2. Axial coding

Table 2 presents some examples of the transition from open coding to axial coding. Axial coding demonstrates systematic relational restructuring by linking categories around conceptual axes. Thus, it shifts from descriptive, decomposed data to abstract thematic mapping.

Table 2
Open and axial codes examples

Open Codes	*Axial Category*
Using visuals to clarify reasoning, AI proposes manipulatives	AI-Supported Representational Scaffolding
The teacher rephrases the AI suggestion to fit the student's level	Adaptive Pedagogical Reasoning
Asking students to justify the pattern	Encouraging Relational Understanding
AI output rejected for being too abstract.	Epistemic Tension

3.3.3. Selective coding and thematic synthesis

Through iterative comparison across cases (PMT1–PMT12), five core themes were identified. While 'Pedagogical Transformation through AI Support' functions as the overall core theme integrating the analytics network, operating distinctly at the macro level, other themes capture specific cognitive and operational changes.

- Pedagogical Transformation through AI Support
- Shifts in Explanatory Strategies
- The Role of AI in Teacher Learning
- Reflective and Epistemological Awareness
- Constraints and Tensions in AI Integration

3.3.4. Narrative construction

Each theme was developed into a within-case narrative (capturing individual transformation) and a cross-case synthesis (identifying collective trends). Video observations were used to triangulate the PMTs' verbal explanations with their enacted representations (e.g., diagrams, digital tools, gestures). Prompt logs provided insight into the meta-cognitive dialogue between teachers and AI, illuminating how the explanation was co-constructed.

4. Findings

To present the findings of the current research in a manner consistent with the dataset's multi-source nature, this section adopts a three-tiered evidence architecture. First, the Participant Opinions Table for each theme includes only participants who have generated relevant evidence in the corresponding data sources. Second, Thematic Interpretation, following the table, elaborates on each theme through an analytical narrative supported by brief, theory-related comments, direct quotations, and—where appropriate—small excerpts from lesson plans, questions, or video transcripts. Third, in the case study, each theme concludes with a detailed story of a focus participant that most clearly illustrates the theme's complexity.

4.1. Theme 1: Pedagogical Transformation through AI Support

This theme analyses the pedagogical transformation experienced by PMTs in their course design and explanations with AI support. The changes in participants' teaching approaches are generally shown in Table 3.

Table 3
Participant views related to pedagogical changes following AI use

Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT2	"AI suggested a real-life example."	Contextual anchoring	Final interview
PMT3	Interactive problem solving is done.	Active learning	Second lesson plan
PMT4	"They saw it more clearly when they overlaid the model."	Manipulative-based visualization	Video recordings of the second lesson
PMT5	"It suggested I use context in the introduction."	Contextual cueing	Second lesson plan
PMT10	"It suggested I start with a real-life context."	Integration of real-life contexts	Prompt logs
PMT12	"AI gave me a suitable, discovery-oriented introduction."	Exploratory entry	Final interview

PMTs consistently shifted from transfer-focused explanations to more exploratory, student-centered, and contextually meaningful instructional designs. Specific contributions of AI included a shift from task sequencing to real life contextualization and a comprehensive recalibration of participants' perceptions of the structure and goals of a mathematics lesson. Rather than simply adding digital input, AI served as a catalyst, enabling teachers to articulate, justify, and reframe their pedagogical decisions. These cross-case trends form the basis of the thematic narrative that follows.

4.1.1. From transmission to exploration

The research revealed that almost all PMTs shifted from a direct, information-transmitting approach to exploratory, constructivist activities after structuring their lessons with AI. Participants' narratives show a noticeable shift from direct, static, explanation-heavy structures to exploratory, student-centered learning experiences. This transformation was particularly evident in two PMT implementations:

PMT2, in its first AI-free lesson, wrote information about the Pythagorean Theorem on the board before continuing with example questions. In its second lesson plan, based on the AI's suggestion for story development, students explored the Pythagorean Theorem through a story about setting up camp using GPS. PMT2, "It provides a more engaging explanation, geared towards their understanding, rather than just a straightforward statement."

PMT4 begins his first lesson on the summation formula by asking, "Children, I put 1 lira into my piggy bank on the first day. Moreover, every day I put in 1 lira more than the previous day. How much money do you think I will have on the 6th day?" He begins his second lesson with a story about Gauss. Then he uses a digital application called Mathigon to guide students through Gauss's method of addition, step by step, using cubes. This repositions explanation as a process to be constructed with the student, not a knowledge to be conveyed.

4.1.2 Integration of contextual and everyday mathematics

PMT6 uses stairs and traffic signs as AI suggestions when explaining the Pythagorean theorem. PMT6 asked questions like, "*Have you heard of the Pythagorean theorem?*" at the beginning of the lesson and showed the figure in Figure 1. *"What is the Pythagorean theorem?"* he stated. He did not use it in their first lesson plan.

Figure 1
Image used by PMT6

Context is a remarkable element in preparing students for mathematical processes. PMTs have noted that they need a real-life reference point, especially before moving on to abstract concepts. The contexts suggested by AI have provided an intuitive basis for explanation. Many PMTs have stated that the everyday examples suggested by AI would not have occurred to them.

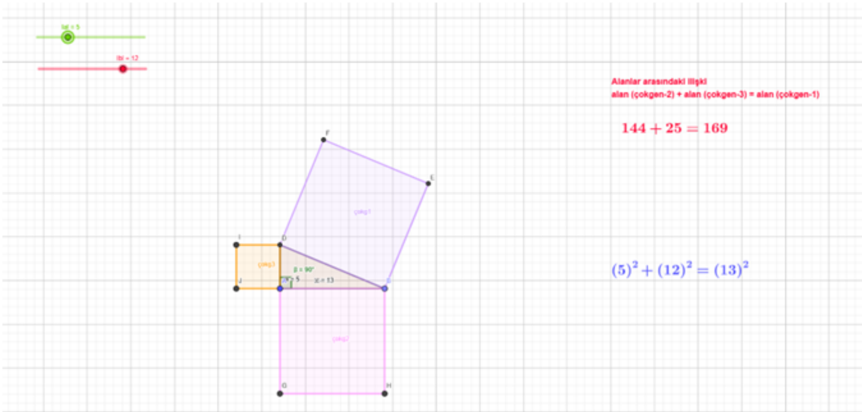
4.1.3. AI as a catalyst for reflective pedagogical thinking

One of the most profound impacts of AI on PMTs is not the direct application of teaching methods, but the transformation of pedagogical thinking. PMT7 questioned where Pythagoras came from and how he could be made meaningful. The following activity was implemented using AI:

Figure 2

Geogebra activity used by PMT7

In the process of students discovering the Pythagorean Theorem, students are first divided into groups of 3-4 and guided to work with concrete materials such as cardboard right triangles and graph paper; appropriately sized squares are drawn on each side, and the areas of these squares are calculated either by counting them using graph paper or by formula. Then, students compare the sum of the areas of the squares of the perpendicular sides with the area of the square of the hypotenuse, discovering the relationship through their own observations. At this stage, students' thoughts are deepened with the teacher's guiding questions, and the effect of changes in side lengths on the square areas is concretized by showing interactive models with digital tools (GeoGebra).



In the interview, PMT7 stated, "They create the formula for the Pythagorean Theorem themselves; thus, knowledge is understood through experience and observation, and student-centered learning is achieved, far from rote memorization."

In the prompt shown in Figure 3, PMT9 asked the AI, "*How can the assessment be different?*" The AI then presented alternative assessment activities.

Figure 3

The prompt that PMT9 asks for is an assessment

Soru 2: Değerlendirme aşamasını daha farklı nasıl olabilir? Harika bir soru! Değerlendirme aşaması sadece klasik soru-cevap değil; aynı zamanda öğrencinin kavrayışını, uygulama becerisini ve hatta akıl yürütmesini test edecek yaratıcı yollarla da yapılabilir. İşte değerlendirme aşamasını daha farklı hale getirebileceğin bazı öneriler: 🎯 Farklı Değerlendirme Yöntemleri: 1. 📏 "Doğru mu Yanlış mı?" Kartları Öğrencilere farklı üçgen kenar uzunlukları verilir (örneğin: 6-8-10, 5-5-7, 9-12-15). Her biri için "Bu bir dik üçgen midir?" sorusu sorulur. Öğrenciler kartları kaldırarak ya da ellerini kaldırarak "Doğru" ya da "Yanlış" şeklinde cevap verir.	**Question 2:** How can the assessment phase be different? Great question! The assessment phase is not just about classic question-and-answer; it can also be done in creative ways that test the student's comprehension, application skills, and even reasoning. Here are some suggestions for how you can make the assessment phase different: Different Assessment Methods: 1. "True or False" Cards Students are given different triangle side lengths (e.g., 6-8-10, 5-5-7, 9-12-15). For each, the question is asked, "Is this a right triangle?" Students answer "True" or "False" by holding up cards or raising their hands.

Note. The original Turkish version is presented on the left, and its English translation is presented on the right.

With this prompt, PMT9 determined alternative measurement and assessment questions using AI.

4.1.4. Case vignette: PMT4 — A shift toward exploratory modeling

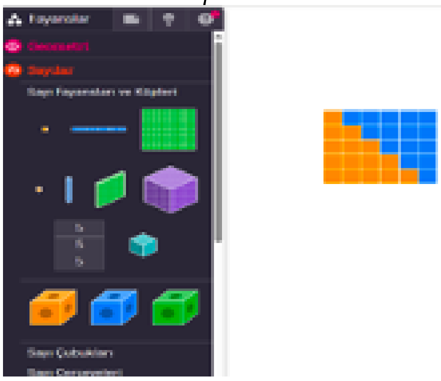
The PMT4 example is a highly representative illustration of how AI functions in pedagogical transformation.

In the first lesson video, PMT4 explained the addition formula using a step-by-step process; students calculated the sums from 1 to 5 and from 1 to 6. This explanation process was linear, teacher-centered, and directed.

In the second video, designed with AI, the lesson was completely transformed:

- PMT, together with the students, created a triangular model using blocks,
- obtained a rectangular area by inverting the blocks,
- enabled students to understand "why $\frac{n(n+1)}{2}$" through the model.

Figure 4
The virtual manipulative tools used by PMT4



This process transformed the nature of the explanation from verbal explanation to visual-exploratory reasoning. The PMT4 described this transformation as "With this method, I wanted the students to see the formula with their own eyes."

The PMT4 case clearly demonstrates the role of AI in pedagogical transformation: AI not only suggested content; it also moved the teacher into a new explanatory ecosystem focused on discovery, construction, modeling, and student-centered reasoning.

4.2. Theme 2: Shifts in Explanatory Strategies

PMTs revealed significant changes in their mathematical explanation practices after interacting with AI. Their initial explanations relied heavily on procedural notation or rule statements. The AI-assisted phase led to a shift toward more sophisticated, relational, and learner-responsive explanation styles. Table 4 shows that participants attributed these changes to the AI's ability to suggest visualizations, restructure the explanation flow, and provide exploratory reasoning steps.

Table 4
Significant changes in the mathematical explanations of PMTs

Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT1	"Mathigon was not in my mind; AI suggested it… the dynamic view made the proof easier to explain."	Expansion of representational repertoire (dynamic geometry)	Final interview
PMT2	"AI focused on prediction tasks. I changed my explanation to let students estimate first, then justify."	Re-sequencing and conceptual scaffolding	Final interview
PMT4	"I always explained straight with a formula. AI reminded me to show *why* it works using a pattern."	Move from formula-centered to pattern-centered reasoning	Second lesson plans
PMT5	"AI showed an analogy with stacking blocks… I used it in class, and students understood the growth pattern."	Use of analogy/metaphor as an explanatory device	Final interview
PMT6	"I added the area-based reasoning only after AI suggested 'use geometric decomposition'."	Inclusion of conceptual justification in explanation	Video recordings of the second lesson

Table 4 continued

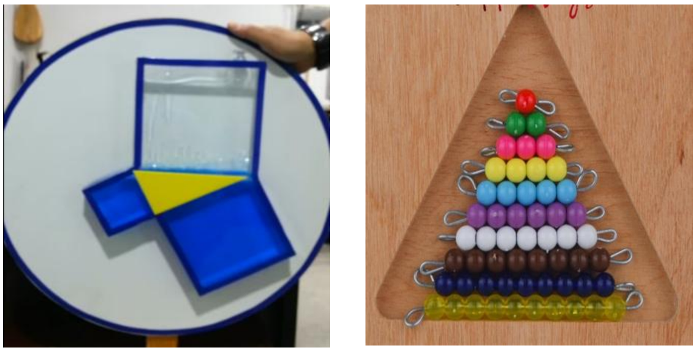
Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT7	"My first explanation was mechanical. With AI, I added the discovery steps—students guessed, tried, then explained."	Shift from direct exposition to discovery-oriented explanation	Video recordings of the second lesson
PMT9	"I used to start with definitions. Now I start with a context or task AI suggested, then formalize."	Context-anchored explanatory redesign	Final interview
PMT10	"The visualization made the structure obvious—my explanation became cleaner and shorter."	Representation improves explanatory clarity	Video recordings of the second lesson
PMT11	"AI helped me see which step confuses students, so I inserted a checkpoint question there."	Embedding checkpoints and micro-scaffolds in explanation	Prompt logs
PMT12	"I changed the whole sequence: first estimation, then manipulatives, then algebraic generalization."	Full restructuring of explanatory trajectory	Final interview

These changes signify a transition from presenting mathematical facts to highlighting mathematical ideas and establishing their conceptual foundations. The ensuing sections will discuss two forms of explanatory change.

4.2.1. From procedural delivery to conceptual elaboration

Before interacting with AI tools, most participants received explanations in a step-by-step, teacher-centered format, with rules and formulas directly conveyed to students. Their goal was to introduce the formula. After the AI, the PMTs took this a step further by using visualizations to help explain the formula and its origins. For example, in PMT1, they used the Pythagorean theorem, and in PMT5, they used the summation formula, as shown in Figure 5.

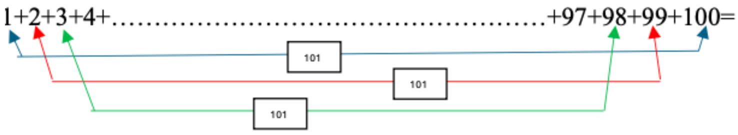
Figure 5
Visualizations PMTs use



AI assistance also influenced the PMTs' discourse and questioning styles. Rather than relying on monological explanations, the PMTs began to adopt a more dialogical tone and encourage exploration. For example, using the AI-generated image obtained by PMT10 and presented in Figure 6, PMT10 formulated questions such as, *"What happens if we add the first and last numbers?"*. These changes reveal that AI not only provides new materials but also models the epistemic applications of reasoning and inquiry. PMT12 achieved this with the question, "*Can you predict before checking together?*"

Figure 6

Visual of PMT10's total formula developed with AI

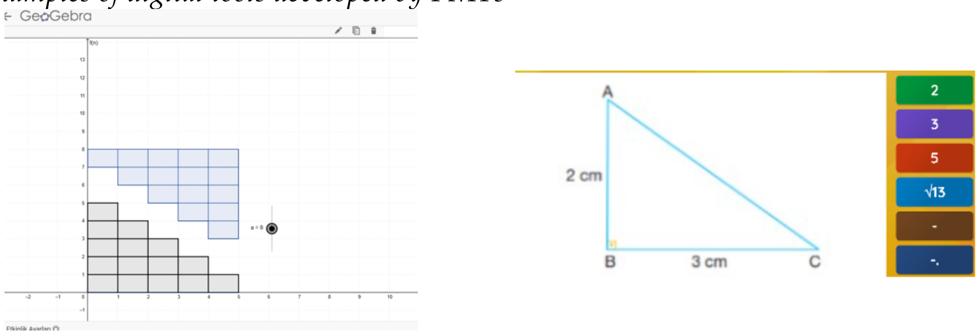


4.2.2. Expansion of representational repertoires

The second common change was the diversification of the tools used to represent explanations. Participants' pre-AI guidance largely relied on verbal narration and static whiteboard exercises. Following AI guidance, many participants incorporated the figures presented in Figure 7: PMT6 dynamic digital tools (e.g., Mathigon, GeoGebra animations); PMT7 manipulatives (e.g., area tiles, cut-and-rearrange sets); PMT9 visual diagrams (ordering diagrams, addition ladders); and the Pythagorean theorem by edge lengths.

Figure 7

Examples of digital tools developed by PMTs



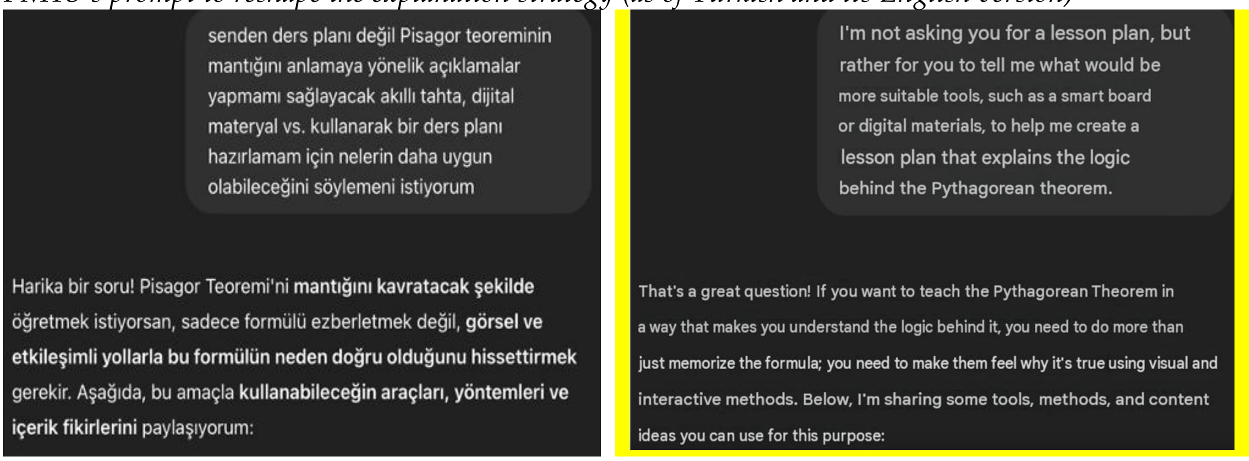
Participants noted that AI often suggests representation choices they would not consider on their own. PMTs emphasized that these representations help students better understand mathematical structures.

4.2.3. Case vignette for Theme 2: "PMT3 – Reconstructing explanation through representation and sequencing"

PMT3 provides a clear example of how AI-assisted planning reshaped the explanation strategy. PMT3 asked the AI to explain the underlying logic of the relationships (see Figure 8).

Figure 8

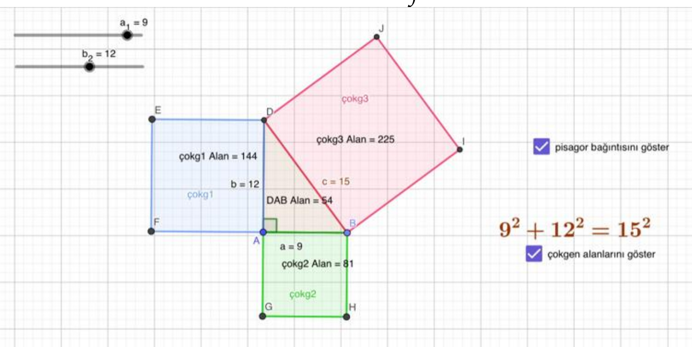
PMT3's prompt to reshape the explanation strategy (as of Turkish and its English version)



During the review of the explanation of the Pythagorean theorem, PMT3 incorporated the digital space decomposition suggested by the AI (an interactive tool that simulates the rearrangement of squares placed on the sides of a triangle) (see Figure 9). This representation allowed PMT3 to gain a deeper understanding of the concept and subsequently convey it to the students more coherently. The teacher also adopted the sequence suggested by the AI, progressing from guesswork tasks, moving to concrete cut-and-fit logic, and then formalizing the theorem.

Figure 9

PMT3's interactive tool is derived from AI



4.3. Theme 3. The Role of AI in Teacher Learning

The third theme concerns how AI functions not only as a source of teaching ideas for PMTs but also as a learning partner that helps them uncover implicit assumptions, recognize gaps in their reasoning, and re-evaluate their own explanatory preferences.

Table 5

The role of AI in PMTs' learning

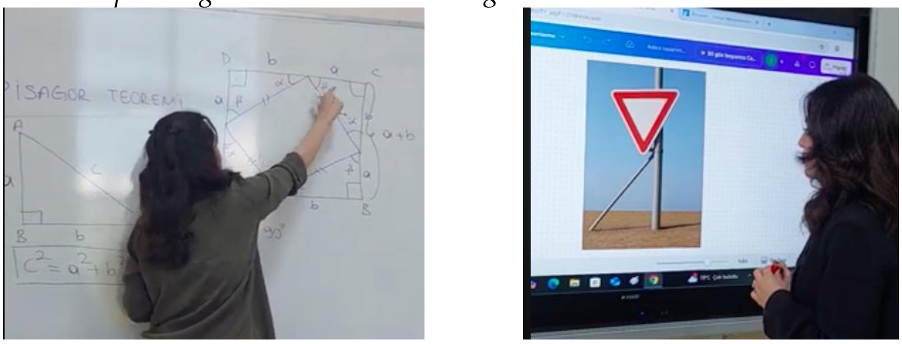
Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT2	“AI made me realize I always start with the rule. I saw that I should first let students predict.”	Epistemic shift in sequencing	Prompt logs; final interview
PMT4	“The AI’s feedback made me justify the steps rather than just show them.”	Transparency of reasoning; justification awareness	Second lesson plans
PMT5	“It pushed me to consider different representations I normally ignore.”	Expansion of representational repertoire	Prompt logs
PMT6	“When AI suggested geometric decomposition, I realized I had been avoiding visual proofs.”	Surfacing blind spots in mathematical reasoning	Final interview
PMT7	“AI made me reflect on why this proof works—not just how to show it.”	Deepening conceptual understanding	Video recordings of the second lesson
PMT9	“It made me notice I always lecture first. I changed that.”	Recognition of default pedagogical mode	Final interview
PMT10	“I learned to check each step for clarity. AI asked me: ‘How will students interpret this?’”	Anticipating interpretive difficulties	Prompt logs
PMT11	“It gave alternative explanations. Comparing them helped my understanding.”	Comparative explanation as a learning mechanism	Final interview
PMT12	“I noticed the importance of transitions. AI highlighted where confusion may appear.”	Attention to coherence and transitions	Second lesson plans

The integration of AI tools has enabled PMTs to reconceptualize mathematical concepts and teaching methods, thus functioning as cognitive and pedagogical collaborators in their own learning processes. Initially, PMTs approached AI as a technical aid to generate ideas or instructional content. However, their involvement gradually evolved into a meta-pedagogical experience. In this experience, they began to observe, evaluate, and improve their own reasoning and instructional explanations. Across the cases, two interconnected aspects of teacher learning emerged: pedagogical restructuring and reflective restructuring of teaching beliefs.

4.3.1. Pedagogical reconstruction through experimentation

PMTs used AI as a kind of pedagogical experiment in lesson design and delivery. For example, in the PMT6 lesson videos, we observed that the explanation strategies shifted from direct procedural narration in the first lessons to conceptual modeling and guided exploration in AI-assisted narration. As seen in the PMT6 lesson videos in Figure 10, a shift has been observed in the use of interactive whiteboards and in the explanation of abstract content through concrete contexts.

Figure 10
PMT6 explaining abstract content through concrete contexts



In PMT7, the first addition lesson replaced the arithmetic listing with AI-inspired story-based reasoning. It demonstrated how AI indirectly supported the shift from telling to showing. As shown in Figure 11, the first lesson plan directly presented the arithmetic listing.

Figure 11
PMT7's arithmetic listing in the first addition lesson

$$\begin{array}{ccccccccccccc}
1 & + & 2 & + & 3 & + & \dots & + & (n-2) & + & (n-1) & + & n = a \\
n & + & (n-1) & + & (n-2) & + & \dots & + & 3 & + & 2 & + & 1 = a \\
| & & | & & | & & & & | & & | & & | \\
n+1 & + & n+1 & + & n+1 & + & \dots & + & n+1 & + & n+1 & + & n+1 = 2a
\end{array}$$

PMT7 used the following statements in its second lesson plan: *"The story of Carl Friedrich Gauss solving this problem as a child is told. A natural transition to the topic is to ask students, 'What do you think Gauss might have done?"* This approach both establishes a historical connection and engages the students, thus preparing them for conceptual exploration.

This process reveals a learning through teaching, in which teachers' understanding improves as they work to make mathematical ideas visible and meaningful to students.

4.3.2. Reflective re-alignment of teaching beliefs

Finally, AI's involvement fostered self-assessment and philosophical reflection. Teachers began to express that "good explanation in mathematics teaching is not about transferring procedures, but about constructing shared meaning."

Post-lesson reflection on PMT1 demonstrated this shift: it realized that abstract proofs "can remain too theoretical" unless supported by visualization or play. In this sense, AI evolved from a mere lesson design aid into a reflective mirror, enabling teachers to perceive the epistemological dimension of their instructional choices.

In the second interview, PMT3 said, "In teaching pure subjects – referring to algebra – I used to give and explain formulas directly. With AI, I tried to start with a thought-provoking question and help them understand the formula."

4.3.3. Case vignette: PMT3 — "From doing mathematics to thinking about teaching mathematics"

PMT3 serves as a compelling illustration of AI-assisted teacher learning. Initially, the lesson plans and explanations were linear and procedural, aimed at demonstrating correct solutions. A notable juncture occurred during its interaction with the AI when the system asked, "*What potential misunderstandings might emerge at this stage?*" PMT3 noted, "I had never asked myself that before." This line of inquiry prompted a comprehensive structural revision of the explanation, including the incorporation of a conceptual checkpoint, a reconfiguration of the step-by-step sequence, and elevating student reasoning to a higher priority than procedural steps.

He also explained that he compared explanations generated by multiple AIs, which helped him recognize alternative ways to construct the idea and assess which approach made mathematical sense. This comparison process sharpened his understanding: he identified where the transitions were weak, where learners might get lost, and which representations clarified the underlying principle.

During his revised lesson plan and final interview, PMT3 demonstrated a qualitatively different understanding of explanation: "Not just showing how it works, but also why it works." His example illustrates how AI acts as a dialogical mirror, revealing blind spots, prompting reflection, and strengthening pedagogical judgment. The changes between the two lessons are summarized in Table 6 by lesson phase.

Table 6
Differences between the first and second lessons of PMT3

Lesson Phases	*Lesson Plan (Without AI)*	*AI-Supported Lesson Plan*
Introduction	The teacher draws a right triangle on the board; students are asked to name the sides and state the location of the right angle.	An animation is shown on the interactive whiteboard, and students are asked to define a triangle using short reminder questions.
Subject Presentation	The teacher directly states the theorem ($a^2 + b^2 = c^2$). Numerical examples are solved. The student is passive.	After a brief explanation, the teacher directs the students to the Wordwall game; interactive problem-solving takes place.
Closing	It is stated that the theorem is valid only for right triangles; a summary is provided.	Evaluation is based on Wordwall results; feedback is provided with a digital badge; online assignments are suggested.
Materials	Whiteboard, pen, eraser	Interactive whiteboard, Wordwall

4.4. Theme 4. Reflective and Epistemological Awareness

In Theme 4, AI prompted PMTs to reflect on the epistemic consequences of their instructional choices regarding the nature of explanation and mathematics. PMTs directly experienced reflective and epistemological awareness of epistemological friction. AI transformed participants from superficial task performers to critical evaluators of digital pedagogy. This phenomenon extends beyond the boundaries of pedagogical organization, representing a profound shift in how participants conceptualize teaching and learning in mathematics. Table 7 shows the reflective and epistemological awareness of PMTs.

Table 7
Reflective and epistemological awareness of PMTs

Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT2	"I never thought about why I started with the formula. Now I question the logic behind my sequence."	Reflective awareness of sequencing	Final interview
PMT4	"Comparing AI's explanation with mine showed me the gaps in my reasoning—not mathematically, but in how I justify steps."	Recognition of justification gaps	Second lesson plans
PMT5	"I became aware that I rely too much on symbolic representation. I saw the limits of that approach."	Epistemic limits of representational choices	Final interview
PMT6	"It made me articulate why the visual method works—not just show it."	Reflective articulation of conceptual rationale	Video recordings of the second lesson
PMT7	"I realized explanation is not only telling but designing thinking paths."	Meta-awareness of explanation as design	Final interview
PMT11	"I compared proofs and realized I did not know why one was more elegant than the other."	Awareness of proof aesthetics and epistemic value	Final interview
PMT12	"It made me think about the nature of understanding—why we need the formula, not just how."	Epistemological stance toward mathematical ideas	Final interview

The AI's feedback prompted reflective episodes among the participating PMTs, during which they confronted assumptions they had not previously expressed. Participants often struggled to provide clear rationales for instructional decisions, including the ordering of activities, the selection of visual aids, and the introduction of formulaic language. The AI may pose the question, "*What is the rationale behind the implementation of this technology in this specific context?*" The following requests were made: "*How does this representation support understanding?*"

These requests compelled the subjects to verbalize their implicit reasoning or to acknowledge its absence. Thus, as they engaged in creating, reviewing, and implementing AI-supported lessons, they began to question the nature of mathematical knowledge and their own role as its explainers.

4.4.1. From "Explaining content" to "Explaining understanding"

Initially, PMTs viewed explanation as a means of communicating formulas and procedures. However, with dialogue with AI and subsequent redesign of the course, the focus shifted to enhancing conceptual understanding and meaning-making.

For example, PMT10 recognized that a purely verbal proof might be *"too abstract"* for students. In its second lesson plan, it considered that AI-generated suggestions, such as Math Antics, Animated Explanation, and Pythagorean theorem water demo videos, would lead to a deeper understanding (see Figure 12). PMT2 also had students play an AI-suggested game during the course assessment phase.

Similarly, the use of AI-generated analogies in PMT2's Farmer problem and PMT5's Theatre activity led to a view of narrative and context as tools for mathematical reasoning, highlighting their explanatory value.

This transformation demonstrates increased epistemic sensitivity to how we explain things, rather than simply to what we explain.

Figure 12
The content PMTs use to create meaning

4.4.2. Epistemic Friction as a Catalyst for Reflection

Instead of passively accepting the AI's pedagogical suggestions, PMTs clashed with their established routines. They observed a productive intellectual conflict in which some of the AI-generated suggestions were either inappropriate or overly general, a phenomenon known as epistemic friction. This situation arises when teachers need to evaluate the quality and validity of AI outputs.

PMT1's reasoning for rejecting a video suggested by AI – "I did not like the video it gave me; I found a more usable one" – demonstrates critical judgment. PMT6's rejection of the AI's visualization for Pythagoras and its subsequent use of a more suitable one with new prompts illustrates this.

These moments of friction did not hinder the learning process; on the contrary, teachers improved their reflective reasoning skills as they learned to leverage AI capabilities and their own pedagogical values.

4.4.3. Case Vignette: PMT10 — "Seeing the blind spots"

PMT10 provides a vivid example of epistemic awareness emerging through AI interaction. The initial lesson plans were technically correct but lacked clear transitions and conceptual justification. During the guidance, the AI asked, "Why this representation now?" PMT10 admitted it had no answer. This question became a turning point, leading it to reevaluate every stage of its explanation.

In his revised plan, he explicitly stated why he chose visual representation over symbolic representation and noted how students could interpret each. He also added transition sentences to clarify conceptual transitions, something he did not realize were missing until the AI highlighted them. In the final interview, he described the experience as "seeing my own blind spots," emphasizing that the AI enabled him to recognize not only the decisions themselves but also the logic behind them.

The PMT10 example demonstrates how AI can foster cognitive self-awareness and shift teachers' thinking from procedural presentation to reflective, conceptually based explanation.

4.5. Theme 5: Constraints and Tensions in AI Integration

The fifth theme highlights the tensions that arise when PMTs attempt to integrate AI-generated explanations or instructional materials into real-world teaching contexts. Participants also expressed concerns about reliability, suitability, and the adequacy of content knowledge (see Table 8).

AI feedback and questions prompted reflective segments in which PMTs confronted assumptions they had not previously articulated. Many realized, sometimes with surprise, that they lacked explicit justification for instructional decisions, such as ordering, selecting a representation, or introducing a formula. AI questions like, "*Why is this being introduced here?*" or "*How does this representation support understanding?*" forced them to articulate the implicit justification or acknowledge its absence.

Table 8
Tensions PMTs experience with AI

Participant	*Representative Quotation*	*Analytical Code*	*Data Source*
PMT1	"The examples AI gave were too general. I still had to find the real video myself."	Limited specificity / low contextual fit	Final interview
PMT2	"The suggested materials were not suitable for the class level, so I had to adapt them."	Misalignment between AI suggestions and curricular demands	Final interview
PMT3	"AI helped but sometimes made me feel dependent… like I might stop thinking for myself."	Perceived cognitive dependency	Final interview
PMT4	"It does not understand the classroom reality… some suggestions are impossible with real students."	Lack of classroom contextualization	Final interview
PMT5	"The plan it created was too long; I had to shorten and reorganize almost everything."	Over-generated content requiring substantial teacher editing	Second lesson plans
PMT7	"The terminology used is too advanced for middle school."	Unlevelled language/mismatch with student level	Final interview
PMT9	"I worried that if I use AI too much, my own voice in the lesson disappears."	Loss of teacher agency/authorship concerns	Final interview
PMT12	"AI cannot determine students' emotional states; that part is still up to me."	Recognition of AI's limits in anticipating student emotions	Final interview

4.5.1. Technical and linguistic barriers

Some PMTs have reported difficulties interacting with AI tools due to language and contextual limitations.

PMT3 stated that "the resources provided by the AI were mostly in English," making it difficult to find suitable materials or activities aligned with local curriculum standards. Similarly, PMT4 and PMT7 expressed concerns about the AI suggesting materials that were "too advanced" or "not appropriate for the class level."

These experiences highlight the gap between AI's general capabilities and the nuanced demands of classroom-level pedagogy.

4.5.2. Professional autonomy

Another recurring tension concerned the balance between AI guidance and teacher autonomy. While AI was often seen as an efficient planner, some participants felt uncomfortable with its over-reliance.

PMT1 explicitly stated: "AI did not add much extra – I would still use digital tools, but it helped organize the flow." PMT8 criticized AI, saying, "The narration was better, which I did myself. AI does not provide anything directly. It suggested a video, but the link does not open."

This dilemma reflects an ongoing negotiation between trusting AI suggestions and defending professional judgment. For some, AI served as scaffolding, while for others it risked restricting creative autonomy.

4.5.3. Epistemological and ethical concerns

The PMTs also raised deeper questions about originality, authorship, and pedagogical integrity regarding the use of ideas generated by AI.

PMT5 questioned whether an explanation was "truly its own" if most of the examples were derived from AI outputs. Similarly, PMT3, PMT9, and PMT10 concerned that students might perceive AI-generated content—especially visual content—as "too polished," "less human," and "lacking in emotion," potentially diminishing the dialogical, collaborative, and constructive nature of the explanation.

4.5.4. Case vignette: PMT8 — "Useful, but never ready-to-use"

PMT8 illustrates the tensions PMTs face in integrating AI. They repeatedly observed a lack of direct applicability in AI's suggestions for generating contextual story problems, developing games, and producing videos and visuals. When AI suggested manipulatives, none were directly relevant to the middle school curriculum. They characterized AI guidance as "inspiring but incomplete."

The PMT8 case demonstrates the dual nature of AI integration: while AI can be a productive source of inspiration, it can also create friction, requiring significant teacher feedback and adaptation. This case study underscores a key finding of Theme 5: AI is pedagogically productive, but rarely pedagogically ready.

5. Discussion

The findings of this study expand upon current discussions regarding the role of AI in mathematics teacher education, illuminating not only changes in instructional design practices but also deeper shifts in pedagogical reasoning and epistemological positioning. Interpreting these findings within explanatory and understanding theoretical frameworks reveals a complex picture of AI in mathematics education, encompassing transformation, tension, and constraints setting. The following sections address these dimensions in detail.

5.1. AI-Supported Pedagogical Transformation

The findings of the present research have revealed that AI-assisted lesson designs for PMTs offer opportunities for profound pedagogical transformation beyond simple planning-process support. They have supported PMTs not only in presenting mathematical ideas but also in structuring them in students' minds. These findings are consistent with previous research arguing that AI will serve as a pedagogical support and mediation rather than a replacement for teacher expertise (Holmes et al., 2019; Kasneci et al., 2023; Yılmazer & Avşar Erümit, 2026; Zawacki-Richter et al., 2019). Instead of directly delivering the lesson, AI has facilitated a student-active lesson structure that encourages exploration. PMTs emphasized inquiry, visual exploration, association, and reasoning based on manipulation in their lessons. This orientation is consistent with student-centered and inquiry-based approaches to mathematics teaching, in which learners engage with mathematical ideas through exploration and reasoning and teachers build instruction around students' developing thinking (Dereje, 2023; Stein et al., 2008).

Participants evaluated the AI-suggested content according to the principles of student level, conceptual appropriateness, or applicability. This finding aligns with reflective decision-making (Engeström, 2001; Luckin et al., 2016). Furthermore, AI, as provided by dynamic geometry software, fostered meaningfulness and relational understanding by promoting dynamic visualizations, simulations, and real-life scenarios (Zengin, 2019). These findings suggest that integrating AI into teaching will be most effective when positioned as a dialogical partner that encourages explanatory choices and epistemological values. AI-supported environments encourage reorientation of teaching with exploration, coherence, and conceptual construction, which are fundamental components of effective mathematics teaching (Dereje, 2023; Mailizar et al., 2026).

The dialogical interaction between teachers and AI suggestions has not only led to pedagogical transformation but also to the development of AI-based tools. Teachers have accepted, rejected, or modified AI suggestions based on student level, conceptual appropriateness, or classroom applicability. This finding supports the view that AI is an intermediary tool that reveals pedagogical possibilities and thus creates a space for reflective decision-making (Engeström, 2001; Vygotsky, 1978). In this sense, AI has not replaced teachers' pedagogical reasoning, but rather made it more open and debatable.

Furthermore, this observed shift from direct explanation to exploratory structuring aligns with research that treats mathematical explanation not as a one-way act of narration, but as an application of epistemological structuring (Hanna & de Villiers, 2012; Prediger et al., 2019). AI-powered planning processes have supported teachers in designing explanations that invite students' meaning-making and relational understanding rather than procedural reproduction, by encouraging multiple representations (e.g., dynamic visuals, simulations, or real-life contexts) (Drijvers & Sinclair, 2024).

These findings suggest that integrating AI into teacher training can contribute to pedagogical transformation when positioned not as an automated planning solution but as a dialogical partner that encourages reflection on instructional goals, explanatory choices, and epistemological values. AI-powered environments appear to promote an instructional reorientation toward exploration, coherence, and conceptual accessibility—essential components of effective mathematics teaching—rather than simply accelerating lesson preparation.

5.2. Epistemic Friction and Reflective Teacher Learning in AI-Mediated Contexts

Current research highlights epistemic friction as the key mechanism by which AI-powered environments foster reflective learning and pedagogical development among PMTs. Epistemic friction refers to moments when the PMTs' existing explanatory or teaching strategies become controversial or are challenged by AI-generated content (Sher, 2016). Instead of providing perfect support, AI has introduced productive tensions that require PMTs to rethink which explanations are valid and how evidence supports understanding (Obiso et al., 2025; Ramsoomair, 2025).

In our findings, PMTs frequently stated that AI not only provided "better explanations" but also challenged the justification of preconceived approaches, particularly those based on procedural or transmission-oriented teaching. This process closely aligns with Shulman's (1987) concept of pedagogical reasoning, which emphasizes the cycles of comprehension, transformation, instruction, evaluation, and reflection. AI primarily functioned as an externalized reasoning partner, intervening during the transformation and reflection phases. For example, when AI proposed alternative representations (e.g., field-based proofs for addition or dynamic visualizations for the Pythagorean theorem), teachers had to assess the epistemological adequacy of these representations in relation to instructional goals and student needs. This assessment constitutes epistemologically grounded pedagogical reasoning rather than mere reactivity.

From a mathematical justification perspective, epistemological friction was particularly evident in participants' interactions with proofs and explanations. Building on Harel and Sowder's (2007) framework of proof schemes, many PMTs initially relied on authoritarian or empirical schemes and accepted them as validated by AI. However, sustained interaction with AI-assisted tasks led some participants to recognize the limitations of these schemes. This shift was not linear; rather, it was often triggered by alternative proof structures generated by AI (Pantsar, 2024; Stylianides, 2007).

Importantly, in this study, epistemic friction was experienced not merely as cognitive dissonance but as a pedagogically meaningful struggle. Participants frequently reported being forced to rethink and question themselves, suggesting that AI-mediated friction creates a space for self-criticism and reflective engagement. This finding aligns with recent discussions in the educational technology literature that caution against positioning AI solely as an optimization tool and instead highlight its potential role in productive struggle and epistemic engagement (Kafai & Proctor, 2022; Luckin et al., 2016).

AI systems function not as content providers, but as dialogical mediators that co-construct understanding through the reorganization of information (Holmes et al., 2022). When AI functioned solely as a ready-made content generator—providing ready-made examples or polished explanations—teachers reported minimal impact on their understanding or instructional reasoning. In contrast, when AI acted as a co-designer by presenting contrasting representations, it created epistemic tension that fostered reflective learning. This distinction aligns with recent educational research emphasizing that the educational value of AI lies not in automation but in its capacity to mediate meaning-making and professional development (Holmes et al., 2019; Williamson & Eynon, 2020).

5.3. Constraints in AI-Supported Pedagogical Reasoning

While the findings demonstrate the productive potential of AI-powered environments for understanding and explanation-oriented pedagogical transformation, they also reveal significant constraints on how PMTs engage in pedagogical reasoning within these contexts. These constraints not only reflect the technical limitations of AI tools but also point to deeper epistemic and pedagogical tensions shaping how AI is addressed in instructional design and explanation practices.

Some PMTs have positioned AI primarily as a resource provider rather than an epistemic interlocutor. In these cases, AI was used to generate examples and activities; however, its role remained instrumental. This finding aligns with current debates in the educational technology literature that distinguish AI as an assistant and a co-designer (e.g., Kasneci et al., 2023; Kohnke et al., 2023). When AI functions solely as a tool to enhance efficiency, participants' pedagogical reasoning largely reproduces pre-existing teaching patterns, limiting opportunities for deeper epistemic interaction.

This study highlights the tensions in mathematical epistemology when teaching proof-based and generalization-focused topics such as the Pythagorean theorem and the summation formula. Building on Sowder and Harel's (1998) framework of proof schemes, many participants continued to rely on classical explanations of the feature summation formula, even when AI provided access to more visual proofs. This demonstrates that AI-mediated environments alone are insufficient to change teachers' fundamental proof schemes without explicit pedagogical support.

These findings suggest that the effectiveness of AI-assisted pedagogical reasoning depends more on teachers' capacity to interact with AI as a dialogical and epistemological partner than on the sophistication of AI tools. This underscores the need for teacher training programs to explicitly address not only how AI can be used, but also how teachers should collaborate with AI in pedagogical decision-making.

6. Implications, Suggestions, and Limitations

The findings of this study hold significant implications for teacher education, particularly regarding how AI is positioned in the pedagogical and epistemological development of PMTs. Rather than conceptualizing AI as a technical tool for optimizing lesson planning, the results demonstrate that AI can serve as an epistemological tool that reshapes how teachers reason about explanation, understanding, and mathematical meaning.

Teacher education programs must move beyond instrumental approaches to AI integration that primarily focus on efficiency, productivity, or content production. The findings show that meaningful learning occurs when PMTs engage in dialogue with AI—questioning, adapting, and sometimes resisting AI-generated suggestions. This highlights the need for teacher education curricula that explicitly frame AI as a thinking partner rather than an authoritative source of pedagogical knowledge. This orientation aligns with contemporary views that see teacher learning as a reflective and iterative process rather than the acquisition of fixed teaching strategies.

The study highlights the importance of fostering epistemic awareness in AI-assisted environments. PMTs were encouraged to rethink what counts as a valid explanation, how to make understanding visible, and where authority lies in mathematics teaching. Therefore, teacher training programs should include structured opportunities for PMTs to evaluate AI outputs, critically question underlying assumptions, and independently justify pedagogical decisions.

Without such support, the use of AI risks inadvertently reinforcing procedural or transfer-oriented teaching practices rather than fostering deeper conceptual engagement.

The concept of epistemic friction observed in this study points to a fruitful design principle for teacher training. Moments of tension where AI-generated ideas clashed with PMTs' prior beliefs or teaching intentions served as catalysts for reflection and pedagogical development. Instead of minimizing these tensions, teacher educators can intentionally design AI-mediated tasks that reveal ambiguity, vagueness, or alternative representations. In this way, AI becomes a resource for the epistemological struggle that supports the development of professional judgment, a fundamental component of pedagogical reasoning.

The current research focused on two topics involving mathematical explanations. This could be expanded by increasing the number and variety of topics. This would make it clearer how AI can explain the inherently diverse nature of mathematical content and how this supports learning. A limitation of the research is the lack of a control group, which prevents a comparison of whether the observed changes in PMTs' explanation skills stem directly from their interaction with AI or from natural learning over time.

A similar study could be conducted with practicing teachers. In particular, examining how experienced teachers interact with AI and adapt its suggestions to classroom settings could provide deeper insights into how this tool can be used to achieve effective understanding.

The participants in this study were only PMTs; this also limits the applicability of the findings to other disciplines. Future research could also examine the thematic patterns observed here, especially in areas where explanations are prominent (e.g., science education). Applying similar processes across various subject areas will enable a comparative analysis of AI's pedagogical flexibility and adaptability.

AI usage and assistance: In this study, an AI-based writing assistant (OpenAI, DeepL) was used only for language editing and translation purposes.

Author contributions: All authors contributed substantially to the conception, design, data collection, analysis, and writing of this study.

Data availability: The data supporting the findings of this study are available from the corresponding author upon reasonable request. To protect participant confidentiality, full transcripts are not publicly available.

Declaration of interest: The authors declare that they have no conflict of interest.

Ethics Declaration: The study was conducted in accordance with the ethical principles applicable to research involving human participants. Ethical approval was obtained from the Bartın University Ethics Committee prior to data collection (approval no. 2026-SBB-0307).

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References

Acuyo Cespedes, A. (2025). Pedagogical shifts in the age of GenAI: Faculty perspectives from a higher education context. *Journal of Pedagogical Sociology and Psychology, 7*(3), 35–48. https://doi.org/10.33902/jpsp.202537244

Agarwal, R., & Bhatnagar, A. (2026). Teacher–AI collaboration: Reimagining pedagogical roles in the AI-infused classroom. In B. Srivastava, M. Tyagi, & M. Tyagi (Eds.), *Redefining innovative practices in the age of AI* (pp. 152–156). CRC Press. https://doi.org/10.1201/9781003679295

Antonenko, P., & Abramowitz, B. (2023). In-service teachers' misconceptions of artificial intelligence in K-12 science education. *Journal of Research on Technology in Education*, *55*(1), 64-78. https://doi.org/10.1080/15391523.2022.2119450

Asare, B., & Boateng, F. O. (2025). Self-awareness and self-regulatory learning as mediators between ChatGPT usage and pre-service mathematics teacher's self-efficacy. *Journal of Pedagogical Research, 9*(2), 38–54. https://doi.org/10.33902/JPR.202530637

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *The Journal of Mathematical Behavior*, *26*(4), 348-370. https://doi.org/10.1016/j.jmathb.2007.11.001

Bergqvist, K. (2010). Think of how you can know that you have reached your goals»–Reflection as self-control in late-modern schooling. *Nordic Studies in Education*, *30*(3), 137-149.

Braun, V., & Clarke, V. (2021). One size fits all? What counts as quality practice in (reflexive) thematic analysis? *Qualitative Research in Psychology*, *18*(3), 328-352. https://doi.org/10.1080/14780887.2020.1769238

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher*, *32*(1), 9-13. https://doi.org/10.3102/0013189X032001009

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Sage.

Crompton, H., & Burke, D. (2024). The educational affordances and challenges of ChatGPT: State of the field. *TechTrends*, *68*(2), 380-392. https://doi.org/10.1007/s11528-024-00939-0

D'Alessandro, W., & Lehet, E. (2024). A noetic account of explanation in mathematics. *The Philosophical Quarterly*, pqae137. https://doi.org/10.1093/pq/pqae137

Dadhich, A., Yadav, R., Yadav, M., Huzooree, G., & Dewasiri, N. J. (2025). Teacher-AI collaboration and the future of the educator's role. In L. Kyei-Blankson & E. Ntuli (Eds.), *Transformative AI practices for personalized learning strategies* (pp. 355–378). IGI Global. https://doi.org/10.4018/979-8-3693-8744-3.ch013

Dai, Y., Liu, A., Qin, J., Guo, Y., Jong, M. S. Y., Chai, C. S., & Lin, Z. (2023). Collaborative construction of artificial intelligence curriculum in primary schools. *Journal of Engineering Education*, *112*(1), 23-42. https://doi.org/10.1002/jee.20503

D'Alessandro, W. (2019). Explanation in mathematics: Proofs and practice. *Philosophy Compass*, *14*(11), e12629. https://doi.org/10.1111/phc3.12629

Dereje, E. W. (2023). The effect of inquiry based technology integration on conceptual and procedural geometry knowledge of preservice mathematics teachers. *Journal of Pedagogical Sociology and Psychology, 5*(4), 12–27. https://doi.org/10.33902/jpsp.202321592

Drijvers, P., & Sinclair, N. (2024). The role of digital technologies in mathematics education: Purposes and perspectives. *ZDM–Mathematics Education*, *56*(2), 239-248. https://doi.org/10.1007/s11858-023-01535-x

Duval, R. (2006). Transformations de représentations sémiotiques et démarches de pensée en mathématiques [Transformations of semiotic representations and thought processes in mathematics.]. In J.-C. Rauscher (Ed.), *Actes du XXXIIe colloque COPIRELEM des professeurs et des formateurs de mathématiques chargés de la formation des maîtres: Strasbourg 2005. Enseigner les mathématiques en France, en Europe et ailleurs* [Proceedings of the 32nd COPIRELEM Colloquium of mathematics teachers and teacher trainers: Strasbourg 2005. Teaching mathematics in France, Europe and elsewhere] (pp. 67–89). IREM de Strasbourg.

Elgin, C. (2009). Is understanding factive? In A. Haddock, A. Millar, & D. Pritchard (Eds.), *Epistemic value* (pp. 322–329). Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199231188.003.0015

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work*, *14*(1), 133-156. https://doi.org/10.1080/13639080020028747

Ernest, P. (1991). Mathematics teacher education and quality. *Assessment and Evaluation in Higher Education*, *16*(1), 56-65. https://doi.org/10.1080/0260293910160107

Fleisher, W. (2022). Understanding, idealization, and explainable AI. *Episteme*, *19*(4), 534-560. https://doi.org/10.1017/epi.2022.39

Gowers, W. T. (2000). The two cultures of mathematics. In V. Arnold, M. Atiyah, P. Lax, & B. Mazur (Eds.), *Mathematics: Frontiers and perspectives* (pp. 65–78). American Mathematical Society.

Guberina, T., & Procházka, F. (2026). Human-AI collaborative teaching: generative artificial intelligence (Gen-AI) as co-teacher. *Social Science Chronicle*, *6*, 1-19. https://doi.org/10.56106/ssc.2026.003

Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange, 21*(1), 6-13. https://doi.org/10.1007/BF01809605

Hanna, G., & de Villiers, M. (Eds.). (2012). *Proof and proving in mathematics education: The 19th ICMI study* (Vol. 15). Springer. https://doi.org/10.1007/978-94-007-2129-6

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 805–842). Information Age Publishing.

Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics 24*(4), 389-399.

Hofer, B. K., & Pintrich, P. R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of Educational Research*, *67*(1), 88-140. https://doi.org/10.2307/1170620

Holmes, W., Bialik, M., & Fadel, C. (2019). *Artificial intelligence in education promises and implications for teaching and learning*. Center for Curriculum Redesign. https://discovery.ucl.ac.uk/id/eprint/10139722

Holmes, W., Persson, J., Chounta, I. A., Wasson, B., & Dimitrova, V. (2022). *Artificial intelligence and education: A critical view through the lens of human rights, democracy, and the rule of law*. Council of Europe.

Inglis, M., & Mejía-Ramos, J. P. (2021). Functional explanation in mathematics. *Synthese*, *198*(Suppl 26), 6369-6392. https://doi.org/10.1007/s11229-019-02234-5

Kafai, Y. B., & Proctor, C. (2022). A revaluation of computational thinking in K–12 education: Moving toward computational literacies. *Educational Researcher*, *51*(2), 146-151. https://doi.org/10.3102/0013189X211057904

Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *The Journal of Mathematical Behavior, 17*(2), 265-281.

Kasneci, E., Seßler, K., Küchemann, S., Bannert, M., Dementieva, D., Fischer, F., Gasser, U., Groh, G., Günnemann, S., Hüllermeier, E., Krusche, S., Kutyniok, G., Michaeli, T., Nerdel, C., Pfeffer, J., Poquet, O., Sailer, M., Schmidt, A., Seidel, T., . . . Kasneci, G. (2023). ChatGPT for good? On opportunities and challenges of large language models for education. *Learning and Individual Differences, 103*, 102274. https://doi.org/10.1016/j.lindif.2023.102274

Katsenou, R., Kotsidis, K., Papadopoulou, A., Anastasiadis, P., & Deliyannis, I. (2025). Beyond assistance: embracing AI as a collaborative co-agent in education. *Education Sciences*, *15*(8), 1006. https://doi.org/10.3390/educsci15081006

Kelp, C. (2016). Towards a knowledge-based account of understanding. In S. R. Grimm, C. Baumberger, & S. Ammon (Eds.), *Explaining understanding: New perspectives from epistemology and philosophy of science* (pp. 251–271). Routledge.

Kim, J. (2024). Leading teachers' perspective on teacher-AI collaboration in education. *Education and Information Technologies*, *29*(7), 8693-8724. https://doi.org/10.1007/s10639-023-12109-5

Kim, J., Lee, H., & Cho, Y. H. (2022). Learning design to support student-AI collaboration: Perspectives of leading teachers for AI in education. *Education and Information Technologies*, *27*(5), 6069-6104. https://doi.org/10.1007/s10639-021-10831-6

Kitcher, P. (1981). Explanatory unification. *Philosophy of Science, 48*(4), 507-531. https://doi.org/10.1086/289019

Kohnke, L., Moorhouse, B. L., & Zou, D. (2023). Exploring generative artificial intelligence preparedness among university language instructors: A case study. *Computers and Education: Artificial Intelligence*, *5*, 100156. https://doi.org/10.1016/j.caeai.2023.100156

Kvanvig, J. (2003). *The value of knowledge and the pursuit of understanding*. Cambridge University Press.

Lange, M. (2014). Aspects of mathematical explanation: Symmetry, unity, and salience. *Philosophical Review, 123*(4), 485-531. https://doi.org/10.1215/00318108-2749730

Lehet, E. (2021). Mathematical explanation in practice. *Axiomathes, 31*, 553–574. https://doi.org/10.1007/s10516-021-09557-4

Li, X., Han, G., Fang, B., & He, J. (2025). Advancing the in-class dialogic quality: Developing an artificial intelligence-supported framework for classroom dialogue analysis. *The Asia-Pacific Education Researcher*, *34*(1), 495-509. https://doi.org/10.1007/s40299-024-00872-z

Ling, J., Zhu, J., Sun, J., Shi, J., Ke, C., Li, Z., Timms, M., & Hwang, G.-J. (2026). What are effective classroom dialogue patterns? A study on classroom dialogue patterns of pre-service and expert teachers based on large language models. *Interactive Learning Environments, 34*(6), 4275–4299. https://doi.org/10.1080/10494820.2025.2592125

Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, *59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Luckin, R. (2018). *Machine learning and human intelligence: The future of education for the 21st century*. UCL IOE press.

Luckin, R., Holmes, W., Griffiths, M. & Forcier, L. B. (2016). *Intelligence unleashed: An argument for AI in Education.* Pearson.

Magne, V., Vince, S., Mace, R., & Hooper, S. (2026). AI as a dialogic partner: Rethinking feedback in higher education. *Assessment & Evaluation in Higher Education, 51*(4), 721–735. https://doi.org/10.1080/02602938.2025.2587247

Mailizar, M., Umam, K., & Elisa, E. (2026). Examining the impact of teachers' knowledge on the integration of dynamic mathematics software into constructivist pedagogy. *Journal of Pedagogical Research, 10*(1), 209–226. https://doi.org/10.33902/JPR.202636113

Matteucci, A., Tortoriello, F. S., & Veronesi, I. (2024). Dialectical chatbots: AI-driven debate in mathematical education. In A. Clark-Wilson, E. Faggiano, M. Tabach, & H.-G. Weigand (Eds.), *Proceedings of the 17th ERME Topic Conference Mathematics Education in the Digital Age – MEDA4* (pp. 271–278). University of Bari Aldo Moro.

McDermott, R., Brown, J. N. A., Daniels, M., & Cajander, Å. (2025). Competency, understanding and the role of explanation in AI-driven education. In B. K. Smith & M. Borge (Eds.), *Learning and collaboration technologies* (Lecture Notes in Computer Science, Vol. 15807, pp. 304–323). Springer. https://doi.org/10.1007/978-3-031-93567-1_21

McKenney, S., & Reeves, T. C. (2025). Educational design research for relevant & robust scholarship. *Journal of Computing in Higher Education, 37*(2), 614–638. https://doi.org/10.1007/s12528-025-09456-2

Mercer, N. (2010). The analysis of classroom talk: Methods and methodologies. *British Journal of Educational Psychology*, *80*(1), 1-14. https://doi.org/10.1348/000709909X479853

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Sage.

Obiso, T., Lai, K., Nath, A., Krishnaswamy, N., & Pustejovsky, J. (2025). Dynamic epistemic friction in dialogue. In G. Boleda & M. Roth (Eds.), *Proceedings of the 29th conference on computational natural language learning* (pp. 323–333). Association for Computational Linguistics. https://doi.org/10.18653/v1/2025.conll-1.21

Okanya, V. A., & Asogwa, J. O. (2026). Evaluating human–AI collaborative pedagogical model for sustainable skill development in technical education workshops. *Sustainable Education and Digital Transformation, 1*(1), e43677. https://doi.org/10.33902/SEDT.202643677

Pantsar, M. (2024). Theorem proving in artificial neural networks: new frontiers in mathematical AI. *European Journal for Philosophy of Science*, *14*(1), 4. https://doi.org/10.1007/s13194-024-00569-6

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice* (4th ed.). Sage.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), *Distributed cognitions: Psychological and educational considerations* (pp. 47–87). Cambridge University Press.

Prediger, S., Erath, K., & Moser Opitz, E. (2019). The language dimension of mathematical difficulties. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the classroom* (pp. 437–455). Springer. https://doi.org/10.1007/978-3-319-97148-3_27

Ramsoomair, N. (2025). Pressing matters: How AI irons out epistemic friction and smooths over diversity. *Atlantis*, *46*(1), 42-55. https://doi.org/10.7202/1119494ar

Robbemond, V., Inel, O., & Gadiraju, U. (2022). Understanding the role of explanation modality in AI-assisted decision-making. In A. Belogin & L. Boratto (Eds.), *Proceedings of the 30th ACM Conference on User Modeling, Adaptation and Personalization* (pp. 223–233). Association for Computing Machinery. https://doi.org/10.1145/3503252.3531311

Schoenfeld, A. H. (2018). Video analyses for research and professional development: The teaching for robust understanding (TRU) framework. *ZDM, 50*(3), *491–506*. https://doi.org/10.1007/s11858-017-0908-y

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Sher, G. (2016). *Epistemic friction: An essay on knowledge, truth, and logic*. Oxford University Press.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, *77*(1), 20-26.

Sowder, L., & Harel, G. (1998). Types of students' justifications. *Mathematics Teacher*, *91*(8), 670-675.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, *10*(4), 313-340. https://doi.org/10.1080/10986060802229675

Steiner, M. (1978). Mathematical explanation. *Philosophical Studies, 34*(2), 135–151. https://www.jstor.org/stable/4319237

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Sage.

Stylianides, A. J., & Stylianides, G. J. (2009). Proof constructions and evaluations. *Educational Studies in Mathematics*, *72*(2), 237-253. https://doi.org/10.1007/s10649-009-9191-3

Stylianides, A. L. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, *38*(3), 289-321.

Stylianides, G. J., Stylianides, A. J., & Moutsios-Rentzos, A. (2024). Proof and proving in school and university mathematics education research: A systematic review. *ZDM–Mathematics Education*, *56*(1), 47-59. https://doi.org/10.1007/s11858-023-01518-y

Tang, K. S., & Putra, G. B. S. (2026). Generative AI as a dialogic partner: Enhancing multiple perspectives, reasoning, and argumentation in science education with customized chatbots. *Journal of Science Education and Technology, 35*(1), 128–140. https://doi.org/10.1007/s10956-025-10240-1

Tusyanah, T., Samsudin, N., Ismiyati, I., Chayati, N., Pramusinto, H., & Suryanto, E. (2026). The pathway from readiness to pedagogy: AI readiness as a catalyst for innovative lesson-designing in preservice teachers. *Journal of Pedagogical Sociology and Psychology, 8*(2), e42865. https://doi.org/10.33902/JPSP.202642865

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard university press.

Wan, P., & Gu, X. (2026). Developing teachers' professional abilities: A systematic review of human-machine dialogic learning for teacher education. *Interactive Learning Environments, 34*(2), 615–639. https://doi.org/10.1080/10494820.2025.2507280

Wang, X., & Yin, M. (2023). Watch out for updates: Understanding the effects of model explanation updates in AI-assisted decision making. In A. Schmidt & K. Vaananen (Eds.), *Proceedings of the 2023 CHI Conference on Human Factors in Computing Systems* (pp. 1–19). Association for Computing Machinery. https://doi.org/10.1145/3544548.3581366

Watson, S., & Shi, S. (2024). Generative AI integration in education: Challenges and approaches. In P. Ilic, I. Casebourne, & R. Wegerif (Eds.), A*rtificial intelligence in education: The intersection of technology and pedagogy* (pp. 59-73). Springer. https://doi.org/10.1007/978-3-031-71232-6_4

Wegerif, R. (2021). A dialogic approach to education for democratic values illustrated with an empirical study of the effect of Internet-mediated dialogue across cultural differences. In P. Iyer, & I. Bhattacharjee (Eds.), *Moral and political discourses in philosophy of education* (pp. 1-11). Routledge. https://doi.org/10.4324/9780429285493

Wertsch, J. V. (1998). *Mind as action*. Oxford University Press.

Wilkenfeld, D. A. (2014). Functional explaining: a new approach to the philosophy of explanation. *Synthese, 191*, 3367–3391. https://doi.org/10.1007/s11229-014-0452-z

Williamson, B., & Eynon, R. (2020). Historical threads, missing links, and future directions in AI in education. *Learning, Media and Technology*, *45*(3), 223-235. https://doi.org/10.1080/17439884.2020.1798995

Yang, X. (2025). AI-empowered pedagogy in ESP curriculum: A constructivist approach to active learning. *International Journal of Social Science, Education and Humanities*, *1*(1), 7-23. https://doi.org/10.63313/IJSSEH.9002

Yılmazer, A., & Avsar Erumit, B. (2026). Redefining lesson planning: Integrating ChatGPT into the teaching practicum experience. *Education and Information Technologies, 31,* 923-952. https://doi.org/10.1007/s10639-025-13830-z

Yin, R. K. (2018). *Case study research and applications* (Vol. 6). Sage.

Zawacki-Richter, O., Marín, V. I., Bond, M., & Gouverneur, F. (2019). Systematic review of research on artificial intelligence applications in higher education–where are the educators?. *International Journal of Educational Technology in Higher Education*, *16*(1), 1-27. https://doi.org/10.1186/s41239-019-0171-0

Zengin, Y. (2019). Development of mathematical connection skills in a dynamic learning environment. *Education and Information Technologies*, *24*(3), 2175-2194. https://doi.org/10.1007/s10639-019-09870-x