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Research Article

Triangle inequality concept teaching: The theory of didactic situations case

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The purpose of this study is to implement an a-didactical activity relating to the acquisition of the triangle inequality, prepared within the framework of the *Didactic Situations Theory* (TDS), and to report the experiences of the students during the implementation process. The study employs a case study method based on a qualitative approach. Six seventh-grade students in a middle-low socioeconomic secondary school in the Marmara Region of Türkiye participated in the study. The researchers prepared an a-didactical activity in which students discovered triangle inequality. Data were collected through observation, video recording, and worksheets. A descriptive analysis was performed on the data obtained. As a result of the analysis, it was concluded that students completed the activity with the correct expressions by experiencing different phases of the a-didactical situation.

Keywords: A-didactical situation; Theory of didactical situations; Triangle inequality

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1. Introduction

The constructivist approach describes both what knowledge is and how individuals acquire it (Fosnot, 2013). With the development of the modern world, it is necessary to transition from traditional education to new educational approaches. In this sense, constructivist education is effective in raising people who are more effective (Saydam, 2009). According to Perkins (1999), the constructivist approach to learning explains how learners transfer knowledge, interpret existing knowledge, create new knowledge, and structure learned knowledge in their minds. Student-centered teaching approaches have emerged through constructivism (Cantürk Günhan, 2006). The constructivist approach can be found in a number of applications in the literature (Bal, 2021; Berkant & Yaren, 2020; Özer, 2019; Yenil, 2020; Yurtyapan et al., 2020). The effectiveness of some approaches and theories is still being researched and tested.

Based on the constructivist approach, TDS is an effective teaching and learning theory (Laborde, 2007). TDS was introduced in the 1960s by the French mathematician Guy Brousseau. This theory is based on the design of a "milieu" and situations that provide students with the opportunity to construct their own knowledge (Erdoğan & Özdemir Erdoğan, 2013). A

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fundamental concept of the theory is the concept of the situation. Based on the theory, there are four types of situations, summarized in Table 1.

Table 1
Four Types of Situations in TDS (adapted from Erdoğan, 2016)

Situation	*Definition*
Didactical Situation	The situation has a clear instructional purpose or function. The subject is explained as a presentation. Students are aware of what will be taught. Additional exercises are given to reinforce the subject.
Non-didactical Situation	The situation does not have a teaching purpose.
Fundamental Situation	A fundamental situation can be constructed for the teaching of each mathematical concept. Fundamental situations exist in the literature only for a few concepts. The fundamental situation was the starting point of the theory of a-didactic situations.
A-didactical Situation	The situation does have a teaching purpose. This purpose is not immediately noticeable to students. The determined goal is attempted to be taught by indirect methods.

Based on these situations, the concept of a-didactic situation is the most basic component of the theory (Erdoğan & Özdemir Erdoğan, 2013). An a-didactic situation consists of four phases: action, formulation, validation, and institutionalization (Warfield, 2006). Prior to these four phases, a devolution phase occurs in an a-didactic learning environment, where the teacher transfers responsibility for the situation to the students (Brousseau, 1997/2002) and encourages them to take responsibility (Warfield, 2006). Table 2 outlines the steps the student takes to assume responsibility.

Table 2
Phases Experienced by Student in an A-didactic Situation

Phases	*Situations experienced*
Action Phase	It is the student's creation of strategies with a trial and error method (Brousseau, 1997/2002).
Formulation Phase	This phase is where the student has a strategy related to the subject and shares the strategy he/she finds with other students (Warfield, 2006).
Validation Phase	This is the phase in which the student proves the strategy they have developed (Warfield, 2006).
Institutionalization	In this phase, the teacher formulates the strategies agreed upon by the whole class and expresses them in the mathematical language (Warfield, 2006).

In an a-didactic environment, students are expected to grasp the fundamental principles of the task (activity, game, problem, etc.) presented to them and come up with the desired strategies (Yavuz et al., 2011). Rather than facilitating student-teacher interaction, an a-didactic environment facilitates students' comprehension of knowledge. Through the feedback they give to the environment and the feedback they receive from the environment, the student solves the problem and discovers the targeted information (Erdoğan et al., 2014). In this way, the teacher facilitates the emergence of knowledge and delegating responsibility to students. This study discusses the triangle inequality in this context. In a triangle, the sum of the lengths of any two sides must be greater than the length of the third side, and the absolute value of the difference in the lengths of any two sides must be less than the length of the third side. This is known as triangle inequality (Erenkuş & Eren Savaşkan, 2019). Based on their study of triangles in an a-didactic environment, Güneş and Tapan Broutin (2017) concluded that eighth-grade students could discover the Pythagorean relations. For their study, Arslan et al. (2011) created an a-didactic environment for finding the center of gravity in triangles. Based on the results, the students were able to determine their center of gravity successfully. An a-didactic environment was created by Ergan (2020) that covered the subject of triangle inequality, triangle drawing, and the Pythagorean relationship. The results of the study showed that a-didactic learning environments allow students to learn independently. It can be concluded from these studies that learning in an a-didactic environment is crucial. In this study, an a-didactic environment was created for the subject of triangle inequality. In this study, the aim was to demonstrate that new environments can facilitate learning and provide insight into how learning occurs in an a-didactic environment.

In the literature, there have been a number of a-didactic environment studies developed in different subjects of mathematics education (Angraini, 2021; Atasay, 2018; Baştürk Şahin et al., 2017; Ercan, 2020; Erdoğan & Özdemir Erdoğan, 2013; Erümit et al., 2012; Gök & Erdoğan, 2017; Rachmiati et al., 2020; Yavuz et al., 2011). According to these studies, it has been found that students learn more effectively in a-didactic environments. This study aims to apply an activity related to achieving triangle inequality within the framework of TDS within an a-didactic environment. We aim to reveal how students' learning takes place in the phases of the a-didactic environment (devolution, action, formulation, validation, and institutionalization). During these phases, the dialogues were examined to learn more about their mental experiences.

In the study, answers were sought for the following problem situation and the following sub-problems.

How do seventh-grade students acquire triangle inequality through a activity prepared according to a-didactic learning environment?

With the activity in an a-didactic environment,

1) What have the students' experienced during the devolution phase?

2) What strategies have been developed by students during the action phase?

3) How have the students shared their strategies with their peers during the formulation phase?

4) What processes have students experienced to verify the correctness of their strategy during the validation phase?

5) What have the students experienced during the institutionalization phase?

2. Method

2.1. Research Design

The case study method was employed for this study, which is based on a qualitative approach. The case study was chosen because it aimed to examine students' processes in depth in light of TDS. Using a holistic approach, the qualitative case study examines the factors (environment, individuals, processes, etc.) related to one or more situations (Creswell, 2021). Through a TDS-based activity, answers were sought regarding how students used their strategies and how peer communication occurred.

2.2. Study Group

Six seventh-grade students (five boys and one girl) from a public secondary school in the Marmara Region participated in this study during the 2021-2022 academic year. Participants were selected using criterion sampling, a form of purposeful sampling. The predetermined criteria included the following: (1) the participants were students who had no prior knowledge of triangle inequality, which was in line with the nature of the study, and (2) they and their parents gave their permission for their participation. The researcher conducted the application rather than the mathematics teacher due to time constraints. Seventh graders who had not yet been introduced to triangle inequality participated in the study. In Table 3, we present the codes of the groups of students without using real names.

Table 3
Group and Participant Codes

Group Codes	*Participant Codes*
Group A	S1, S2, S3
Group B	S4, S5, S6

2.3. Research Design Process

According to the TDS, teaching the Triangle Inequality does not involve the transfer of knowledge from teacher to student. Rather, students must learn to cope with the challenges presented by the designed environment while making sense of the situations they face. As part of this activity, seventh-grade students who were encountering the Triangle Inequality for the first time were introduced to the activity and responsible for learning. During the action phase, the students were able to build their own knowledge by sharing the situations that they encountered. As ideas arise from this sharing, the accuracy of these ideas is discussed, and once the correctness of the ideas has been accepted, the teacher is reintroduced into the activity.

2.4. Data Collection Tools and Data Collection Process

Observations and activities conducted by the researcher were used to collect data in this study. In order to clarify the occurrence of events in their natural environments, observations were conducted (Çepni, 2018). In this study, the observation form prepared by Ergan (2020) was revised, and semi-structured observations were conducted using it (see Appendix 1). Five phases were included in the observation form: devolution, action, formulation, validation, and institutionalization. These categories were created by Ergan (2020) based on student behavior, teacher behavior, and environmental interaction behavior. As a result, we coded the results as 'Y' if they were observed, 'N' if they weren't observed, or 'M' if they were partially observed. Throughout the observation process, the researcher observed participants. Observation by participants, which involves collecting data from first-hand experiences in a research environment, was used (Çepni, 2018). An analysis of the expected behaviors during the activity process was conducted before creating the observation form. For each phase of the TDS, separate observation forms and items were created. An observation form was filled out by the researcher for each student and group during the observation.

The activity, which was another data collection tool, was presented to a faculty member who was an expert in the TDS. In line with the expert's opinion, the necessary changes were made, and the activity was rearranged. In addition, the activity was presented to a secondary school mathematics teacher to ensure that it would be easily understood by students. The points in the plan that were not understood by the teacher (in terms of the language used and examples provided) were also rearranged, and the activity was finalized. The activities related to the acquisition of -*The relation between the two sides' lengths' sum or difference of a triangle and the length of the third side*- are listed in Table 4.

Before the data collection phase, the classroom environment was designed to be suitable for an a-didactic learning environment. The tools (pipettes, ropes, scissors, and rulers) and materials (pencils, paper, erasers, and blank A4 papers) were prepared separately for each group and left on the tables. Students were divided into two groups of three during the application phase. The data collection process began with the implementation of the activity and ended with the collection of information obtained through observation. The entire process was recorded on video to prevent data loss. It took 65 minutes to complete the application.

Table 4
Phases and activities

Phases	*Activities*
Devolution	The teacher determined the lengths of two of the sides of the three sides required to form a triangle (5 cm and 8 cm). Pipettes prepared from 1 to 16 cm have been distributed to each group. After that, the pipettes have been representing the two sides that have determined by the teacher were placed on the student's table with a string threaded in the middle (in that situation, the string was passed through the pipettes to help the corners of the pipettes come together and create a closed area). Students were instructed to form a triangle by trial and error, selecting the third side length from the other given lengths. They have taken notes and were asked to form triangles in addition to that the students were asked to come up with ideas about why they formed triangles.
Action	Students were taking notes by measuring the lengths of the pipettes that form the triangles, and the triangle that weren't formed, by trying the pipettes placed next to the two pipettes that have been determined. They are expected to discover the relationship needed to form a triangle.
Formulation	Students are expected to come together as groups in order to express their individual strategies in a verbal way or in a written way.
Validation	Students were expected to validate whether their ideas were correct about forming triangles different from the pipettes. Allow time for each group to form their own ideas. After each group has reached a consensus, they were asked to come together with other groups and present their ideas to the others.
Institutionalization	The ideas that have been created by the groups were institutionalized by the teacher.

2.5. Data Analysis

In this study, which was prepared according to the TDS, the data collected through observation, video recording, and answer sheets of the students were analyzed using the descriptive analysis method in order to determine the experiences of students in devolution, action, formulation, validation, and institutionalization. A descriptive analysis was conducted to communicate the findings to the reader in an organized and interpretive format; direct quotations are included to express the views of the observed individuals (Yıldırım & Şimşek, 2016). A descriptive analysis method was then used to analyze the data and themes revealing students' behavior in didactic situations.

2.6. Reliability and Validity

Study validity and reliability were ensured using different strategies. In describing the research process, the aim was to include both the characteristics of the participants as well as the characteristics of the environment in a way that would allow for the creation of a similar environment. In keeping with ethical rules, the characteristics of the participants were conveyed. To collect reliable and detailed data, participants were informed that their data would not be shared with anyone or their identities disclosed. Data interpretation was supported by direct quotations from participants' answers.

3. Findings

This section examines student behaviors exhibited during the a-didactic phases of the a-didactic learning environment. Phases of devolution, action, formulation, validation, and institutionalization are outlined.

3.1. Devolution Phase

In the devolution phase, the researcher explained the activity to the students. The steps of the devolution phase were as follows:

- Asking to students measure the lengths of two ropes given to them
- Noting of the lengths of these two sides
- Determining the pipettes from 1 cm to 16 cm as the third side and try to form a triangle
- Noting the sides that form triangles and those that do not
- Coming up with an idea about the causes of triangles having three and sides non-triangles having more or fewer than three sides.
- Forming triangles of different sizes in order to prove the accuracy of these ideas
- Asking students to convey their ideas to the other group members.

The data from the observation form in the devolution phase are listed in Table 5.

Table 5
Observation Form Data in a Devolution Phase

Expected Behaviors	*Group A*	*Group B*
Asking questions to understand the problem	Y	Y
Expressing the problem situation in their own words	Y	Y

Note. Y: Observed.

Table 5 shows that both groups were asked questions in order to understand the problem. S6 was concerned with the pipettes given as the third side, asking, "In the case of these pipettes, is it like this long or very short?" He asked if generalizations would be made, and the researcher said that they could form general ideas. S2 stated, "We will note that these pipettes happen/do not happen in the first step, so..." and wanted to confirm that those who do or do not need to be noted in the first step. To ensure that the students did not have any other questions, the researcher asked if they had any other questions, but the students answered no. A random person from each group was asked to describe the activity. S3 explained the activity, saying, "First of all, there will be a sequence of operations (the 3rd side shows the pipettes), and we will try to find out which one broke the triangle by arranging them from small to large. If this does not form a triangle, we will share our ideas about it." A random student from Group B described the activity in a similar way. Figure 1 shows Group B starting to sort the pipettes after S3, indicating that they needed to sort the pipettes.

Figure 1
The pipettes of Grop A in the devolution phase

Based on their feedback, the researcher assumed the students understood the activity. When the students in Group A began the activity, it became apparent they did not understand they had to form a triangle with the two sides given. During the devolution phase, the researcher asked the students to measure the length of the two sides given as a rope and emphasized that a triangle should be formed according to these sides. Group A did not seem to pay attention to the instructions carefully when it came to forming a triangle according to the two sides provided, indicating that they did not listen to the researcher during the explanation. This may be due to the students not paying attention to the explanation, but rather to the activity materials in front of them. This careless behavior led to the need to reiterate the importance of forming a triangle based on the two given sides, which was emphasized during the devolution phase.

3.2. Action Phase

Students worked willingly throughout the action phase, interacting with media elements (pipette, ruler, scissors) and attempting to find the triangle-forming parts through trial and error. By measuring the lengths of triangular and nontriangular pipettes, students took notes in this phase. Table 6 presents the observation data regarding the action phase.

Table 6

Observation Form Data in Action Phase

Expected Behaviors	*S1*	*S2*	*S3*	*S4*	*S5*	*S6*
Try and see if it works through trial and error	Y	Y	Y	Y	Y	Y
Making suggestions and predictions	M	Y	Y	Y	M	Y
Developing specific strategies for problem-solving	Y	Y	Y	Y	Y	Y
Interact with the environment	Y	Y	Y	Y	Y	Y

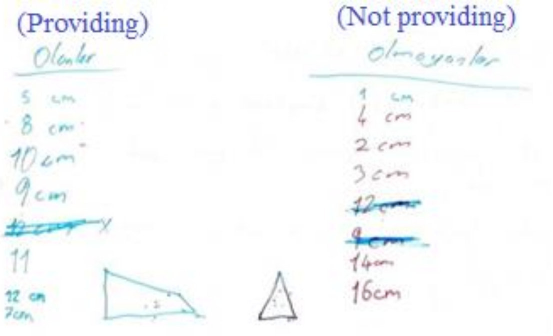
Note. Y: Observed; M: Partially observed.

Table 6 shows that all students used the trial and error strategy in their groups. The students in some groups began making predictions and suggestions sooner than their groupmates. They wanted their groupmates to approve of their suggestions. All students attempted to develop a strategy and did this in interaction with the environment.

Group A began ordering the pipettes from smallest to largest by measuring the third edge of a triangle created randomly with the pipettes provided. After wrapping the rope around the triangle, they attempted to perform the activity. After this process, Group A switched to the trial-and-error method. Figure 2 shows the first trial and error results for Group A.

Figure 2

Trial and error strategy of the Group A



The students did not record some values first in Figure 2, which led to incorrect results. S1 attempted to convince his friends by saying, “Since the short side is 5 cm, it should not be less than 5 cm.” During this process, it was observed that the students tried to connect the sides by pulling ropes. S2 suggested 8 cm by stating “Then it must be 8 cm.” S1 wanted to have his friends confirm what he observed by saying "They are both isosceles triangles."

> S2: Can the triangles exist on different sides?
>
> S3: It can be a right triangle, an equilateral triangle, or a scalene triangle. So, yes, it can be.

The students realized that there could be a scalene triangle and continued the experiment; an image of this process is shown in Figure 3.

Figure 3
An image from the action phase of Group A

Eventually, Group A was able to connect the corners without a rope. After having difficulty connecting the corners to the ropes, Group A asked the researcher if they could remove the rope. They were told that they could do this without the rope since it wouldn't affect the activity. Through trial and error, Group A eventually succeeded in connecting the corners without rope.

S2: It cannot be 12 cm or greater.
S1: Let us try this again
S2: (5-8-12 cm sides attempted to form a triangle) Was it a triangle? The angle at the top is very wide.
S3: The angle at the top may be wide and why not?

Group A was unsure about 12 cm, so S3 suggested to his friends 11 and 13 cm pipettes. Attempts were made on both sides of 11 and 13 cm.

Starting with the shortest pipette, Group B ordered their pipettes from largest to smallest. As a result of sharing the work within the group, S4 performed trial and error with the threaded sides, S5 measured the third side and gave it to his friend, and S6 started to separate the threaded and non-threaded sides. While performing trial and error, S6 took a longer pipette (lengths from 4 to 12 cm) after switching to pipettes that provided the third side and reasoned, "If this happens, so will the others." S4 took the pipette shown in S6, measured it (12 cm), and checked whether it formed a triangle by threading it on the string.

S5: This value is quite large.
S4: But the triangle (counting the sides) 1 2 3
S6: Try this (Takes a 13cm pipette)
S5: Does this happen?(Shows 12 cm pipette)
S4: Can I address this question? (Takes a 1 cm pipette)

S4 tried the 1 cm pipette again without considering what his other friends said and accepted that it was not successful. He then showed this to his friends and confirmed that it had not worked. Meanwhile, S6 wanted to take other pipettes smaller than 12 cm (not including 1, 2, and 3 cm, which they had assumed were not available in the beginning) and put them in the group of pipettes. His groupmates tried to dissuade S6 by saying that perhaps it would not be successful.

S6: As a result, it is larger than this (3 cm) and smaller than this (12 cm)
S5: Let us try this by selecting the middle one among them.

S4 started to try all of them, and his group friends joined him. S4 put forward a suggestion that by adding 5 and 8, 13 is not equal to 13. In response to S6's desire to generate ideas and make judgments, his groupmates continued to make trial-and-error guesses.

3.3. Formulation Phase

In formulation, the students shared their ideas in groups to come up with a solution. Formulation and action took place simultaneously. Table 7 shows the data from the observation form during the formulation phase.

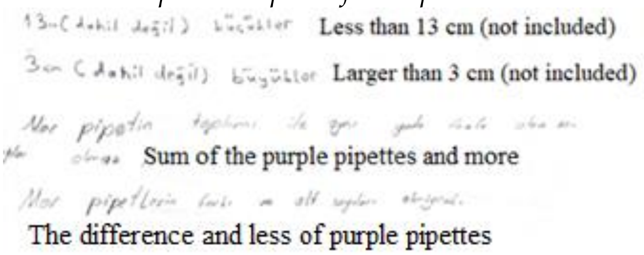
Table 7
Observation Form Data in a Formulation Phase

Expected Behaviors	*S1*	*S2*	*S3*	*S4*	*S5*	*S6*
Expressing individual strategies verbally or in writing	M	Y	M	Y	M	Y
Suggesting verbal strategies for group problem solving	Y	Y	Y	Y	Y	Y
Suggesting algebraic strategies for group problem solving	N	N	N	N	N	N

Note. Y: Observed; M: partially observed; N: Not observed.

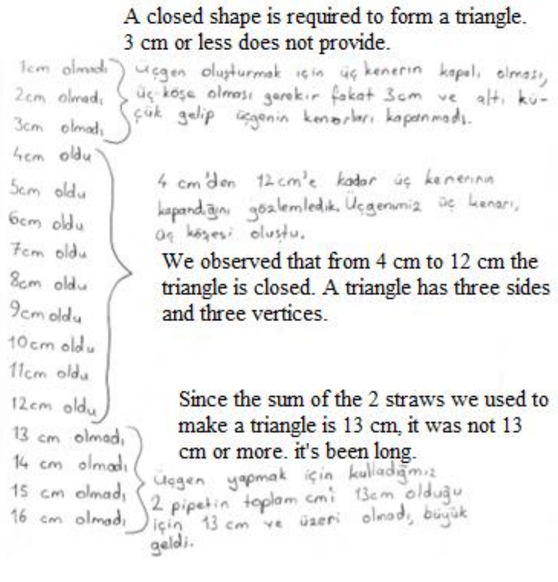
Table 7 shows that students generally suggested strategies individually. Both groups developed verbal strategies by sharing these strategies. Figure 4 illustrates the verbal expressions reached by Group A through trial and error strategy.

Figure 4
Formulation phase outputs of Group A



As seen in Figure 4, Group A concluded that the smaller ones, excluding 13 cm, and the larger ones, excluding 3 cm, could be the third side. It was observed that Group A verbally expressed the triangle inequality correctly. At had forming first, difficulty they triangles, but they quickly resolved the issue. Afterward, it took them a short time to reach a solution. Figure 5 shows the paper on which Group B wrote providing/not predictions of their providing.

Figure 5
Predictions of Group B



In Figure 5, the students in Group B wrote that a triangle must have all three sides closed. According to them, lengths of 1, 2, and 3 cm did not meet this requirement. The students developed a strategy by writing that the sum of 8 and 5 is 13 for lengths greater than 13 cm.

3.4. Validation Phase

To validate their own ideas, each group applied the strategies they had discovered in new situations after the formulation phase. Following their final conclusion on their own ideas, another group was allowed to discuss their ideas. Validation and formulation took place simultaneously. Table 8 presents the observation data.

Table 8
Observation Form Data in a Validation Phase

Expected Behaviors	*S1*	*S2*	*S3*	*S4*	*S5*	*S6*
Engage with the environment	Y	Y	Y	Y	Y	Y
Defend your hypothesis	M	Y	Y	Y	Y	Y
Adapting the strategy to other situations	Y	Y	Y	Y	Y	Y
Experimenting in a group to examine the validity of the hypothesis	Y	Y	Y	Y	Y	Y
Experimenting to examine the validity of the hypothesis from the other group	N	N	N	N	N	N
Accept or reject the proposed strategy	Y	Y	Y	Y	Y	Y
Expecting support from the teacher in the process of accepting and rejecting strategies	Y	Y	Y	Y	Y	Y

Note. Y: Observed; N: Not observed.

A review of Table 8 revealed that all students participated actively in the validation phase and were working on the strategies found. As shown in Table 8, the students had not attempted to confirm the hypothesis of the other group. The reason for this was that they accepted that they had achieved the same results.

According to Group A, the third side should be shorter than the sum of 5 and 8 cm and longer than the difference. As S1 was hesitating, his groupmates took the pipettes and formed a triangle again in order to prove what they had proposed.

S2: Look at how it works. When it’s 13, the shape is incomplete. (They showed 5-8-13 cm lengths)
S3: Equals the size of the other two pipettes. (Showing pipettes)
S1: What you mean is that it will not happen if it is greater than the sum of the other two pipettes, but if it is less.
S3: Think like that, okay? (With pipettes in hand showing that the sum of 5 and 8 cm is equal to 13 cm) This was equal to its height. (S1 approves S3) (Trying to create angles with 5 and 8 cm pipettes) Each time we removed them, the pipette remained short. That is why this does not occur.
S1: Therefore, it will be shorter than 13.
S2: It is going to be shorter than 13; it is going to be, like 12.

Groupmates convinced their friends through trial and error, using pipettes. S1 told his friends that he was convinced by saying, “Then it has to be greater than 3 cm and will be less than 13 cm.” Group A students wanted to discuss these strategies with the researcher and have the researcher confirm what they said. The researcher asked the students whether the strategy they found was valid for new situations. The students then turned to the uncut pipettes to try out a new situation..

S1: Let’s cut 7 cm from the purple pipette.
S2: Let the pipettes be 8 and 10 cm.
S3: I think there should be some differences between the lengths of them.
S1: Let them to be 8 and 12 cm.

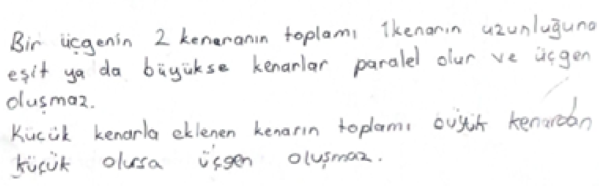
S2 and S3 should agree on the sides. S3 cut the pipettes; however, S2 insisted that the sides should be 10 and 12 cm. Group A created a new situation by defining new sides of 10 cm and 12 cm. Before S2 starts trial and error, S2 and S3 should agree on the sides,

S2: The difference of the pipettes will not be taken, their total will be taken.
S1: Can we try?
S3: It can be a pipette with 3 cm lenght, but not 2 cm.
S1: Let's try 2 and 1 cm pipettes. If that doesn't work, we'll have proven our theory to be correct.
S2: Yes, it is definitely not working. (By trying 1 and 2 cm pipettes)
S3: I think the pipette with 22 cm lenght also doesn’t work.

Having made repeated trials and errors, they informed the researcher that they had found their theory and that they were ready to share it with the other group.

The students in Group B concluded that, in the formulation phase, the length of the third side should be less than the sum of 5 and 8 cm. However, they developed a different perspective on the differences between 5 cm and 8 cm. The conclusions drawn by the group from this point of view are shown in Figure 6.

Figure 6
Final Statement of Group B



(If the sum of the 2 sides of a triangle is greater than or equal to the length of the third side, the sides are parallel and no triangle is formed.)

(If the sum of the side added by the short side is shorter than the long side, then a triangle is not formed.)

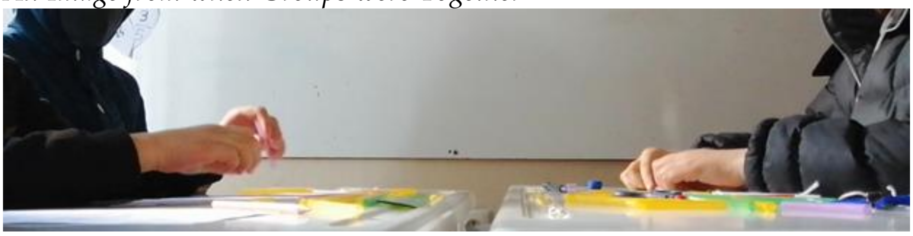
As can be seen in Figure 6, for the lower bound of the triangle inequality, the students have formed a correct proposition by stating that the sum of the added third side and the 5 cm long side is not less than 8 cm. They began creating new situations to test the theories they found.

S6: We can choose one of them as four and the other as five.
S4: Shall that we make smaller or bigger?
S5: But let us do this. How many of these are there? (S4 starts counting what he wrote on the paper) Look, let's make it bigger than 16 and make it smaller than 9 cm.
S4: (Cuts a 5 cm pipette from a pipette) let the other side be 5 cm, (takes the 5 cm pipette from the given pipettes)
S6: Why did you choose both of them of the same length? The lengths were 5, 5, and 10.

Having been told by S6 that it should not exceed 10 cm, Group B began trial and error. Following trial and error, they concluded that it cannot exceed 10 cm. Their conclusion was that it should not be longer than their total, so they told the researcher. For the new situation, the proposition they developed in Figure 6 showed that adding the third side to the existing 5 cm long side will make it longer than the other 5 cm long side. The researcher then suggested they choose between two sides of different lengths. Group B chose two sides of different lengths and stated that the sum of the other two sides could not exceed the longest side. Group B was sure that their theory was correct. They told the researcher that they had found their theory and were ready to share it with the other group.

Once both groups had completed their processes, they came together to share their ideas. In this process, Group A wanted to start by saying they had finished first. Figure 7 shows the moment they come together.

Figure 7
An Image from when Groups were Together



Group A began explaining their theories on the board.

S2: Two pipettes were provided. These are 5 cm and 8 cm; for example, a pipette is 8 cm, right? How many centimeters is the other pipette? (Everyone says, 5) The sum of this and this is 13; their difference is 3 cm. (Draws lengths representing 5 and 8 cm on the board) If the third side is 3 cm, a triangle will not form.
Group B: That's right, we found that there won't be 3 either. (all say at the same time)

S2: Because their difference is 3 cm, there must be something higher than that (other group members approve of). Now, our friend will explain you the sum of lengths (he gives the word to S1, but S3 interrupts).
S3: We found that it must be larger than 3 cm. If it is shorter than 3 cm, the other shapes are not close. (He gives the word to S1 by pointing with his hand)
S1: We sum the lengths of 8 and 5 to 13 cm. It must be shorter than 13 cm, as 13 cm is the sum of these.
S4: Yes. This is how we found it.

Group A explained their experience to Group B. They stated that the third side should be less than the sum of the lengths of the two sides and greater than the difference. The other group claimed that they found something different and asked for permission to explain.

S6: You said the sum; we found something else.
S4: If the long side of the triangle is equal to the sum of the other two sides, the triangle will not form. This is how we wrote. (lays on the paper and looks) If the sum of the short side and the added side is shorter than the long side, then it does not form a triangle. So it should be bigger. We found results similar to ours or even the same.

Group B also transferred their ideas to other group. The group accepted the opinions of other group. Both groups agreed that their opinions were identical. Students verbally developed the correct strategies. These verbal expressions were then transformed into formal forms by the researcher during the institutionalization phase.

3.5. Institutionalization Phase

During this phase, the researcher explained verbally the situations expressed by the students. It was asked of the students whether they could express verbally expressed strategies algebraically. It was generally agreed by the students that they could be, in general. The researcher then said "Let a and b be given as two sides. You have said that the third side must be greater than the difference. Since the length cannot be negative, can we take it to absolute value?" After all students answered yes, the researcher continued: “It had to be shorter than their sum. Let the third side be c. Then we can express as follows (writing on the board $|a - b| < c < a + b$).” The observation data for the institutionalization phase are presented in Table 9.

Table *9*
Observation Form Data in the Institutionalization Phase

Expected Behaviors	*S1*	*S2*	*S3*	*S4*	*S5*	*S6*
Making sense of the information that has been transformed into a formal form in one's mind	Y	Y	Y	Y	Y	Y
Expressing the target information in their own words	M	M	Y	Y	M	M

Note. Y: Observed; M: partially observed.

As shown in Table 9, all students expressed that they made sense of the formulated information. In each group, one person summarised the event again, and their friends confirmed their friends' conclusions by confirming their own. With the students' acceptance, the activity came to a close.

4. Discussion and Conclusion

Using an activity prepared within the TDS framework to acquire triangle inequality in an a-didactic environment, the purpose of this study was to report on the experiences during the application process. Students successfully completed the triangle inequality activity by using the correct expressions. During the devolution phase of TDS, necessary preparations were made, students were left face-to-face with the activity, and they were assigned tasks. During the research process, the devolution phase was not fully accomplished. Today's children are used to either a purely constructivist or a testing environment. Devolution lies between these two phases. Due to the fact that it was a-didactic engagement that students were not used to, this phase may have taken place along with the action phase. A similar problem was reported by Arslan et al. (2013).

The action phase involved trial-and-error using different strategies used by the groups. However, the devolution phase was not fully accomplished during the research process. It was observed that with the successful realization of the action phase, the formulation phase was also realized correctly. Not all the strategies expected in the formulation phase emerged. When the formulation phase began, the formulation and validation phases were realized as the students quickly validated their ideas about triangle inequality. In the validation phase, the groups correctly confirmed their strategies by choosing different values. After the groups were sure that their ideas were correct, they came together and transferred their strategies to the other group. In this phase, although the students had to present evidence and try to convince the other group, one of the expected behaviors in this case was skipped because of similar results.

During institutionalization, the researcher formalized the students' statements. All of these processes were successfully completed. Students have successfully performed similar situations in similar studies (Aktaş, 2019; Ergan, 2020; Gök & Erdoğan, 2017). Although the students achieved successful results, similar to studies in the literature, the fact that they did not exchange ideas during the validation phase revealed a different outcome. The reason for this is that both groups completed the activity at the same level in this study, which was conducted with intermediate- and high-level students. This indicates that, in some cases, learning can be achieved effectively without expectations in the validation phase.

Although the phases of TDS occurred separately and independently, they progressed together in this study. Previous studies have reported similar results. It was noted by Erdoğan and Özdemir (2013) that the processes of action and formulation were simultaneous. Baştürk Şahin et al. (2017) revealed in their study that devolution, action, and formulation phases occur together. In this study, part of the devolution was repeated during the action phase. In addition, it was concluded that students, in which the action, formulation, and validation phases occur together, need to look back and test their ideas. Similar studies have shown that students transition between situations (Aktaş, 2019; Arslan et al., 2011; Erdoğan et al., 2014).

Based on all situations, it was observed that all expected student behaviors were realized as expected. Similar results were obtained by Ergan (2020), who concluded that student behavior met the expected situations. It is also similar to the findings of this study that informal information produced by students during the validation phase was formalized by the teacher during the institutionalization phase. It is evident from the fact that the researcher was only active during the devolution and institutionalization phases that students can construct their own knowledge. As a result, it has been demonstrated that students can discover triangle inequality through prophylactic activity conducted within the framework of TDS.

In our observation, students had no problems forming triangle inequality. In contrast, Ergan (2020) attempted to find a triangle inequality within the framework of TDS with different activities. However, the groups could not conclude whether the length of the third side should be greater than the difference between the two sides. In her study, Türnüklü (2009) found that students who lack the concept of a triangle have difficulties forming triangle inequality. The students participating in this study generally recognized the triangle and knew its properties during the activity period. As a result, the triangle inequality was not difficult to form for the students. The success of the students in the designed educational environment is in line with the findings of Çiftci's (2018) study, where a learning environment effectively prevented students' difficulties and increased their academic success.

Upon reviewing the results of the study, the researchers highlighted some limitations. First, only six secondary school students were included. Due to the presence of two groups in the study, different opinions may not have emerged during validation. By increasing the study group, different results could be obtained from different groups, and this may allow the behavior of conducting experiments to examine the validity of the hypothesis from the other group during validation. Secondly, different activity environments can be prepared to test students' understanding in an artificial environment.

Author contributions: At the completion of the study, the contribution rate of the first author is 40%, the contribution rate of the second author is 30% and the contribution rate of the third author is 30%.

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Appendix 1. Observation Form

Student code:

The behavior of the students in the a-didactic phases will be examined in a classroom environment created with an a-didactic learning environment. The behavior patterns below reveal possible student behaviors that are expected to occur in a-didactic phases to find determinations suitable for the purpose of the research. If the behaviors in the form are observed, 'Y' will be marked; if they are not observed, 'N' will be marked; and if they are partially observed, 'M' will be marked.

A-didactical Situation	*Expected Behaviors*	*Y*	*N*	*M*	*Explanation*
Devolution Phase	Asking questions to understand the problem				
	Expressing the problem situation in their own words				
Action Phase	Try and see if it works through trial and error				
	Making suggestions and predictions				
	Developing specific strategies for problem-solving				
	Interact with the environment				
Formulation Phase	Expressing individual strategies verbally or in writing				
	Suggesting verbal strategies for group problem solving				
	Suggesting algebraic strategies for group problem solving				
Validation Phase	Engage with the environment				
	Defend your hypothesis				
	Adapting the strategy to other situations				
	Experimenting in a group to examine the validity of the hypothesis				
	Experimenting to examine the validity of the hypothesis from the other group				
	Accept or reject the proposed strategy				
	Expecting support from the teacher in the process of accepting and rejecting strategies				
Institutionalization Phase	Making sense of the information that has been transformed into a formal form in one's mind				
	Expressing the target information in their own words				