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Research Article

Investigating effective teaching practices in advanced placement calculus AB: A qualitative exploration of student recounts

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It is evident that more qualitative research on Advanced Placement [AP] classrooms for students from marginalized backgrounds is needed to uncover how AP practices and pedagogies might better serve students who had been historically excluded from the AP Program. I asked my former calculus students (32 total – 22 participated) from my last three years of teaching to complete a survey that was developed with open and closed-ended questions to explore the student experience in their AP Calculus AB course. I aimed to investigate the specific practices and encouragements that are most impactful in helping students meet the demands of the AP Calculus curriculum. Using this AP program as a phenomenological study, my intention was to gain insights that could support educators in helping students meet the demands of a challenging course. I used a six-phase thematic data analysis to make sense of and create a report on the corpus of data collected. After conducting the thematic analysis of the coded excerpts, seven themes and a variety of subthemes emerged: Participants felt supported to meet the demands of the course; they reported that the extended class time provided the necessary duration to master the course concepts; they expressed value in the variety of In-Class Tasks; they found the Out-of-Class-Time Tasks challenging but helpful to their learning; they conveyed great benefit from the overall organization of the course; they expressed gains in transferable skills; and they described a variety of struggles or barriers to success in the course. The findings are presented in conjunction with representative excerpts that were extracted verbatim from the survey and indicate that an AP course should provide targeted support to be successful on the AP Exam, but it should also provide opportunities to develop deep understanding of concepts and procedures through a variety of modalities to positively impact students' future success in college. The results are discussed in connection to relevant mathematics education literature and to implications and future directions of related research.

Keywords: Advanced Placement Calculus; Pedagogical practices; Mathematics teaching and learning; Curriculum; High school to college transition

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1. Introduction

It is well documented that low-income students tend to underperform on Advanced Placement [AP] examinations when compared to their more affluent peers (e.g., College Board, 2014; Dougherty et al., 2006; Hallet & Venegas, 2011), but what is less clear is how AP teachers can help improve these student outcomes in underserved urban communities. Importantly, we are missing

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the voice of former AP students and the instructional practices that they believe to have been greatly beneficial or detrimental in their preparation for AP Exams. While there's been some reports on general best practices for AP programs (e.g., Hanover Research, 2014), there is less research that focuses on the student experience, particularly what low-income students encounter in these courses, despite the recent growth of AP in marginalized communities (Kolluri, 2018).

The College Board's AP Program enables high school students to pursue college-level studies by enrolling in AP courses. These classes have been developed by college faculty and secondary teachers, and "emphasize challenging, research-based curricula aligned with higher education expectations" (College Board, 2020, p. 1). While the Course and Exam Description for each AP class present a framework of the content and skills that are the focus of the corresponding college course and that appear on the optional culminating AP Exam, individual teachers are responsible for designing their own instructional materials – selecting a suitable textbook and choosing or creating appropriate college-level assignments, assessments, and resources. Prospective teachers must also submit a course syllabus to the College Board for approval before being allowed to teach an AP course (Finn & Scanlan, 2019).

My primary intention is to try and determine what's helping and missing – what do AP Calculus AB[2] courses, particularly in urban low-income neighborhoods, need to do more or less of, to simultaneously prepare students for the AP Exam and college-level work? The more specific research question that I will be exploring is: Which classroom structures and in/out-of-class supports do students perceive to best prepare and engage them to meet the expectations of the AP Calculus curriculum and exam, as defined by the College Board?

2. Relevant Literature

In addition to evaluating syllabi of AP courses, the College Board also regularly reviews and revises their own AP course frameworks by conducting a full-scale college curriculum survey of up to 200 colleges and universities, at least every four years, to ensure that the AP classes reflect the changing content and instructional practices of the associated college classes (Rothschild, 1999). AP Exams undergo extensive review, revision, piloting, and analysis in a multi-year endeavor by the AP Test Development Committees (Finn & Scanlan, 2019) to "ensure that questions are accurate, fair, and valid, and that there is an appropriate spread of difficulty across the questions" (College Board, 2020, p. 2). The College Board and Educational Testing Service (ETS) jointly put much effort into the psychometrics of the AP Exams by administrating questions to samples of actual college students to ensure that a passing score of 3 (scored 1-5) is equivalent to passing the corresponding freshman college course – although many universities now require a 4 or even a 5 to earn college credit (Finn & Scanlan, 2019). What may be more significant about passing an AP Exam, however, is the fact that "research indicates that students who score a 3 or higher on an AP Exam typically experience greater academic success in college and are more likely to earn a college degree than non-AP students" (College Board, 2020, p. 1).

2.1. The Role of the Advanced Placement (AP) Program

Sadler and Sonnert (2010) provide some evidence behind the research that the College Board may be referring to – they used survey data from 4,207 undergraduates in 124 different first-semester introductory college science courses from a nationally representative sample of 55 small liberal arts colleges and large state universities, and concluded that students who "reported passing their AP exam earned college grades that were significantly higher than those of students with other experiences" (p. 133). In addition, their findings showed that for students not passing AP science exams, AP offered little to no benefit to students beyond regular or honors science courses, which should caution schools from overly offering AP courses if students are not passing the associated

[2] I will refer to the AP Calculus AB course as AP Calculus for fluency.

AP Exams. It appears then, that to truly benefit from the AP Program, students must not just be given access to an AP course, but to one where students are consistently passing the AP Exam.

Geiser and Santelices' (2006) regression analysis similarly showed that "whereas AP coursework, by itself, contributes almost nothing to the prediction of college performance, AP examination scores are relatively strong predictors" (p. 93). Their results indicated that AP Exam scores held greater predictive weight of college performance than any other factor (school academic performance index quintile, parents' education, SAT I and II scores, and number of AP/Honors courses taken) except high school grades. Their findings likewise suggest that to improve college performance among low-income urban students, it is necessary not only to expose them to AP courses but also to ensure they gain the necessary knowledge and skills needed to pass the corresponding AP Exams.

Early exposure to the high expectations of college classes through AP courses and being able to pass the corresponding AP Exams might not only improve college performance for students but also promote their college persistence. In a longitudinal study, Dougherty et al. (2006) found that "the percent of a school's students who take and pass AP exams is the best AP-related indicator of whether the school is preparing increasing percentages of its students to graduate from college" (p. 13). Their findings indicate that in ensuring students enroll in AP courses and pass the associated AP tests, high schools can improve the college graduation rates among its students, which is particularly important for schools in low-income urban communities. Unfortunately, they concluded "that although the percentage of low-income and minority students taking Advanced Placement courses and exams has risen encouragingly, the percent of those students passing AP exams is still disappointingly low" (p. 14). In their data, for instance, only about one in eight low-income students who took one or more AP courses in English, mathematics, science, or social science actually passed any of the accompanying AP tests. AP Calculus is of particular importance because approximately 20% of high school students now take some form of calculus in high school, primarily through the AP Program, and this course is especially impactful on their views and preparation for a science, technology, engineering, and mathematics [STEM] education (Bressoud, 2021).

2.2. Equitable Access to Advanced Placement (AP)

While AP has vastly expanded to serve a wide range of marginalized students, inequities persist. Studying a panel dataset of all California public high schools from 1997 to 2006, for instance, Klugman (2013) found that despite AP access across all races and income levels in California, schools serving advantaged students have expanded their AP participation at greater rates than schools serving traditionally underserved populations. Relatedly, in an exploratory examination of College Board data from 1996 to 2012, Judson and Hobson (2015) found that the expansion of the AP Program has been associated with notable decreases in AP Exam pass rates – the percent of students passing with a score of 3 or higher has decreased from the 1992 level of 65.5% to the 2012 level of 59.2%, which was a statistically significant downward trend. In addition, the proportion of students receiving a 1 on AP Exams has doubled from 10% in 1992 to 20% in 2012. The decline in pass rates begs the question of which groups of students are not passing AP Exams.

Judson and Hobson's analysis show that between 1997 and 2012, the AP Exam pass rates for White and Asian students have remained fairly steady with a pass rate of about 65% and 68%, respectively. There have been statistically significant declines in the AP Exam pass rates for Hispanic, Black, and American Indian students, however, especially for Hispanic students whose AP Exam pass rate has decreased from 61.1% in 1997 to 42.8% in 2012. These findings suggest that while there's been large increases in the participation in the AP Program among the groups of students who have typically been considered underrepresented in higher education, many of them are unable to achieve the benefits of passing an AP Exam.

Similar inequities persist for low-income students more broadly. In their statistical exploration of AP participation, Theokas and Saaris (2013) found that a 10.1% national gap in AP Exam

participation rates exists between low-income students and students who are not from low-income families. What's arguably more troubling, is the quality of AP courses that low-income students have access to. Across the nation, the College Board (2014) noted that although the expansion of AP has more than quadrupled the number of low-income students from 58,489 AP examinees in 2003 to 275,864 in 2013, only 21.7% of AP exam takers scoring a 3 or higher during high school were of low-income background – despite making up nearly half of the U.S. public school population (48.1%). In his review of educational research related to the expansion of the AP Program over the past several decades, Kolluri (2018) summarized that the "expansion of AP to serve more students from groups who have historically been excluded from the program has led to curricular challenges. Many students fail to pass the test. The failure rate presents troublesome realities for AP as a means for improved college readiness" (p. 704).

2.3. The Advanced Placement (AP) Course Experience

As Kolluri (2018) points out in his educational review, all of the statistical findings suggest that more underrepresented students are engaging with AP but may be unable to truly benefit from the passing of AP Exams. He admits that the qualitative research of AP pedagogies at schools serving low-income students is limited, urging for new approaches to AP research that investigates the challenges of AP support for students of marginalized backgrounds and the ways in which AP pedagogy aligns with the development of college readiness among these students. In one of the few qualitative investigations of AP classrooms, Hallet and Vanegas (2011) interviewed 48 highly motivated, college-bound students from 15 different low-performing high schools in the Los Angeles metropolitan area and concluded that:

> Academic preparation offered within these classroom settings did not lead to exam success. There were meaningful differences between doing well in an AP class and passing an AP exam. Finally, these differences may be explained by teacher preparation, school structures, and a general lack of support for the AP program within these urban school settings. (p. 485)

More specifically, their data showed that students received a 4 or above on only 16% of the attempted exams. In comparison, the College Board reported that 33% of students earned a 4 or above on AP Exams nationwide. Further, students in their study averaged a course grade of 4.31 (equivalent to a grade of B/B+) in their AP courses; however, their AP Exam scores were significantly lower, with an average score of 2.42 (or a grade of D+).

Using a national random survey of 1,024 participating AP teachers and four focus groups with AP teachers, Duffett and Farkas (2009) similarly found that "in high-poverty schools, only 25% of AP teachers say most of their students score a 3 or better; 70% of AP teachers in low-poverty schools report likewise" (p. 18). This outcome, the survey shows, may be attributed to the fact that high-poverty schools are also less likely to field highly experienced AP teachers in their classrooms.

This issue is perhaps a large reason why Kolluri's (2018) scholarship review of the recent expansion of the AP Program yielded evidenced that "strongly suggests that the AP program is yet to achieve its dual goals of equitable access and effective college-level skill development" (p. 698). Kolluri's analysis indicates that although access has been admirably expanding to increase participation among low-income students and students of color, inequities continue to persist. It is evident that more research on AP classrooms for students from marginalized backgrounds is needed to "uncover how AP practices and pedagogies might better serve underrepresented students" (p. 701). Teacher experience and preparedness to teach AP, then, seems critical to ensuring low-income urban students are successful in passing AP Exams. What is less clear, however, is what exactly constitutes effective AP teaching.

2.4. Studies on Advanced Placement Instruction

Kyburg et al.'s (2007) more broad investigation of best practices in AP programs found that in classrooms that fostered the growth of academic talent among students of diverse backgrounds,

teachers seemed to recognize the importance of demonstrating an exemplary level of commitment to their practice. They understood that supplemental scaffolding was necessary for students in their classes who enter less well-prepared for the rigors of advanced study. Teachers maintained high academic standards, for example, but recognized that some students might require more and different kinds of assistance. Teachers adopted strategies to nurture achievement by extending class time, holding one-on-one conferences, providing timely feedback, treating students as adults, engaging in culturally sensitive teaching practices where possible, and helping students develop learning strategies and missing skills (pp. 199-200).

In one of the few published case studies of AP instruction, specifically, Henderson et al. (1996) compared more (2) and less effective (2) AP American History teachers and indicated that the more effective teachers, measured by student AP Exam scores, made slightly higher use of instructional time (42.7 vs. 39.3 minutes), had a much higher percentage of student engagement (96.9% vs. 66.2%), asked considerably more questions (57.1 vs. 11.8) with a higher success rate (93.9% vs. 81.9%), had a particularly higher participation rate (62.2% vs. 22%), and had markedly more graded assignments (42.5 vs. 28.5). The authors conclude that the results "suggest that more effective teachers think about and organize their subject knowledge differently, and induce their students to think differently as well" (p. 34).

2.5. Related Mathematics Education Programs and Research

Although there is very little research on AP Calculus instruction, specifically, there are relevant calculus intervention programs, related mathematics education research, and studies of ambitious teaching and learning in high school that are important to discuss. In the 1970s, for instance, the very successful Emerging Scholars Program [ESP] model, as described by Hsu et al. (2008), was developed in response to ethnographic research at the University of California, Berkeley, which described the contrasting study habits of Black and Chinese calculus students. The Black students, who were underachieving in calculus, tended to study in isolation, while their higher achieving Chinese classmates, on the other hand, were more likely to form "cohesive support groups that factored heavily in the ability of these students to navigate the system" (p. 207). The primary goal of the ESP is to increase student achievement among minority students by creating a collaborative community of active learners who work on challenging mathematics together in workshops of 12-20 students, for 75-120 minutes, 2-3 times per week.

While it may seem that after so many years of work on the ESP, there might now be a recipe to optimally design a local ESP. Unfortunately, because of the diversity of local situations, there does not appear to be any concrete findings for the successes and failures at different sites (Hsu et al., 2008). Even though there may not be a simple formula for designing a sustainable ESP – from studying a branch of an ESP, Adiredja et al. (2008) hypothesize that what may lead to the increase in student achievement is the fact that the program: (1) *dedicates time* for students to work on challenging tasks to develop fluency with mathematical language and gain both a conceptual and procedural understanding of calculus; (2) the sections *support students' self-efficacy*, leading to more effort and persistence with mathematics; (3) students *work actively* in small groups, where they discuss and engage with one another's ideas; and (4) the sections provide *a safe space* for students to incorporate their personal identity in developing their academic identity. By emphasizing excellence, diversity, and community, the ESP aims not to fix students, but rather to provide them with a welcoming environment where they can thrive academically by working together.

The complexity of the ESP model is in line with Schoenfeld's (2022) exploration of why teaching and learning mathematics is so difficult. To better understand these intricacies, Schoenfeld presents the Teaching for Robust Understanding [TRU] Framework, which identifies five dimensions of powerful mathematics classrooms: The Mathematics; Cognitive Demand; Equitable Access to Content; Agency, Ownership, and Identity; and Formative Assessment (Schoenfeld, 2022, p. 777). I will later use the TRU Framework to discuss the findings by relating them to relevant literature and studies. For example, in their investigation of current research about

assessment, Suurtamm and Arden (2017) convey several important aspects surrounding the meanings and importance of assessment. First, an assessment cannot be viewed in isolation because it is a multidimensional process for examining what students know and can do – and then using this examination to further enhance teaching and learning, which should be the central purpose of assessment (pp. 141-142). Second, because views on classroom assessment have shifted from viewing assessment as an event that measures the acquisition of knowledge toward a view of assessment as a social practice that provides continual information to support learning, teachers must provide a range of opportunities (a variety of assessment strategies and tasks) for students to show the complex way that they engage in mathematics, and pay close attention to their thinking throughout the learning process (pp. 142-143). Finally, and relatedly, classrooms should engage students in a range of mathematical practices and processes to support their development of conceptual understanding, procedural fluency, and mathematical reasoning (p. 144). Upon presenting my findings, I will discuss them with relation to the TRU framework and to related research studies, such as the one on assessment by Suurtamm and Arden (2017), to connect my results to related literature.

3. Motivation and Researcher Positionality

I asked my former calculus students (32 total – 22 participated) from my last three years of teaching (12,8, and 12 students, respectively, were enrolled in the class during these three years) to complete a survey, that I developed, to explore the student experience in this course. By surveying former AP students of mine, this study aims to capture student's experience before and while taking AP Calculus to explore how these experiences shaped their preparation for the challenge of meeting the demands of the course. The primary goal was to systematically identify AP instructional practices that are associated with increased test performance because, as Kolluri (2018) notes, while many studies have documented the challenges of AP programs at urban schools, only one study (Hallett & Venegas, 2011) actually spoke to students from those schools. "Students may be able to articulate how their academic abilities and school resources affect their ability to succeed in AP classes. In particular, investigating students from underrepresented backgrounds who find success in AP programs might suggest whether their success can be replicated for other students in similar circumstances" (Kolluri, 2018, p. 703). Through his analysis, Kolluri identifies a research gap in the qualitative study of AP courses in low-income communities and I believe that this study helps contribute to this research gap and may simultaneously help improve the effectiveness of AP courses in low-income urban communities.

3.1. Researcher Positionality

During my seven years of high school teaching (2013-2020), I developed a relatively successful AP Calculus program in a low-income urban community of Southern California. The program was successful, in one sense, in that the large majority of students passed the AP Exam (at least 80% during each of my last four years of teaching). Many AP programs in these types of neighborhoods, however, are not having the same type of success (Finn & Scanlan, 2019; Hallet & Vanegas, 2011), and I aim to investigate the specific practices and encouragements that are most impactful in helping students meet the demands of the AP Calculus curriculum. Using this AP program as a phenomenological study, my intention is to gain insights that could support educators in helping students meet the demands of a historically difficult course.

With my effort and dedication to make appropriate modifications to the activities, assignments, and assessments in my curriculum over the years, I developed a great sense of what it takes to run an AP program at a low-performing small public school that leads to passing scores on the AP Exam. By surveying my former calculus students, I intend to more closely investigate what students experienced in their AP Calculus course and how these experiences shaped their preparation for the AP test. Since these participants are former students of mine, I believed that it would be easy for them to trust my purpose with the information they provide, as they've

witnessed my commitment to adequately prepare some of our most motivated underserved students for the rigor of college.

4. Methods

Members of the Middle-school Mathematics and the Institutional Setting of Teaching [MIST] project collaborated for four years with four urban school districts that had set ambitious goals for reforming mathematics instruction. Through an analysis of the teacher interview transcripts from the MIST project, Munter (2014) developed rubrics for assessing a teacher's level of Vision of High-Quality Mathematics Instruction [VHQMI]. The VHQMI rubrics were developed through an informed review of the relevant literature and provide sufficient details to distinguish between the three proposed related dimensions of classroom instruction: Role of the Teacher, Classroom Discourse, and Mathematical Tasks. I drew from these three dimensions and associated rubrics to help create a survey that explores the practices, structures, and supports that students find to be most impactful in helping them meet the expectations of the AP Calculus course framework. The rubrics aided in focusing my questions on the three different concepts that Munter (2014) proposed. I also created an additional dimension, Out-of-Class-Time Practices, to include out-of-class-time supports because AP classes typically require students to spend substantial time practicing, learning, and mastering the course content outside of classroom time. Figure 1 displays a visual representation of the four constructs, and related questions to each construct, that I used to help categorize the different AP structures and supports of an AP Calculus program. I also made sure to incorporate two important elements that also affect student learning: Prior Experience and External Factors.

4.1. Survey Development

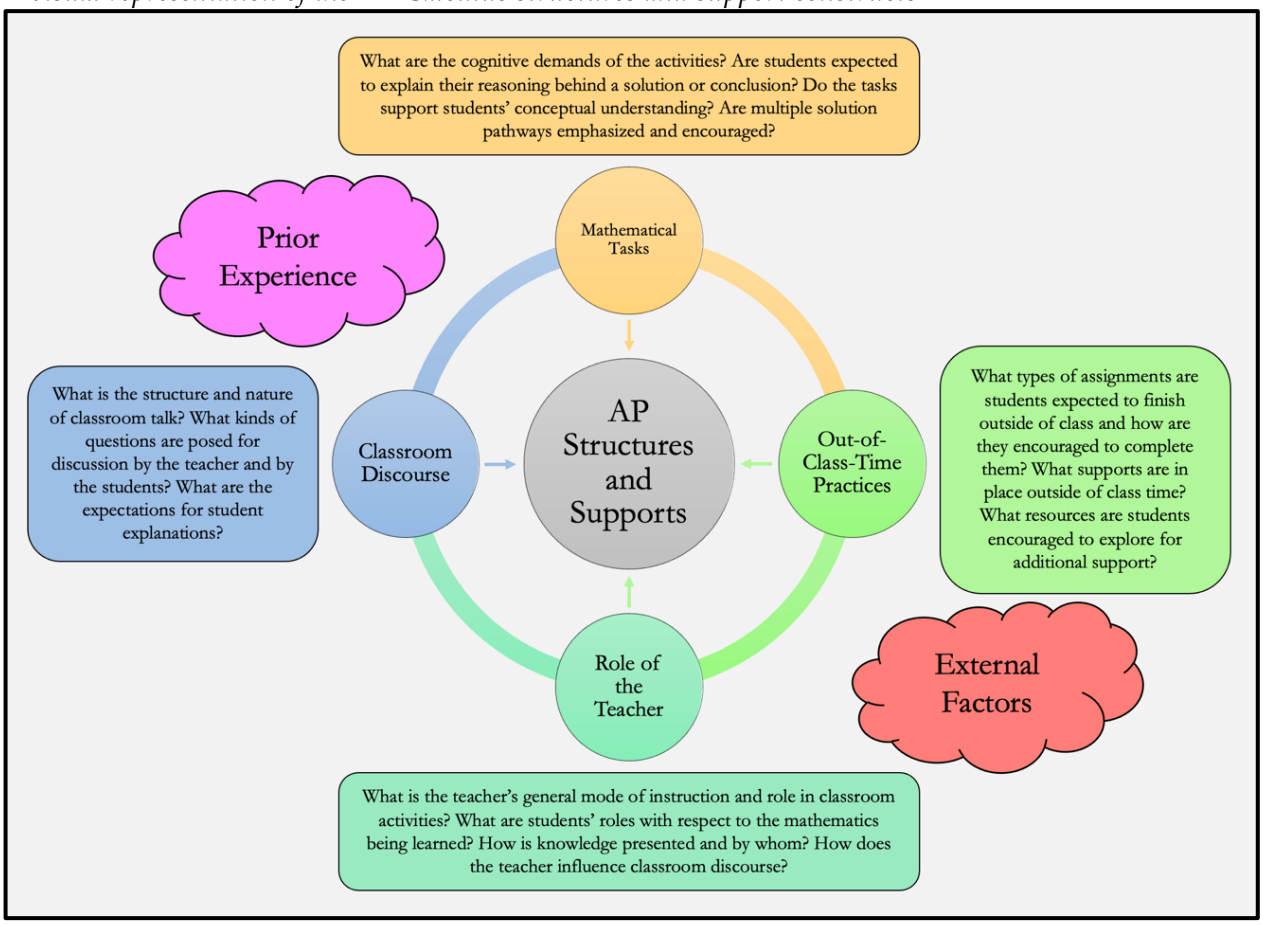
The Participant Survey was created through a graduate-level workshop/course at Standord University. The 10-week course draws on relevant cognitive processing theories and research related to the development of good survey questions. The constructs were iteratively designed using Munter's (2014) VHQMI rubrics as inspiration, which eventually led to the Concept Map, shown in Figure 1, that became the guiding framework. The survey took several revisions through peer and instructor feedback, and conversations in the class. In addition, the survey was reviewed and revised upon presenting it to specialists in the field of mathematics education and a current AP Calculus teacher, and pilot-tested with a former student from my fourth year of teaching. Finally, a variety of mathematics education surveys, used to capture the student voice, were carefully examined to ensure the integrity of the Participant Survey and to gather ideas for questions to include or exclude.

Ultimately, the final Participant Survey contained mostly Likert scale questions together with reminders and pictures of the different course components. The primary purpose of these questions and supporting materials was to help the respondents with their memory recall, but also to help develop and corroborate the analytic themes that would be later identified. The responses to the open-ended questions (see Table A.1 in the Appendix for a list of the questions) were the main objective, as this study intends to capture and synthesize the student voice of former AP Calculus students.

4.2. Aims of Exploration

Through this study, I aimed to investigate the research question: Which classroom structures and in/out-of-class supports do students perceive to best prepare and engage them to meet the expectations of the AP Calculus curriculum and exam, as defined by the College Board?. Using the four constructs, shown in Figure 1, I developed the survey questions to start with one particular construct and ask follow-up questions with regard to the others. For example, I typically started class with a collaborative Warm-Up activity that consisted of two or three AP style multiple-choice questions, which falls under mathematical tasks (see Figure A.1 in the Appendix for a sample

Figure 1
A visual representation of the AP Calculus structures and support constructs



Note. The Concept Map was adapted from Munter (2014).

Warm-Up activity). How effective did students feel this activity was for their learning – were these questions overly challenging, or might they have alternatively benefited more from reviewing a homework problem or two during the beginning of class? During the Warm-Up task, I usually circulated, assisted, gave feedback, and took note of what students were doing and saying as we segued to a classroom discussion to review the problems. This classroom discourse varied – sometimes I would present the answers while calling on students randomly or purposefully, while other times I would have students present their results. These in-class decisions fall under the Role of the Teacher, and I wondered how students felt about this variety – did they feel that they benefited from presenting and listening to their peers' explanations? Even further, there were times when I would assign the problems ahead of time for students to complete outside of class time and begin instruction with the student-led presentations of the problems in the Warm-Up. I was curious how students felt about this use of out-of-class-time – did it motivate them to learn how to clearly explain and show the solution process to their classmates, or did it seem like an excessive burden? Having answers to these types of questions from former AP Calculus students should help convey what students feel works for them and what does not. In my experience, students are very open and honest about their learning. By taking the time to analyze their perspectives, we can begin to gather information on which AP Calculus structures and supports should be utilized and minimized to best prepare low-income urban students to meet the rigorous demands of the AP course and accompanying test.

4.3. Participants

For this study, I surveyed my former AP Calculus students from my last three years of teaching at Central Academy[3], a small public high school in a major metropolitan area of Southern California. Central Academy serves between 550 and 600 students on a learning complex that is shared with three other similar public high schools. Approximately 95% of the students are Hispanic and 4% are Black, with 99% of the student body being eligible for free or reduce-price lunch (Ed-Data, 2023).

The California Assessment of Student Performance and Progress [CAASPP]/Smarter Balanced is taken by all 11th-grade public school students in the state. In the 2017-2018 and 2018-2019 school years, 25% and 27% of the Central Academy students, respectively, met or exceeded the mathematics standards of the CAASPP – slightly underperforming compared to California public school students more broadly, where 31% and 32% met or exceeded the CAASPP mathematics standards (Ed-Data, 2023). The AP Calculus course, which students are free to self-select into, had strikingly different results: 92% of the enrolled students passed the AP Exam in the 2017-2018 school year, 88% the following year, and 92% during the 2019-2020 school year. The passing rates for all of the AP Calculus test-takers during these three years were: 57.6%, 58.4, and 61.4%, respectively (College Board, 2023a.) It is worth noting, however, that only about 5-10% of students from each graduating class would take the course, either during their junior or senior year.

I attempted to invite all 32 students from the three cohorts to complete my survey, although I was unable to obtain the contact information of several students while others did not respond. In total, 22 students completed the survey (69%). Even though I did my best to encourage participation by explaining the exploratory nature of the study and the importance of capturing the student voice in hopes of improving the teaching and learning of calculus in high school, the missing surveys cause a concern that the samples may be biased toward the students who may have had a better experience in the course. However, I used snowball sampling and provided the participants a $25 Amazon gift card (provided by an external fellowship without any interest in the study) for completing the 30-45 minute survey to minimize this threat to validity.

All the respondents come from disadvantaged backgrounds – none had a parent with a bachelor's degree (most had parents who did not attend high school) and the majority of the participants came from homes with a household income between $0 and $50,000, as shown in Table 1. Their dedication to education, however, is undeniable – 62% of the participants were pursuing a STEM degree at the time of the survey, and only one was not enrolled in higher education. At least in terms of gender and AP Exam score, the sample participants appear comparable to the population of my former students: 66% males and 34% females, with AP Exam scores of five (38%), four (25%), three (28%), two (6%), and one (3%).

It is important to note that the gender disparity is unlike that of the broader test-taking community of AP Calculus – approximately 49% of students who took the AP Calculus Exam between 2018 and 2020 were female (College Board, 2013c). The unfortunate underrepresentation of female students in my AP student population may be due to the strange 2018-2019 school year where all eight of the students were male. During the 2017-2018 school year, 6 of the 12 students were female, and 5 of the 12 students were female during the 2019-2020 school year.

4.4. Data Analysis

This research project was approved by the Stanford University Institutional Review Board [IRB]. I used a six-phase thematic data analysis to make sense of and create a report on the corpus of data collected. First, I successfully transferred the data from the Qualtrics online survey platform into the Dedoose (9.0.62) software, where I completed the majority of analysis. Although I collected participant names and emails to provide them with their gift card, I anonymized the data upon

[3] Pseudonyms are used for both the school and the participants.

Table 1
Participant demographics

Respondent Characteristics	*No. (% of 22)*
Gender	
Male	13 (59)
Female	8 (36)
School Year	
2017 – 2018	7 (32)
2018 – 2019	3 (14)
2019 – 2020	12 (55)
Household Income	
$0 - 24,999	8 (36)
$25,000 – 49,999	9 (41)
$50,000 – 74,999	3 (14)
$75,000 – 99,999	1 (5)
Race/Ethnicity	
Hispanic	21 (95)
Black	1 (5)
AP Calculus Exam Score	
One	1 (5)
Two	1 (5)
Three	5 (23)
Four	4 (18)
Five	10 (45)

Note. Percentages were rounded; one participant did not finish the survey.

transferring to Dedoose and assigned pseudonyms by identification number. I conducted several close readings of the survey report, by individual Item results and by individual participant responses, to familiarize myself with the data. I then used the Concept Map (shown in Figure 1) as a guide to inductively create relevant codes and subcodes (Saldaña, 2009). I developed my codebook by creating, describing, providing inclusion and exclusion criteria, and incorporating examples of text to codes (Fonteyn et al., 2008). To account for my reflexivity and potential research bias, I wrote analytic memos to document my reasoning for the creation and refinement of the codebook (Malterud, 2001).

I used the data from three of the participants to adjust my preliminary codebook. I also partnered with a colleague to review and revise the codebook by separately coding representative transcripts of the data and reaching interrater agreement. I then used the finalized codebook on the entire qualitative dataset and Ryan and Bernard's (2003) framework to identify themes. Finally, I used the quantitative data from the survey to both support the qualitative findings and explore within group differences. Unfortunately, the sample size was too small to identify any major differences of the results among different groups of individuals (e.g., male and female responses did not drastically differ).

5. Findings

This study aimed to investigate the teaching practices and supports in AP Calculus that students report as being the most impactful and motivating for their learning. I began the analysis of the survey data by using the Concept Map, shown in Figure 1, as a guide to iteratively develop a codebook and then used it to assign codes to excerpts in Dedoose. The quotes were extracted verbatim from the responses to the open-ended questions in the survey. After conducting a thematic analysis of the coded excerpts, seven themes emerged: (i) Feeling Supported; (ii) Extended Class Time; (iii) In-Class Tasks; (iv) Out-of-Class-Time Tasks; (v) Organization of the Course; (vi) Transferable Skills Gained; and (vii) Struggles or Barriers to Success. In the following sections, I will describe each theme and associated subthemes using participant responses. For a

comprehensive description of the themes and subthemes, including example data segments, see Table A.2 in the Appendix.

5.1. Feeling Supported

Many participants commented on feeling highly supported to meet the demands of the course because of particular components of the class (see Table A.3 in the Appendix for a list of the participants; see Table A.4 and Figures A.1-A.7 in the Appendix for descriptions of course components and sample tasks). First, participants appreciated the multitude of peer and teacher feedback throughout the course: "Participating in leading problem solving on a board and receiving feedback in real-time from peers and the instructor also (Mr.Ramirez) helped paved the way for my success in the course" (Jorge)[4];

> What I really liked about our in class tasks were how we reviewed every single task warmups, group exams, math essays, and how there was communication about everything. The teacher did an amazing job when it would come to questions he would always ask and make sure we understood. (Mary)

Second, participants conveyed that they felt supported by the amount of collaboration during class time. Participants described benefitting from and enjoying group work – some specifically pointing to Group Exams as sincerely making them feel supported: "I thoroughly enjoyed the group exams because it required us to work together and hear different angles of attack that peers would use for different problems" (Cris); "Adding the group exams working together with your peer was fun because you get to analyze the different ideas everyone else has or the shortcuts people would use to counter that problem and take every bit of that for your use" (Trey).

Third, participants expressed feeling supported by the availability to engage with the teacher and students outside of class time. Students were encouraged to attend tutoring during lunch or before- and after-school to ask questions or work collaboratively with their peers. Participants expressed great benefit from this convenience: "I think the availability that you had for students greatly supported my ability to learn because I felt that whenever I got stuck that I could always swing right by" (Cris).

Finally, participants reported feeling encouraged and supported by the nurturing classroom environment. Respondents commented that the small class size provided intimacy and space to discuss, make mistakes, explore multiple solution pathways, and ask questions: "Well since there was only a little of us in the class, learning from each other and having those whole class discussions definitely supported my learning. Knowing I was able to ask Mr. Ramirez for help that was probably needed for the whole class was also very helpful" (Ruben). Participants also described feeling supported and motivated by the teacher's dedication, passion, and belief in student success: "The classroom environment and professionalism that you brought everyday was something that always motivated me to work harder" (Cris); "In my opinion having a teacher that was interested in teaching us something helpful everyday to help us understand and succeed in our ap exam was my motivation" (Mary); "I believe the greatest asset to support my learning was my teacher during the course. Not only did his passion for math rub off on me but his hard work and true belief in my abilities also showed me that I could do it too" (Jorge).

5.2. Extended Class Time

All of my calculus students had a Mandatory Support Course that extended the class time to two daily periods. Participants strongly conveyed benefitting from the extended class time and that it should be a requirement of AP Calculus courses. Respondents reported that two hours was needed to understand the vast amount of content, and that having an assortment of activities kept engagement high and promoted conceptual understanding: "Well AP Calculus AB has many ideas and concepts to go over so having the 2 hours gave us plenty of time to ask questions and actually

[4] The quotes were extracted verbatim from the survey and may contain minor grammatical or spelling mistakes.

understand the concept being introduced. The more time we had left over after being taught a new concept gave us more time to dig deeper towards its understanding" (Ruben); "These concepts were so foreign and complex to me that I don't see how we could've learned everything we did in a shorter time period. I felt like in understood math pretty well before calculus and once I was introduced to these new concepts it was nothing like I was used to. I struggled but it wouldve been much worse if we had half the time we did" (Luciano).

Participants also described gaining inspiration to learn from the additional in-class time because they knew help and space was available to develop a deep understanding of the course concepts: "What motivated me to learn was the Mandatory Support Course. I felt like I had a lot of time to learn everything in Calc AB and was a motivation for later in the future when the AP Calc AB Exam was coming up" (Isaac).

Finally, participants reported that although the extended class time was sometimes a strenuous burden, it was an essential component because it eased the pressure to learn and made them realize the value of extended practice: "I do think the 2 hours is beneficial because there is a lot of material you have to cover and prepare you for the AP test, 1 hour of lecture is not going to be enough. I feel that if I were to take 1 hour of calculus class I would not have passed the AP test although 2 hours can be quite challenging your effort put into the class will not be taken away" (Trey);

> I'd say the mandatory support course (at the time) seemed excessive because spending two hours in the same class learning hard math problems was grueling for my younger self, but I understand it was necessary. Otherwise, my peers and I would have probably struggled in the course. My point is that to students in the course in the moment it may seem excessive and may detract them from learning, especially if they get easily distracted or have a hard time grasping the concept at hand. (Bryan)

5.3. In-Class Tasks

The theme of Extended Class Time is critical because without the extra period, the activities in the class would have been substantially reduced. The theme of In-Class Tasks emerged because many participants expressed great appreciation for the activities in class. In particular, participants reported that all or most of these varied tasks were helpful because they kept them engaged and motivated to learn, forced them to reason and think conceptually, and provided frequent practice that was invaluable: "The course was filled with many ways to help was succeed from pop quizzes to warm ups to practices exams. I feel that all was was great practice" (Hugo); "During these times I found myself critically thinking about the concepts and reasoning behind my answers" (Jorge).

More specifically, participants reported that the review activities in the form of academic games provided the opportunity to deepen understanding in a very engaging manner: "Some of the features that excelled my learning of AP Calculus AB were the interactive games we had in the format of Jeopardy!" (Jorge).

Participants also remarked that projects helped better understand the concepts and were quite instrumental to both learning and building community: "I do think the projects where the best tasks as for the most part they were fun tasks which made them more impactful to me personally" (Flavid); "Looking back I remember enjoying the posters and group work. This is because i got to work on things with my friends and also be a bit artistic in creating the posters" (Vanessa).

Participants described the Warm-Up activity as a very useful way to start class because it helped review and build toward gaining mastery of the concepts: "I liked the warm ups because I was able to do them and have the right answers explained right after which would help me either confirm I understood the material or help me understand it better" (Bryan). Participants also described the structure of reviewing the Warm-Up to be particularly impactful: "Doing the warm-ups in the board really helped everyone engage and motivated" (George).

In addition, respondents conveyed that the course Practice and Group Exams were valuable preparatory tools because they helped ensure individual success: "He would assign "group exams" which were exams taken in little groups to prepare you and give one an idea of what to expect and

what to prepare/study for the "solo exam", which was extremely helpful in my opinion" (Perla). Furthermore, respondents reported high praises for the full-length Mock AP Exams that they took toward the end of the school year. Participants reported that the Mock AP Exams provided vivid awareness of the expectations of the actual AP Exam, that they reduced anxiety on the day of the actual test, that they allowed them to learn from their mistakes, and that they should be an integral part of an AP Calculus course: "The exams being treated like the AP exam was great, it really helped me get a feel of how it will be when the final exam will be taken" (Hugo);

Finally, participants recounted that it was very helpful to have ample review time at the end of the school year to complete mastery activities: "Creating flashcards and a list with every formula by the end of a class also helped with an overall review" (Elsa); "Also the fact that we did finish early with the material and has the last months before the exam to study up and practice" (Hugo).

5.4. Out-of-Class-Time Tasks

The theme of Out-of-Class-Time Tasks, or "homework," contained four subthemes. First, participants reported that the Out-of-Class-Time Tasks were the right amount of practice, and that they were very challenging but that being encouraged to work on them collaboratively and seek help from the teacher made them both encouraging and feasible:

> These out-of-class tasks were sometimes some of the most difficult things that we did in the class for me. This pushed me to seek out help from my peers and they sometimes asked me for help to. Something that amazed me was the attempt to learn from your peer instead of simply just copying. Everyone would try to get a better understanding of the work we needed to complete. (Luciano)

In addition, participants recounted how helping each other on the Out-of-Class-Time Tasks made it fun, that explaining to others improved understanding, that collaboration helped overcome challenges, and that it was motivating to recognize peers were also struggling: "A task that really stood out to me were the problem sets and assignment packets. These tasks were a PAIN!!! But I can't lie about how beneficial they were to not only understanding lessons but working together with classmates and challenging yourself to make time out of your day to study a little harder" (Perla);

> I found it very helpful to be able to work on the problem sets and assignments with my peers outside of class. I'd try them on my own and anything I didn't understand or couldn't solve I'd go to my peers for help and found it very motivating because I'd be able to understand the solutions better correcting my mistakes. It also helped just knowing that my peers would struggle with the same problems. (Bryan)

Second, participants reported that the logistics on submission removed pressure – having a week to complete Out-of-Class-Time Tasks provided space for productive struggle and promoted collaboration: "The time limit for each assignment was perfect and I really liked that because it teaches you about discipline" (Mary); "I loved that he emphasized more time on homework activities which allowed us to learn how to manage our time and work together as a class to figure out these challenging problems" (Perla).

Third, participants conveyed that most or all of the Out-of-Class-Time Tasks were helpful because they promoted mastery, they supported learning through review of previous concepts, and that it was especially valuable to peer- or self-correct them in class: "I really liked how we had our packets for the week I think that really helped with trying to understand everything" (Nancy); "The homework packets also helped put what we learn into practice and helped retain that information. Going back and correcting them was very beneficial for the learning process" (Elsa).

Finally, respondents indicated that the contents of the Out-of-Class-Time Tasks, which were primarily made up of AP-style questions, helped prepare for the AP Exam, and made space for improvement: "The features of the class that stand out to me as having impacted my learning of AP Calculus AB were having past AP exam questions as homework" (Bryan). Participants also recounted that the amount and structure of the Problem Sets was extremely beneficial because of the challenge they provided: "Something that stood out to me about the AP calculus was the

constant drilling we did on subjects. The number of examples and problems sets we did helped tremendously in understanding the concepts that we were introduced to" (Luciano).

5.5. Organization of the Course

Perhaps one of the most profound theme that emerged was the Organization of the Course, which contained a total of seven subthemes. First, participants described how the overall organization of the class provided a feeling of support and motivation to learn a challenging subject through the amount of class time and the format or types of tasks which reduced anxiety/stress:

> One of the main things that stood out to me while taking this course was how organized and serious this class was. For me this was a positive thing because during this time of my life I was slowly losing interest in school, this class really helped me appreciate the effort my teacher was putting into the class and made me fall in love with school once again. (Hugo)

Participants also reported that the activities supported the learning process through different forms of assessments, that the tasks increased their confidence and understanding, that they provided motivation to learn, and that they felt thoroughly engaged by the teacher's practices: "I believe that reviewing and having different methods of practicing supported me through the learning process" (Elsa).

Second, participants described how the course focus on teamwork helped deepen understanding through collaborative tasks and projects: "I found working with my peers and then being able to ask you for help when we couldn't figure something out together was very helpful" (Nancy);

> There was no aspect of the course I could say I necessarily didn't enjoy. Even the group projects while not being the most helpful for me, allowed me to interact with peers and view their understandings as well. This allowed me to fill in the gaps of concepts I lacked understanding in or gave me the opportunity to assist my fellow students in a given subject to better propel their reasonings. (Jorge)

Third, participants recounted valuing how there was an alignment of the expectations of the course to the format and demands of the AP Exam, which were met through significant practice with an emphasis on knowing and understanding: "Also, the way you would quiz us on basic things like the Unite Circle and equations. It helped a lot in the long run when we took our AP Calc AB Exam" (Isaac); "Mr.Ramirez was always willing to help and answer questions we also went over many many examples in class. These examples were actually harder than the ones on the exam. Of course we also went through very similar structured problems that were on the exam" (Sonia).

Fourth, participants commented that the constant review activities fostered conceptual understanding and motivation: "I found the activities, exams and pop quizzes to be extremely helpful as I found numerous drills to be super helpful. These assignments contained reviews from a peer or the whole class, which made it helpful to understand where my responses were wrong" (Karla); "Additionally, the constant practice of new concepts and old concepts that we did in this class really helped integrate the formulas and methods deep into my memory" (Quin). In addition, participants conveyed that the tasks provided helpful resources, made the amount of content manageable, and promoted engagement: "I don't believe I had any outside experiences that helped me in my learning. It was more on the resources that the course provided" (Karla); "Well to be honest the class was beyond our time. As a junior I didn't really get to appreciate how organized and accessible the class was made for us" (Ruben).

Fifth, participants reported that the delivery of new concepts through traditional note-taking and heavy discussion helped develop understanding, encouraged learning, provided useful references, was concise, and reduced overwhelmingness with cohesive units and sections: "Introduction of new concepts was really helpful because we always had our notes and they really came in handy when we needed more time understanding something" (Nancy);

> The introduction of new concepts was always Interesting to me because on the board will be the description of the concept, how it will be done, special circumstances and sometimes history about the concept. Being able to do problems with us and explain step by step how each step is done really gave me the motivation to learn the topic. (Hugo)

Sixth, participants expressed relief from the Grading Scheme (a 70% grade corresponded to an A, which aligned with how a 70% score on the AP Calculus Exam results in a score of a 5), and that it stimulated persistence, determination, and confidence: "The grading scheme eased my mind a bit for the course" (Sonia); "The grading scheme was very motivating because it got rid of an outrageous expectation of us to perform at such a high level for a very demanding class" (Cris). In addition, respondents described how they appreciated the Grading Scheme because it was aligned to the AP Exam expectations, which afforded a focus on understanding and not performance: "The grading scheme was extremely helpful because I feel it reflected my AP score. It was motivating to know that we were working towards passing the AP exam and not just passing the class" (Olivia); "The Mandatory Support Course and Grading Scheme helped me feel less stressed about the course since they both allowed me to spend more time understanding the material rather than feeling stressed about grade percentages and scores" (Diego).

Finally, participants noted that a few additional resources, that were either encouraged or required, helped meet the demands of the course: "Khan academy was a very good tool to use sometimes when I didn't understand after class" (Vanessa); "AP readiness was a great help. Sometimes it wasn't even what was being taught it was more of has calculus on your mind for an extra day" (Luciano).

5.6. Transferable Skills Gained

An unexpected theme that emerged was Transferable Skills Gained. Participants described how the fast pace of the class promoted studying more at home, how the course impacted their study habits and increased their inclination to work with others while in college, and opened future possibilities by being challenged and finding success:

> An impact it had on my learning was the way I study for my classes in college for math. Like taking notes and doing demonstrations to friends to see if I understand the concept of the lecture by explaining the concepts to them, also way we would write down the equations on note cards and quiz each other as a way to study. Also, the fact that we would have to memorize the equations, rules, graphs, and the unite circle was also helpful. (Isaac)

Participants also recounted how the course gave them a preview of the way college courses are organized, and that the challenge of their AP class provided support to succeed in a college setting:

> Not only did the course take on the same structure as the AP exam, but it took the structure of a college course. I see a lot of similarities between Mr. Ramirez's AP Calc AB class and my current college courses. Mr. Ramirez even had after school time where we can drop in, ask questions and work collaboratively on our take home worksheets, and I think that that was his take on a professor's office hours. (Quin)

In addition, participants described transferable benefits from the discomfort of being forced to present in class: "The assignment that I did not find helpful was the presentation since I had issues presenting the material, but I did find this experience to be beneficial during my undergraduate years" (Karla).

Finally, participants expressed future benefits of the intimidating Math Essay activity, and that the course overall encouraged and prepared them to pursue a STEM degree: "An assignment that I found challenging but fun was the math essay. I found it difficult to complete this assignment as I have never written an essay explaining math practice. But with this experience, I was prepared to explain STEM-related concepts in the form of writing in my undergraduate years" (Karla).

5.7. Struggles or Barriers to Success

Although the majority of participant comments expressed positive outcomes from the different instructional strategies of the course, the open-ended questions also prompted them to convey

what was unhelpful or detracted them from learning, which allowed for the Struggle or Barriers to Success theme to emerge. Participants recounted that presenting work was overwhelming, challenging but enjoyable, and helpful but nerve-wracking: "One of the aspects I struggled with were the presentations but that is simply because I am more of an introverted person and any public speaking makes me nervous" (Aidan); "I enjoyed the presentations as they challenging to present my understanding of a topic without the use of tests or homework, which is a typical approach for a math subject" (Karla).

Participants also expressed discomfort with the fast pace of the course and that it detracted their motivation and made it difficult to keep up: "Definitely the pace and intro to new concepts since at that time it was the fastest class I've ever taken and well information just flew left and right" (Ruben);

> The only component that I slightly believe may feel was detracting me from my motivation to learn was the pace of the course. Although I felt comfortable with the pace of the course, I have heard from peers that it may have been to quick since some of the material may have felt confusing one day and then more confusing with the limited time they had since they had other responsibilities to attend to. (Diego)

As mentioned in the previous section, several participants found great benefits from the Math Essay activity, however others found it overly challenging, unmotivating, and that it did not support their learning: "The math essays I feel weren't really helpful, they seemed to cause stress and take away from the math" (Olivia); "The math essays were the only thing I felt did not support my learning, at least for me. Although I understand it may have been to better our understanding of the concepts we were learning through a hands-off approach and that it may work for some students" (Bryan).

Respondents also described how some aspects of the course raised anxiety levels or created stressful situations. First, being randomly called on was helpful but stressful: "[Detracted Learning] Probably being randomly called on. I don't specifically remember being anxious about being called on, but I can see how it can be anxiety inducing for someone else" (Quin). Second, participants recounted that the number and rigor of Pop Quizzes was exhausting: "Sometimes the pop quizzes were the only thing that did seem excessive, but it kept us on our toes and ready for any calculus problem thrown our way" (Ruben); "The quiz to me was personally very excessive for me 3 times a week can be a lot and overwhelming" (Trey). Finally, participants stated that the Problem Sets were overly demanding and that there was an outrageous amount of work to complete outside of class, albeit helpful: "The problem sets were vastly different compared to other things that we did in class and tackling one felt dreadful" (Cris).

A few participants also described how personal conflicts made it difficult for them to remain committed to learning: "Unfortunately, during the year when I was taking this course I had a lot of conflicts within myself. I was in a situation where bad Influences were always around me. Which made me lose interest in school which resulted in not being able to take studying as serious as I would have liked" (Hugo); "Once I started struggling and falling behind I found myself losing motivation" (Sonia).

Finally, respondents reported having mixed feelings about their ability to meet the demands of the course because of their diverse prior mathematical experiences. Most of the participants took either my Algebra 2 or Honors Algebra 2 and Trigonometry course and some respondents commented on finding it to be an excellent preparation for the class: "I believe after taking Mr. Ramirez's Algebra 2 I was well off for AP Calculus AB, as the two courses transitioned well into each other" (Jorge). A few respondents, however, conveyed feeling underprepared for the course or expressed the need for certain components to be more prevalent in prior mathematics classes:

> I wished that Pre-Calc was more helpful, unfortunately when I was taking Pre-Calc Professor [Teacher 1] received a promotion which resulted in her leaving the class. There was time we had subs after subs until a permanent teacher was found. By the time they found the teacher. I did not fine the class serious and more as a pass time. I wish these events did not happen or a replacement was found quicker before I lost interest in learning the subject. (Hugo)

To make better sense of the obstacles students recounted, I will present a few of the quantitative results from the survey questions. First, Figure 2 shows the responses to the Likert scale matrix question that asked respondents to indicate their thoughts on the rate of occurrence of each type of common task or activity. The results for Problem Sets were slightly differing – the majority of respondents (14 or 67%) indicated that it was "just the right amount," while four (19%) respondents would have liked to "have done them a little less often," and three (14%) participants would have like to "have done them a little more often." The results for Assignments were much more decisive – nineteen (90%) of the respondents indicated that it was "just the right amount." As for Projects, the results were much more varied – three (14%) respondents would have liked for them to be a little less frequent, while six (29%) would have liked to have done them a little or a lot more often. Students were required to take three Exams (and three Group Exams) and one Final Exam per semester – participants indicated that the number of Exams was either adequate or that they would have appreciated doing more of them. As for Group Exams, the number of respondents (9 or 43%) who would have liked for them to be a more common occurrence was more pronounced, compared to the Exam results. Finally, four (19%) participants indicated that they would have liked to have done a little less Pop Quizzes – these may be the respondents who commented that they found them to be somewhat excessive.

Figure 2

Results from the question: What did you think of the frequency of the following activities/tasks?

Activity / Task	We should have done them a lot less often		We should have done them a little less often		It was just the right amount		We should have done them a little more often		We should have done them a lot more often		Total
Problem Sets	0%	0	19%	4	67%	14	14 %	3	0 %	0	21
Assignments	0%	0	5%	1	90 %	19	0 %	0	5%	1	21
Projects	0%	0	14%	3	57 %	12	19 %	4	10%	2	21
Exams	0%	0	0%	0	71%	15	24%	5	5%	1	21
Group Exams	0%	0	5%	1	52 %	11	33%	7	10%	2	21
Pop Quizzes	0%	0	19%	4	71 %	15	5%	1	5%	1	21

Note. The percentages were rounded to the nearest whole number.

The instructional practice of randomly calling on students to participate in whole-class discussions was a school norm and a daily routine in my class. A very small number of participants, however, discussed the impact of being randomly called on to share their thoughts or present their work on the board. Results from the question: "How anxious was it knowing you could be randomly called upon to participate in Whole-Class Discussions?," shows that this teaching strategy caused most participants to either feel "quite anxious" or "very anxious" (80%). However, results from the question: "How was your learning affected by the act of being randomly called upon to participate in Whole-Class Discussions?," shows that this instructional practice positively affected the majority (75%) of the respondent's learning. Overall, the quantitative results from the survey were used to both inform the development of the themes, and to corroborate the themes and their descriptions, as presented at the end of this section.

6. Discussion, Implications, and Future Directions

This study aimed to investigate the instructional practices that students perceive to best prepare and engage them to meet the expectations of the College Board's AP Calculus course framework. The thematic analysis revealed seven impactful themes as a result of the course: Feeling

Supported; Extended Class Time; In-Class Tasks; Out-of-Class-Time Tasks; Organization of the Course; Transferable Skills Gained; and Struggles or Barriers to Success. The complexity of these results aligns well with Schoenfeld's (2022) attempt to answer the question of why teaching and learning mathematics is so difficult. In this section, I will structure the discussion around the themes but use Schoenfeld's Teaching for Robust Understanding [TRU] Framework to guide the discussion, link the results to related research, and provide implications or opportunities for future work.

6.1. Feeling Supported

The theme of Feeling Supported most closely relates to the third dimension (Equitable Access to Content) and the fourth dimension (Agency, Ownership, and Identity) of the TRU Framework. Participants reported the important impact of peer and teacher feedback, encouragement to collaborate during and outside of class time, the ability to effortlessly seek teacher support, and a rigorous but intimate classroom environment where all students are expected to actively participate. These findings are consistent with Kyburg et al.'s (2007) more broad investigation of best practices in AP programs. The active participatory nature of the classroom is vital because "students' opportunities to author, justify, and debate mathematical ideas position them with mathematical authority. These subject positions, when experienced over time, support the construction of mathematics-linked identities that include belonging and a sense of intellectual power" (Langer-Osuna & Esmonde, 2017, p. 645). The findings of Boaler and Greeno's (2000) classroom study indicate that narrow mathematical practices within schools are problematic, not only because they disenfranchise many students, but because they encourage forms of knowing and ways of working that are inconsistent with the discipline. The data in my study maintain the conjecture that when mathematics learning practices support students to place themselves in positions with more significant conceptual agency, it is much easier for many of them to author their identities as learners and mathematical doers with that kind of agency.

As suggested by the research on AP and International Baccalaureate [IB] courses, simply providing access to advanced coursework does not seem to consistently foster students' success – students still require substantial support to meet the demands of these classes (Renbarger & Long, 2019). The findings on the theme of Feeling Supported provides opportunities for future research that may investigate what exactly does it mean to "feel supported" by students with different personalities and needs. Additionally, students who have been previously successful in mathematics courses through independent study and practice may be inclined to avoid collaboration and participation in AP Calculus courses that embrace active learning. It would be an important study to determine how teachers can effectively convey to successfully independent students the value of cooperation both within and outside of the classroom.

6.2. Extended Class Time

Participants intensely reported that the extended class time provided the necessary duration to practice and master the course concepts, eased the pressure to learn, and made them realize the importance and value of extensive preparation. The theme of Extended Class time most closely relates to dimension two of the TRU Framework (Cognitive Demand) because students were given the time and encouragement to wrestle with challenging ideas and concepts. This finding is consistent with Prong's (2018) dissertation study that aimed to determine whether there were gender and ethnic AP Calculus achievement gaps in the state of Michigan, and to test whether increasing the length of the instructional period from one hour to two hours might remedy the problem. Prong's analysis showed statistically significant gender and ethnic differences – males outperformed females in the years of study, and White and Asian students outscored Hispanic and Black students, in all of the years, on the AP Exam. The results of the study suggest that increasing the length of the instructional period may "ameliorate the confluence of factors" that negatively affect student achievement in AP Calculus, such as gender, ethnicity, and socioeconomic status (pp. 114-115).

Fisher et al.'s (2020) large-scale analysis of AP science teachers similarly revealed that after reducing their sample to the 638 teachers who taught in schools with at least 50% of their student body enrolled in free- or reduce-price lunch programs, increasing the number of days in the school year was significantly associated with AP performance gains. Unfortunately, providing extended class time to a small population of calculus students in schools that have larger academic needs may be unfeasible. However, if schools – especially those serving low-income and/or minoritized youth – wish to truly give their students the opportunity to be successful in AP Calculus, then they should provide extended class time within the school day to support them in meeting the cognitively challenging demands of the course. In addition, considerable time should be spent planning activities to keep students engaged and motivated to learn with the extra time, and not simply expecting students to work productively independently. If schools, especially those who primarily serve historically disadvantaged students, are unable to provide extended time in the school day for their calculus students to learn, they may consider offering the new AP Precalculus course (College Board, 2023b) instead of AP Calculus. Afterall, mastery of the mathematics considered preparatory for calculus has been shown to be more critical for future success in college mathematics than a shallow introduction to the abundance of topics found in AP Calculus (Sadler & Sonnert, 2018). More research is needed to verify the important need of lengthening the in-class/school time of AP Calculus students to ensure their success in the course.

6.3. In-Class Tasks

Participants conveyed the benefit of varied tasks in supporting their learning, how interactive activities provided joy and encouragement, and how the different forms of collaborative tasks helped guarantee individual success. The theme of In-Class Tasks most closely relates to dimension five of the TRU Framework (Formative Assessment) because students are given ample opportunities to express their understanding, assess that understanding, and work to deepen it by addressing their misconceptions. Including a variety of assessments is important not only from a measurement perspective but as a matter of sensitivity to varied learners (Tomlinson & McTighe, 2006). Having a variety of assessments and different forms of feedback provides students with an important need: to receive information about their learning – rather than achievement – which accelerates pathways to success and gives students powerful growth mindset messages about mathematics and understanding (Boaler, 2016). In addition, the participatory nature of the majority of these in-class tasks is of great importance because active learning in STEM courses not only increases examination performance and lowers course failure rates (Freeman et al., 2014) but also has a disproportionately beneficial impact for students from minoritized groups in STEM and for individuals from low-income backgrounds (Theobald et al., 2020).

The theme of In-Class Tasks indicates that although one of the primary goals of an AP Calculus course should be to prepare students to be successful on the summative test, teachers should not rely solely on individual exam scores throughout the year to support their students and adjust their instruction. It is imperative that assessment is varied where students are given multiple opportunities to demonstrate their understanding through diverse forms of modalities. In addition, and relatedly, the findings suggest teachers should find time and space for students to complete and review at least one full-length Mock AP Exam – although ideally two, one that is completed collaboratively in groups. With the wide array of topics to cover in AP Calculus, future research should examine the benefits and drawbacks of having sufficient time at the end of the school year to review the learning objectives of the course, and how to best support and motivate students to study outside of class to be able to efficiently address the entire course content.

6.4. Out-of-Class-Time Tasks

Participants reported that the Out-of-Class-Time Tasks were challenging but provided the necessary practice to meet the goals of the course. In addition, the format of these tasks and the encouragement to complete them collaboratively helped improve their understanding by providing and receiving explanations to the problems. The theme of Out-of-Class-Time Tasks most

closely relates to dimension four of the TRU Framework (Agency, Ownership, and Identity) because students were given frequent opportunities to have conversations about important mathematical ideas and challenge each other's thinking.

Consistent with the Emerging Scholars Program [ESP] model, the theme of Out-of-Class-Time Tasks highlights the importance of providing a space for students at their high school to complete their tasks in a collaborative and supportive environment. In addition, the favoring of participants in this study to have a week to complete these tasks indicate that teachers should consider providing ample time for students to complete their assigned work, but make sure to check-in on their progress regularly. More research is needed on homework policies and practices in AP courses, particularly with low-income and/or minoritized student populations who may have substantial responsibilities outside of school, as well as how to leverage technology to improve the way students complete these tasks (e.g., Zoom, ASSISTments).

6.5. Organization of the Course

Participants conveyed that the overall organization of the course supported the learning process, increased confidence and conceptual understanding, provided motivation to learn, and helped ensure success on the AP Exam. In addition, the delivery of new concepts through traditional note-taking and heavy discussion provided useful references, was concise, and reduced confusion with cohesive units and sections. The theme of Organization of the Course most closely relates to dimension one of the TRU Framework (The Mathematics) because of the importance of having focused and coherent discussions that connect to prior learning and encourage multiple perspectives or solution pathways.

Hallett and Venegas (2011) identified three themes that recently graduated high-achieving seniors identified for their lack of success on their AP Exams: "(a) teachers were unprepared or unmotivated, (b) course material did not match the national exam, and (c) school-based structural issues negatively influenced the AP class experience and later test performance" (p. 478). The fact that these themes were completely absent in the participants' responses in my study, points to the ways the organization of the course paved the way for their success in the class and on the AP Exam. The findings indicate the vital importance of carefully structuring and organizing the course to both support student understanding of class content and their future learning.

6.6. Transferable Skills Gained

Participants mentioned several personal outcomes of the course that extended beyond the mathematical contents of the class. The rapid pace of the course, for instance, prompted them to study more outside of class. In addition, the collaborative focus of the course increased their inclination to work with others while in college. The class also provided an enduring challenge that made them realize their full potential and opened future possibilities. Finally, the organization of the course provided a preview of the structure of college courses and supported their success in a college environment. The theme of Transferable Skills Gained most closely relates to dimension four of the TRU Framework (Agency, Ownership, and Identity) because of the way the class provided opportunities for students to enhance their understanding of concepts through conversation and how the organization of the course contributed to their development of agency and positive identity as thinkers and learners.

Using survey data, Sonnert et al.'s (2020) national study showed that while having a standardized testing focus in the AP Calculus class had an unequivocal positive impact on AP Exam scores, when it came to the long-term impact on college calculus grades, the results appeared "harder to reconcile" (p. 10). The findings indicate that an AP course should provide targeted support to be successful on the AP Exam, but it should also provide opportunities to develop deep understanding of concepts and procedures through a variety of modalities to positively impact students' future success in college. Participants in this study, for example, recounted transferable benefits from being required to present work and explain their reasoning to others – an activity that would likely not be categorized by students as "standardized preparation"

– yet simultaneously seemed to have helped in both preparing for the AP Exam and the rigor of college instruction. More research is needed, however, on how we can prepare secondary calculus students for both the AP Calculus Exam and for tertiary calculus, since these two type of classroom settings and expectations may be vastly different; in addition, future studies should explore how to connect mathematical discourse to increase student performance while supporting the learning of all students (Wade et al., 2018).

The results of Cribbs et al.'s (2021) study suggests that mathematics identity can be used as a way of explaining student career intentions in mathematics-related fields. The authors contend that it is the combination of students' self-perceptions related to interest, recognition, and competence/performance in mathematics that provides a picture of students' mathematics identity. These theories and prior research stress the importance of supporting the learning experiences that students have with mathematics, both in and outside of the classroom, because they have the potential to influence students' agency and mathematics identity development, which in turn may influence their career intentions. Their study found that the primary factors that are positive predictors for mathematics identity were: (a) classroom interactions or discussions, (b) focus on mathematics connections, (c) activities involving conceptual learning, and (d) the availability of tutoring. All of these instructional practices corroborate the findings of my study and helps explain how the course impacted the participants beyond the learning of calculus.

The theme of Transferable Skills Gained has important implications for the preparation and motivation to pursue a STEM major in college. It is incredibly important to help students find success on the AP Exam because AP scores have strong influence in students' choice of college major – students are more likely to major in the AP subject in which they scored a 4 or a 5 on the associated AP Exam (Avery et al., 2018). As mentioned earlier, however, teachers should use caution in overemphasizing standardized testing preparation to avoid making students feel like STEM subjects are all about performance on tests. More research is needed on how the AP Calculus experience and AP Exam score affect a student's motivation to pursue a STEM degree. Nonetheless, this study has identified practices and supports that appear to help highly motivated first-generation students to be appropriately prepared for the style and rigor of college.

6.7. Struggles or Barriers to Success

While the vast majority of participant comments were positive in supporting and motivating student learning, the theme of Struggles or Barriers to Success also emerged. A few participants claimed that presenting work was stressful and disengaging, that the pace of the course was overwhelming fast, that the Math Essay was not a very helpful activity, that some aspects of the course created stressful conditions, that personal circumstances hampered their commitment to the course, and that their prior mathematics experience either enhanced or hindered their learning. The theme of Struggles or Barriers to Success most closely relates to dimensions two and three of the TRU Framework (Cognitive Demand and Equitable Access to Content) because students were being challenged to make sense of important mathematical ideas and connect concepts or strategies in new ways to solve nonroutine problems. In addition, students were required to be active learners in the classroom and frequently participate in small group and whole-class discussions. It is not surprising that these structures were uncomfortable struggles for some students because they are not typical of many American classrooms (Hiebert, 2013).

A few participants reported feeling nervous, overwhelmed, or stressed by having to present work to others despite the nurturing classroom environment. This finding indicates the need to provide more opportunities for students to explain their reasoning and solution strategies to others, both formally and informally, to make students comfortable in doing so in mathematics classrooms because many of the participants conveyed strong positive benefits of being required to actively engage in class through presentations. More research is needed on the benefits of student-led discussions and effective strategies, especially for introverted students, to help pupils deliver coherent explanations without feeling overly anxious.

In their investigation of predictors of success among high school students in AP and IB programs, Suldo et al. (2018) similarly identified several stressors that trigger students as a result of the demands of these courses. Findings from their study support student engagement as a factor associated with success of AP/IB students – specifically, motivation and cognitive engagement predicted superior academic outcomes. Although the majority of participants in my study found the pace of their AP Calculus course to be quite or very helpful in both supporting their learning and instilling confidence, a few found it difficult to keep up and lost motivation to learn. This is an important finding because AP Calculus teachers must cover an abundance of topics throughout the year and may feel inclined to slow down their pace to help meet the needs of all students. The results of my study, however, indicate several positive outcomes as a result of the rapid pace of the course. Nevertheless, "there is an unmet need for educational supports specific to the growing population of AP/IB students" (Suldo et al., 2018, p. 368). More research is needed on how to reduce stress and how to support students who are not accustomed to the pace and wealth of information that is presented in AP Calculus courses.

Through interviews with 30 AP or IB students in Florida, Shaunessy-Dedrick et al. (2015) relatedly found that students faced a variety of stressors because of their advanced courses. The participants in their study indicated feeling overwhelmed with academic issues (e.g., volume of assignments, overlapping due dates, worries about upcoming exams, the need to study for tests), constant efforts to balance time demands (e.g., balancing after-school commitments, finding time to complete different assignments), and other personal challenges (e.g., family obligations, preparing/applying to college). These findings corroborate the struggles and barriers that the respondents of my study reported. In particular, participants recounted the uneasiness of completing tasks or problems that required high levels of cognitive demand, perhaps because they were accustomed to simply reproducing procedures that had been introduced to them. The Math Essay, for instance, required students to convey their conceptual understanding of topics in a narrative form, which was a novel and unnatural task for them to complete in a mathematics classroom. The Items on the Problem Sets similarly challenged students to apply their understanding of concepts to solve nonroutine problems and provide explanations or justifications, which made them feel uncomfortable.

Peterson and Viramontes (2017) describe productive struggle occurring when students are given complex mathematics problems that are focused on underlying concepts, which are accessible to the students because the mathematics is within reach and they have the tools to solve them, but the problems are complex enough that there are no clear paths to the solution (p. 75). More research is needed to investigate how to best support AP Calculus students to struggle productively in tasks, which is an experience that they are unlikely to have felt in their previous mathematics courses. In addition, several participants reported the positive impact of having to write a mathematics essay, while others found it unhelpful and stressful – future work should examine the benefits and drawbacks of having students complete extended writing responses that connect or distinguish different concepts or ideas, particularly because students in AP Calculus may dislike reading and writing.

Finally, participants reported mixed feelings on their preparedness for AP Calculus because of their diverse prior mathematical experiences. An abundance of prerequisite skills and knowledge are essential to ensure student success in AP courses (Dougherty & Mellor, 2009). Respondents indicated benefitting from a vertically aligned curriculum for their success in AP Calculus, and from participating in a week-long summer review course – both practices that have been shown to help improve student outcomes in AP courses (Hanover Research, 2014). A few participants, however, indicated feeling underprepared or pointed to important practices that should be more prevalent in mathematics courses such as mathematical reasoning and productive struggle. The findings of this study indicate that educators should carefully examine their high school mathematics curriculum to certify that there is vertical alignment that will adequately prepare students for AP Calculus.

7. Limitations and Conclusion

There are several limitations of this study that are important to discuss. First, the participants of this study were my former students who may have been inclined to provide pleasing recounts over honest critiques. I intentionally decided to survey my former students, as opposed to interviewing, and anonymize the data to minimize this threat to validity. I am confident that the respondents provided their truthful opinions of their experience, as they have witnessed my sincere dedication to helping students, and to education, more broadly. Second, the sample size of 22 participants was too small to detect any within group differences that may be important for educators to attend to and understand. Nevertheless, this is an important phenomenological study exploration that provides vivid details of student recounts and their opinions of the different course structures and supports. Third, the respondents were providing details several years after taking the class, which certainly affects their memory recall. I made careful decisions when crafting the survey to help the participants with their recollection by providing brief descriptions and pictures of tasks and structures. I also avoided questions that were overly detailed or that focused on very minor aspects of the course that they may have forgotten. Finally, while I had personal ideas of what are effective instructional practices in AP Calculus, which may have influenced the investigation, the thematic analysis allowed me to focus on the respondent data and place the weight on the student voice. My deep knowledge of the course contents was an important asset in conducting this study because I observed their activity on a daily basis, but it also affected the creation of the survey and, in turn, the data that was gathered.

7.1. Conclusion

This study sought to explore how the AP Calculus practices and pedagogies support and affect student learning and encouragement. The thematic analysis revealed themes and subthemes that were difficult to distinguish because of their overlapping nature. Through several rounds of analyzing and organizing, the data revealed that participants felt supported to meet the demands of the course, that the extended class time provided the necessary duration to master the wealth of course contents, that the course tasks supported and motivated their learning, that the organization of the course was especially inspirational and impactful, that the class fostered transferable skills, and that there were certain struggles or barriers that they needed to overcome. The complexity of these findings points to the difficulty of teaching and learning mathematics in general, and of AP courses in particular because, as Finn and Scanlan (2019) contend,

> It isn't easy, though, to teach AP courses well. Besides mastering the content – and keeping up as frameworks and exams are revised – the pedagogy is challenging, the more so as the "new AP" seeks deeper understanding on the part of students, not just acquisition of a body of knowledge. Helping kids to learn to reason, to analyze, to argue, to write, to work with fellow pupils…to explain why something happened the way it did or how to solve a complex problem – none of this is simple, little of it is instinctive, and much of it goes well beyond typical teacher preparation in the United States (p. 163).

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Appendix

Table A.1
Open-ended questions in the Participant Survey.

	Question
1.	Before diving into some of the specific components of the course, what **feature(s) of the class stand out to you** as having really impacted your learning of AP Calculus AB?
2.	After reflecting on these common **in-class tasks**, what stood out to you in **helping and/or preventing you** from being engaged and motivated to learn? In other words, what did you like, what didn't you like? And why?
3.	After reflecting on these common **out-of-class tasks**, what was especially **helpful or unhelpful** in keeping you engaged and motivated to learn through these tasks?
4.	Are there **any other experiences** that you had outside of class that helped you learn? Or that prevented you from learning?
5.	What **reasoning** led to your decision to choose "optional" or "required" in the previous question [Do you think the *Mandatory Support Course* should be an **optional** or **required** part of AP Calculus AB?]?
6.	After reflecting on some of the structures of the course (*Introduction of New Concepts*, *Mandatory Support Course*, *Grading Scheme*, and *Pace of the Course*), did any of these components **contribute** to your motivation to learn? If so, how/why?
7.	After reflecting on some of the structures of the course (*Introduction of New Concepts*, *Mandatory Support Course*, *Grading Scheme*, and *Pace of the Course*), did any of these components **detract** from your motivation to learn? If so, how/why?
8.	Looking back at your AP Calculus AB course and many of the questions taken up in this survey… (a) What are some of the things about the course that you feel **supported** your learning? (b) What are some of the things about the course that you feel **did not support or detracted from** your learning?

Note. The bold and underlined emphasis was also used in the Participant Survey.

Table A.2
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
Feeling Supported: Participants felt supported to meet the demands of the course through: (a) peer and teacher feedback; (b) class time collaboration; (c) teacher and student engagement outside of class time; and (d) a nurturing classroom environment	Discussing solutions; problem solving on the board; teacher and peer tutoring; reviewing or having access to all of the solutions of given tasks	**Jorge**: "Finally, another crucial aspect that wasn't in the survey was the tutoring hours after school and during lunch. These moments were pivotal to my success in the course as they allowed me to check my reasoning on topics. I found myself using these opportunities often for Problem Sets, Assignments, and Pop Quizzes."
	Building on each other's thinking; collaborating on Group Exams to learn from others, deepen understanding by explaining to peers, and use the results to prepare for the Individual Exam	**Bryan**: "The exams were also very helpful for the same reasons. I liked being able to do these activities with my peers because I could see their thought processes and how they differed from my own which allowed me to see where I would go wrong in a particular problem or allow me to explain where they were going wrong."
	Ability to reach out to the teacher for help; encouraged to collaborate with peers in the classroom outside of class time	**Perla**: "What was helpful in keeping me engaged and motivated to learn through these tasks was knowing that I had a support system around the corner at all times. Ramirez always offered tutoring before and after school and even during lunch time. Not only was Ramirez there as a support system, but so were my classmates. We all had our challenges and were all looking for some guidance from each other."
	Intimacy and encouragement; mistakes are part of learning; discussing multiple solution pathways; small class size; teacher passion and belief in student success	**Luciano**: "Something that i think was helpful to me was the intimacy of the classroom. The environment made me feel like mistakes were a part of the learning process and it was an encouraging space to be in. Being in a room where its okay to start a dialogue about what could be done differently or what was done correctly was key to me."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
Extended Class Time: Participants reported that the extended class time provided the necessary duration to practice and master the course concepts, eased the pressure to learn, and made them realize the importance and value of extensive preparation	Two hours is needed to understand the content; multiplicity of tasks kept engagement high; allowed time to deepen understanding by having the opportunity to collaborate with peers, ask questions, and practice solving problems	**Jorge**: "AP Calculus simply requires that much time to understand. The two hours flew by since the entire time you were practicing some sort of concept. Remaining focused for 2 hours straight is somewhat of a difficulty however the course made sure to do a multiplicity of tasks that kept you both active and engaged in the course. If the 2-hour chunk wasn't required I feel students would miss out on a pivotal time to practice with peers and ask the instructor questions that could be explained clearly. It only made sense that AP Calculus is a 2-hour long course." **Quin**: "I believe that that support course should be required, because there was A LOT of material that needed to be covered, and I don't think that there would have been sufficient time to cover all the material with only one 1-hour class each day. I like to think as Period 1 as the "definitions and new concepts" class and Period 2 as the "practice and collaborate" class."
	Made the pace of the course manageable; realized the importance of extended practice	**Luciano**: "The mandatory support course definitely helped as I didn't feel too much pressure to have to learn things within a set time period. This allowed me to be able to handle the pace at which we worked. It kept my mind thinking 24/7 about calculus so it helped me retain more information easier." **Mary**: "The feature that really stood out to me was taking a class in high school for two hours. That really stood out to me because the idea of taking a class for more than one hour really shows how important it was to fully be ready to learn everything that dealt with AP calculus AB. It impacted my learning in a good way because now I appreciate sitting and learning ap calculus ab for those two hours it helped me succeed."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
In-Class Tasks: Participants felt that most or all in-class tasks were helpful – specifically citing that interactive activities or games supported learning, that projects were impactful, the Warm-Ups were a useful form of review, Practice and Group Exams were valuable preparatory tools, Mock AP Exams provided excellent awareness, and having final review activities was very beneficial	Varied tasks supported learning by helping stay engaged and motivated, think critically, and frequently practice and review concepts	**Diego**: "The in-class tasks that stood out to me in helping me be engaged and motivated to learn include the warm-ups, pop quizzes, exams, and poster presentations. These tasks had helped me feel more engaged and motivated to learn since they emphasized the main topics of a section and reflected on the ideas to be mindful."
	Interactive activities and/or games provided the opportunity to review in a fun and stimulating manner	**Aidan**: "I also believe that the gmaes such as family feaud and kahoot as well as jopardy were very fun and allowed fo r extra review in a very fun and motivational way."
	Projects helped better understand the concepts and were quite impactful to both learning and building community	**Flavid**: "Something that I can recall as having impacted my learning of AP Calculus AB, are the projects we did that involved the subjects of the course as it helped me better understand what we were being taught beyond the routine instruction we received on the subjects."
	The Warm-Ups were a useful way to start class by gaining a refresher	**Trey**: "The warm-ups were the most I personally liked the most because They will help my day get started and give my brain a refresher as to what we had learned yesterday."
	Practice and Group Exams helped ensure individual success	**Ruben**: "Having the practice exams were also a great addition because not only were they more notes to look back on but it showed us exactly what to expect on upcoming exams."
	Mock AP Exams provided vivid awareness of the expectations, reduced anxiety, allowed to learn from mistakes, and should be an integral part of the course	**Diego**: "Practices that I frequently used that I consider important in preparing me for the AP Calculus AB Exam were to take the all mock exams seriously, both those in-class and take-home, since they offered a good experience on how the AP Calculus AB exam will be and reduced anxiety for the day of the exam."
	Having a lot of time to complete review activities at the end of the year was very helpful	**Flavid**: Another feature that I have strong memories of are the note cards we created ourselves, as it caused me to process the information we were noting down especially from the earlier parts of the class which I otherwise would have had trouble remembering, and to this day I have the notecards in my closet in case I for whatever reason need them."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
Out-of-Class-Time Tasks: Participants reported that most or all Out-of-Class-Time Tasks were helpful and provided a difficult challenge but made the expectations of the AP Exam apparent and manageable because of their alignment to the style, language, and rigor of the exam; in addition, the ability to collaborate on these tasks was both helpful and motivating, the logistics on submission removed pressure, and the content of the tasks provided opportunities to develop deep understanding of the concepts	The out-of-class-time tasks were the right amount of practice; they were very challenging but being encouraged to work on them collaboratively and seek help from the teacher made them comforting and feasible; helping each other made it fun; explaining to others improved understanding; collaboration helped overcome challenges; motivating to recognize peers were also struggling	**Elsa**: "The problem sets and assignments were the right amount of practice and would help me feel supported. There were times I was challenged by them and sometimes that would cause me to feel overwhelmed but, looking back, it was the right amount of practice." **Perla**: "Some things that made me feel extremely supported in my learning in this course were definitely the homework assignments because they were challenging but in a good way. They allowed me to reach out and ask for help and I realized that all my classmates felt the same way I did, and I didn't feel so alone after all."
	Having a week to complete provided time for productive struggle, promoted collaboration, and removed pressure to finish	**Flavid**: "The ability to do them out of class was helpful I believe, as since they were due after a week I wasn't feeling the pressure of a time limit one usually has while working on an assignment in class. Furthermore having the ability to ask for assistance when needed from peers allowed us to compare the assignment to see if we are all on the right track or on a completely different road. Lastly, the ability to ask you for help was especially useful for when not even my peers and I could come to a collective agreement on what was right and what as wrong."
	Most or all were helpful because they promoted mastery; supported learning through review; correcting them was beneficial	**Mary**: "Homework supported my learning a lot because it was practice of what we learned about during our class time. It was a review that helped my brain remember what happened hours ago and it was just great having an assignment to support your learning."
	AP-style questions helped prepare for the AP Exam; multiple-choice questions on Assignments allowed for correction and understanding; the amount and structure of Problem Sets was extremely beneficial because of the challenge they provided	**Jorge**: "I had great enjoyment of the Assignments we were given. Multiple choices allowed me to analyze and reflect on my understanding of given questions. The choices listed were often very close to one another or explored a different route that students may have taken which would lead them down a false rabbit hole. This alone allowed me as a student to correct minor mistakes in my calculations and understandings of the properties of the function which ultimately allowed me to perform well in the class."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
Organization of the Course: Participants conveyed that organization of the course provided: (a) support to learn a challenging subject; (b) opportunities to deepen understanding of concepts through collaboration; (c) an alignment to the content and practices assessed on the AP Exam;	The overall structure provided a feeling of support with the amount of class time and the format or types of tasks which reduced anxiety/stress; supported the learning process through different forms of practice (assessments); increased confidence and conceptual understanding; provided motivation to learn; feeling of engagement through the teacher's practices	**Nancy**: "I honestly feel the whole course was organized was really supporting from how much time we were in class, the assignments we were given, and how much we were about to work with each other as well as on our own was really good." **Karla**: "I believe that all these components did contribute to my motivation to learn, as I felt that math was no longer complicated. The way the course was structured and the assignments students had to do made math less of a difficult challenge and more of an exciting mystery waiting to be solved. With all this support coming from one educator, I found my confidence in math to have increased." **Diego**: "The amount of out-of-class tasks were mostly manageable, but this was due through the instructor's practices that made me feel engaged and passionate about the topics."
	Projects provided opportunities to deepen understanding; tasks encouraged communication; Group Exams allowed learning from mistakes; working together first helped when working alone later; frequent change of seats enhanced teamwork	**Quin**: "What I found very beneficial to my learning was the ability to work collaboratively with other people in the class, and THEN having exams/pop quizzes where we were required to work on our own." **Jorge**: "Warm-ups and Group Projects/Exams gave way to communicate with my peers. The frequent change of seats also kept things refreshing and allowed me to work with all students to gain a better connection and experience working with others. This helped build my teamwork while also assisting with the communication of others."
	Expectations and activities were aligned with the format and demands of the AP Exam; significant practice with an emphasis on understanding	**Ruben**: "This class was designed to help you succeed in your final AP test and all the games and practice exams do that exactly." **Elsa**: "I believe that I was able to perform well in the AP Exam and still to this day retain that information because we were putting a significant amount of time into reviewing, and making sure we understood each concept."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
(d) a variety of activities that promoted understanding, engagement, and motivation; (e) a concise and cohesive delivery of new concepts; (f) a fair Grading Scheme that fostered persistence and confidence; and (g) additional resources and supports to help meet the demands of the course	Constant review activities promoted conceptual understanding and motivation; tasks provided helpful resources, made the content accessible, and promoted engagement	**Karla**: "I believe a feature of the class that did stand out to me was the continuous drills and exams. My previous math courses prepared me to learn the information and be able to recall it for a short period of time. However, the AP course that I took contained more engaging learning experiences, and I found myself yearning to learn more and practice math concepts."
	The delivery of new concepts through traditional note-taking and heavy discussion helped develop understanding, inspiration to learn, provided useful references, was concise, and reduced overwhelmingness with cohesive units and sections	**Vanessa**: "The way notes were structured helped aswell. I recall holding on to my notes from the class for a while after graduation because I put a Iot of work into them but also because they were helpful. This class was the most thought out course I've taken during all of highschool and I think it worked out well." **Diego**: "Although the pace of the course was rather quick, the introduction of new concepts were cohesive and utilized familiar ideas the day prior that it felt relatively easy to absorb and create connections between them."
	The grading scale was relieving; promoted persistence, encouragement, and confidence; aligned to the AP Exam expectations; allowed for a focus on understanding	**Luciano**: "The grading scheme also helped with the anxiety that the class put on me. It allowed me to work without the worry of my grade looming over me. I honestly think that it contributed to the success I had in class." **Quin**: "Because there was such a huge range for getting an A-letter grade, this allowed me to be less focused on my grade for the class and more focused on practicing furthering my learning in preparation for the AP exam."
	Online options for additional support; outside tutoring; mandatory monthly Saturday review sessions	**Bryan**: "If all else failed and we couldn't solve them between ourselves we would come to you for help and that was very beneficial to us all as well." **Isaac**: "Also how it was mandatory to go to the AP readiness I believe was called. To learn more about curtain sections of Calc AB."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
	Fast pace promoted studying more at home; impacted study habits and increased inclination to work with others while in college; opened future possibilities by being challenged and finding success	**Jorge**: "Sometimes the pace of the course was a bit too fast, as a week would go by with me unable to grasp the latest concept introduced. However, this merely motivated me to study harder at home. I had become passionate about math and grew to accept that in order to perform well I must put in the work." **George**: "Presentating in front of the whole class and engaging. It really preps you for class presentations in other subjects and in college. Gets you out your comfort zone."
Transferable Skills Gained: The participants reported gaining transferable skills as a result of the course, describing how the fast pace of the course promoted studying outside of class time and that it impacted their future behavior or beliefs; participants also recounted how the course gave them a preview of the structure and rigor of college courses	Preview of how college courses are organized; challenge of the course provided support to succeed in a college environment; found transferable benefits of being forced to present in class; Math Essay was helpful for college; encouraged to pursue a STEM degree	**Jorge**: "Each aspect of the course had a special way of preparing me for college, as the homework load gave way to building up my time management and study habits. Test prepared me for not only the AP test but the Midterms and Finals that were new to me in college. Mentally preparing me for how to take tests and perform to the best of my ability." **Olivia**: "The problem sets impacted my learning because it was the first time I had needed to dedicate over three hours on an assignment. It was a great foreshadowing to college coursework." **Isaac**: I like the math essay because at the time it seem like it was something useless but now that I think about it and reflect on it it was help full. When other classmates would present their essays I got to understand concepts I was having trouble with and also it with my own essay helped me understand more. The presentation just helped because it makes you think and put yourself in a way they you had to give instructions on how to solve a problem that would be easy to solve but would have to go into depth to explain to someone younger than you which was a great way for me to understand some concepts now that I am in college and most of the time is spent reviewing notes or reading the book. Then explain it to someone to see if I really understand the notes or work." **Vanessa**: "I believe that my desire to continue in STEM came from my highschool calculus AB course. This supported my decision to pursue Mechanical Engineering."

Table A.2 (continued)
Description of the subthemes and example data segments

Theme Description	*Subthemes*	*Example Data Segment*
Struggles or Barriers to Success: The participants described a variety of struggles or barriers to success in the course: (a) presenting work was stressful and disengaging; (b) the pace of the course was overwhelmingly fast; (c) the Math Essay was not very helpful and demotivating; (d) some aspects of the course raised anxiety levels or created stressful situations; (e) personal conflicts made it difficult to remain committed to learning; and (f) varied prior mathematical experiences	Caused nervousness; felt overwhelming; challenging but enjoyable; somewhat helpful but stressful	**Hugo**: "On the other hand, something that would prevent me from engaging in the work would be my shyness. Presenting my work or projects would be an overwhelming task as I am not an open person."
	The introduction of new concepts was too fast; fast pace detracted motivation and made it difficult to keep up	**Ruben**: "So, the pace of class was definitely fast, and it was sometimes difficult to wrap my head around more complex concepts. The amount of math there is to AP CALC AB was also extremely vast and felt like there was never an end."
	Math Essay was unhelpful, stressful, not impactful, and did not support learning	**Diego**: "Although most of the tasks were engaging and motivated me to learn, the math essays somewhat prevented me in motivating to learn since it felt out of place in comparison to the other in-class tasks. Nevertheless, I could understand its usefulness in discussing the role of certain ideas in calculus."
	Random calling was helpful but stressful; Pop Quizzes were overwhelming; Problem Sets were overly challenging; excessive amount of work to do outside of class, albeit helpful	**Aidan**: "The random calling was helpful however it was somewhat anxiety inducing as you never knew if youd be called." **Trey**: "The problem set and assignments were by far hard and all the time I would have to connect with my peers and work out the problem with them."
	Bad influences; mental health issues; personal problems	**Nancy**: "My mental health and a lot of problems at home with my family that really unmotivated me and prevented me from being able to learn and stay committed to Ap calculus."
	Diverse mathematical experiences affected the transition to AP Calculus	**Ruben:** "Pre-Calc should definitely have its own class. I know many schools do have that, but I took my pre-Calc class afterschool 2 times a week for like a whole 4 months. I felt prepared for Ap Calc AB, but it definitely wasn't enough."

Table A.3
Individual participant demographics

Name	*Gender*	*Grade*	*Parent Education*	*AP Exam Score*
Aidan	Male	Junior	HS Graduate	5
Bryan	Male	Junior	No HS Completed	5
Cris	Male	Junior	No HS Completed	5
Diego	Male	Junior	No HS Completed	5
Elsa	Female	Senior	No HS Completed	5
Flavid	Male	Junior	HS Graduate	4
George	Male	Junior	No HS Completed	5
Hugo	Male	Senior	No HS Completed	3
Isaac	Male	Junior	No HS Completed	5
Jorge	Male	Junior	HS Graduate	5
Karla	Female	Senior	HS Graduate	4
Luciano	Male	Senior	No HS Completed	4
Mary	Female	Junior	HS Graduate	3
Nancy	Female	Junior	HS Graduate	2
Olivia	Female	Junior	No HS Completed	3
Perla	Female	Senior	Some College	3
Quin	Male	Junior	No HS Completed	5
Ruben	Male	Junior	Associate Degree	1
Sonia	Female	Junior	HS Graduate	4
Trey	Male	Junior	No HS Completed	3
Uriel	Unreported	Junior	Unreported	Unreported
Vanessa	Female	Senior	No HS Completed	5

Note. The highest parental education reported is presented in the table; HS = High School.

Table A.4
Descriptions of the prevalent course components

Course Component	*Description*
Mathematical Tasks: Warm-Ups	A 15-20 minute daily activity during which students typically solved 2-3 multiple-choice problems or 1 multi-part free response problem. These Items were either: (a) taken verbatim from released AP Exams or College Board materials; or (b) created to mimic the same style and rigor of the Items on the AP Exam. Students were encouraged to work collaboratively and use supporting materials before the solutions were presented and discussed. Facilitation of the solutions was either done by the teacher or a student at the board.
Pop Quizzes	A 30-40 minute activity (85 throughout the school year) during which students typically solved 5 multiple-choice problems (80% of the time) or 1 multi-part free response problem (20% of the time). These Items were either: (a) taken verbatim from released AP Exams or College Board materials; or (b) created to mimic the same style as the Items on the AP Exam, but were often more rigorous or took longer to complete. Students were usually encouraged to work collaboratively but they could not use any supporting materials nor ask the teacher for help. On occasion, students were asked to work independently. The solutions were normally discussed the next day through a whole-class discussion.
Exams	There were three types of exams: Practice Exams (completed collaboratively in- and out-of-class with supporting materials), Group Exams (completed collaboratively in-class without supports), Individual Exams (completed independently in-class without supports). All three exams usually contained the same number of Items and assessed the same content but were certainly not identical with a mere change of numbers or equations. Four exams (12 total) were completed each semester. The Exams were balanced between multiple-choice and free-response problems (e.g., 15 multiple choice problems, worth 2 points each, and 3 free response problems, worth 9 points each). The solutions to each Exam were discussed through a whole-class discussion. Students were allowed to re-take a modified version of an Individual Exam.
Projects	There were three main types of projects: Hosted Gallery Walks (a combination of a group poster and jigsaw presentation), Math Essays (individual written accounts comparing different techniques or concepts within a section), Presentations at the Board (partner or individual presentation of work, usually an entire free response problem, to the whole class). Typically, at least one Hosted Gallery Walk was completed per month and two Math Essays per semester (started and revisited in class but mostly worked on outside of class time), which were presented one per day upon completion. Presentations at the Board were more common in the second half of the second semester when all of the course concepts had been presented.

Table A.4 (continued)
Descriptions of the prevalent course components

Course Component	*Description*
Out-of-Class-Time Practices:	
Assignments	A weekly task consisting of 30 multiple-choice problems that mainly focused on a central theme such as Related Rates or Volume in the Plane, but also included several review problems from prior topics. The Items on the Assignments were created to mimic the same style as those on the AP Exam but were often more challenging. Students were encouraged to work collaboratively, use supports, and seek teacher help throughout the week.
Problem Sets	A weekly task consisting of 6 multi-part free response problems that mainly focused on a central theme such as Optimization or Differential Equations, but often contained parts from prior topics. The Items on the Problem Sets were created to mimic the same style as those on the AP Exam but were often more involved. Students were encouraged to work collaboratively, use supports, and seek teacher help throughout the week.
Course Structures:	
Introduction and Discussion of New Concepts	An almost daily activity during which the teacher introduced new concepts, definitions, ideas, and procedures in writing at the whiteboard for about 40 minutes. Students were expected to write the information in a notebook, ask questions, pair-discuss, and anticipate being randomly called upon to answer questions or provide a summary.
Mandatory Support Course	Students were required to enroll in a support course that provided extended time (almost two hours each morning) to work on learning the calculus concepts of the class by practicing, presenting, and discussing ideas.
Grading Scheme	The Grading Scheme for the course was not very typical of high school mathematics classes but was reflective of the expectations of the AP Exam: A percentage grade between 70 and 100 corresponded to an A, a percentage grade in the 60s corresponded to a B, a percentage grade in the 50s corresponded to a C, a percentage grade in the 40s corresponded to a D, and a percentage grade below 40 corresponded to an F.
Pace of the Course	The Pace of the Course was rather quick – with new concepts being introduced almost every day. This strategy provided the opportunity to cover all the learning objectives of the course by the middle of March and have ample time to review and achieve mastery of the topics before the AP Exam.
AP Readiness	Students were required to attend monthly Saturday sessions at UCLA, provided by the AP Readiness program, to gain up to 4 hours of additional exposure to calculus concepts, procedures, and students.

Note. Brief descriptions of the course components and pictures were included in the Participant Survey.

Figure A.1
Sample Warm-Up activity

AP Calculus AB
Warm-Ups #23 – 24: Continuity and Differentiation

Name _____
Date _____ Per. ____

Directions: You must show all work leading to your answer. No calculator is allowed for these problems.

Warm-Up #23

1. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

   (A) -3 (B) -1 (C) 1 (D) 3

---

2. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

   (A) $y = 7x - 3$ (B) $y = 7x + 11$ (C) $y = -5x - 1$ (D) $y = -5x - 5$

$$f(x) = \begin{cases} x + 2 & \text{for } x \leq 3 \\ 4x - 7 & \text{for } x > 3 \end{cases}$$

3. Let f be the function given above. Which of the following statements about f are true?

   I. $\lim_{x \to 3} f(x)$ exists.

   II. f is continuous at $x = 3$.

   III. f is differentiable at $x = 3$.

   (A) I only

   (B) II only

   (C) I and II only

   (D) I, II, and III

1

Note. Warm-Ups were used as a formative assessment and were only checked for completion monthly.

Figure A.2
Sample Pop Quiz activity

AP Calculus AB
Pop Quiz #9: Riemann Sums and Definite Integrals

Name _____
Date _____ Per. ____

Directions: You may use scratch paper, if necessary. No notes, text, or calculator.

1. Let f be the function given by $f(x) = x^2 + 1$. If four subintervals of equal length are used, what is the value of the right Riemann sum approximation to $\int_{-1}^{3} f(x)\,dx$?

   (A) 10 (B) 14 (C) 16 (D) 18

x	0	1	4	8	10
$f(x)$	4	5	10	12	8

2. The values of a continuous function f for selected values of x are given in the table above. What is the value of the right Riemann sum approximation to $\int_{0}^{10} f(x)\,dx$ using the subintervals [0,1], [1,4], [4,8], and [8,10] ?

   (A) 70 (B) 62 (C) 83 (D) 99

$$\lim_{n\to\infty} \sum_{k=1}^{n} 2\left(4 + \frac{2k}{n}\right)^2 \frac{2}{n}$$

3. The limit above is equal to which of the following definite integrals?

   (A) $\int_{2}^{4} 2x^2\,dx$

   (B) $\int_{2}^{4} 4x^2\,dx$

   (C) $\int_{4}^{6} 2x^2\,dx$

   (D) $\int_{4}^{6} 4x^2\,dx$

1

4. The expression $\frac{2}{25}\left(\sqrt{\frac{2}{25}} + \sqrt{\frac{4}{25}} + \sqrt{\frac{6}{25}} + \cdots + \sqrt{\frac{50}{25}}\right)$ is a Riemann sum approximation for

   (A) $\frac{1}{25}\int_{0}^{2} \sqrt{x}\,dx$

   (B) $\int_{0}^{2} \sqrt{x}\,dx$

   (C) $\int_{0}^{50} \sqrt{x}\,dx$

   (D) $\int_{0}^{1} \sqrt{2x}\,dx$

5. If f is the function given by $f(x) = 2x$, which of the following statements is true about a Riemann sum approximation to $\int_{2}^{10} f(x)\,dx$?

   I. A left Riemann sum approximation using four subintervals of equal length is equal to 80.
   II. A right Riemann sum approximation using four subintervals of equal length is equal to 112.
   III. A midpoint Riemann sum approximation using two subintervals of equal length is equal to 96.

   (A) I only
   (B) II only
   (C) I and II only
   (D) I, II, and III

2

Note. Students typically spent 30-40 minutes collaborating on Pop Quizzes without any additional resources.

Figure A.3
Sample pages from an Individual Exam

14. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative minimum at $x = 2$.

II. A relative minimum at $x = 3$.

III. A relative minimum at $x = 4$.

(A) I only (B) III only (C) I and III only (D) II and III only

15. Let f be a function that is continuous on the closed interval $[2,4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

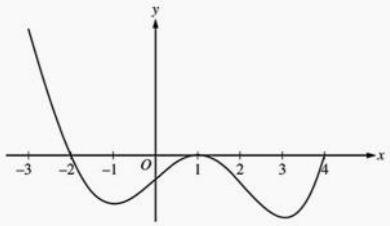
(A) $f(x) = 13$ has at least one solution in the open interval $(2,4)$.

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2,4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2,4)$.

8



Graph of f'

16. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $[-3,4]$. The graph of f has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$.

(a) Find all x-values, on the open interval $-3 < x < 4$, at which f has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

(b) Given that $f(1) = 3$, write an equation for the line tangent to the graph of f at $x = 1$.

9

Figure A.4
Sample pages from an Assignment

AP Calculus AB
Assignment #9: Motion and Potpourri

Name _____
Date _____ Per. ____

Directions: You may use scratch paper, if necessary. No calculator is allowed for these problems.

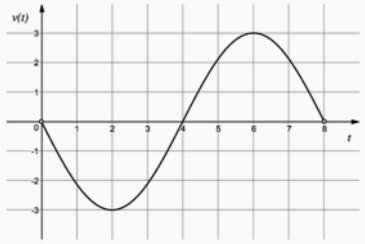
1. A particle moves along the x-axis such that its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$ for time $t \geq 0$. At what values of t is the particle at rest?

   (A) 3 only (B) $\frac{7}{2}$ only (C) 3 and $\frac{7}{2}$ (D) 3 and 4

2. For time $t \geq 0$, the velocity of an object moving along a straight line is given by $v(t) = (t-5)(t-2)^2$. At what values of t is the acceleration of the object zero?

   (A) 2 only (B) 4 only (C) 2 and 4 (D) 2 and 5

1



3. Over the time interval $0 < t < 8$, a particle moves along the x-axis. The graph of the particle's velocity, v, is shown above. On which of the following time interval is the particle's speed decreasing?

   (A) $(0,2) \cup (4,6)$ (B) $(6,8)$ only (C) $(0,2) \cup (6,8)$ (D) $(2,4) \cup (6,8)$

4. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = -5t^2$. What is the particle's average velocity on the time interval $0 \leq t \leq 3$?

   (A) -45 (B) -30 (C) -15 (D) -10

5. A particle moves along a straight line so that at time $t \geq 0$ its position is given by $x(t) = (t-a)(t-b)$, where a and b are constants. If $a \neq b$, for which of the following values of t is the particle at rest?

   (A) $a \cdot b$

   (B) $\frac{a+b}{2}$

   (C) $a + b$

   (D) $2(a+b)$

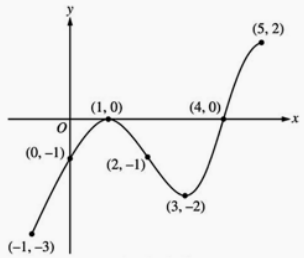
2

Figure A.5
Sample pages from a Problem Set

AP Calculus AB
Problem Set #11: Graphical Analysis and Potpourri

Name _____
Date _____ Period ____

Directions: Show all steps leading to your answer. No calculator is allowed for these problems.



Graph of f'

1. The figure above shows the graph of f' on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

   (a) For what values of x is the graph of f decreasing and concave down? Justify your answer.

   (b) Find the x-values at which f has a critical point. Classify each critical point as the location of a local minimum, a local maximum, or neither. Justify your answers.

1

   (c) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

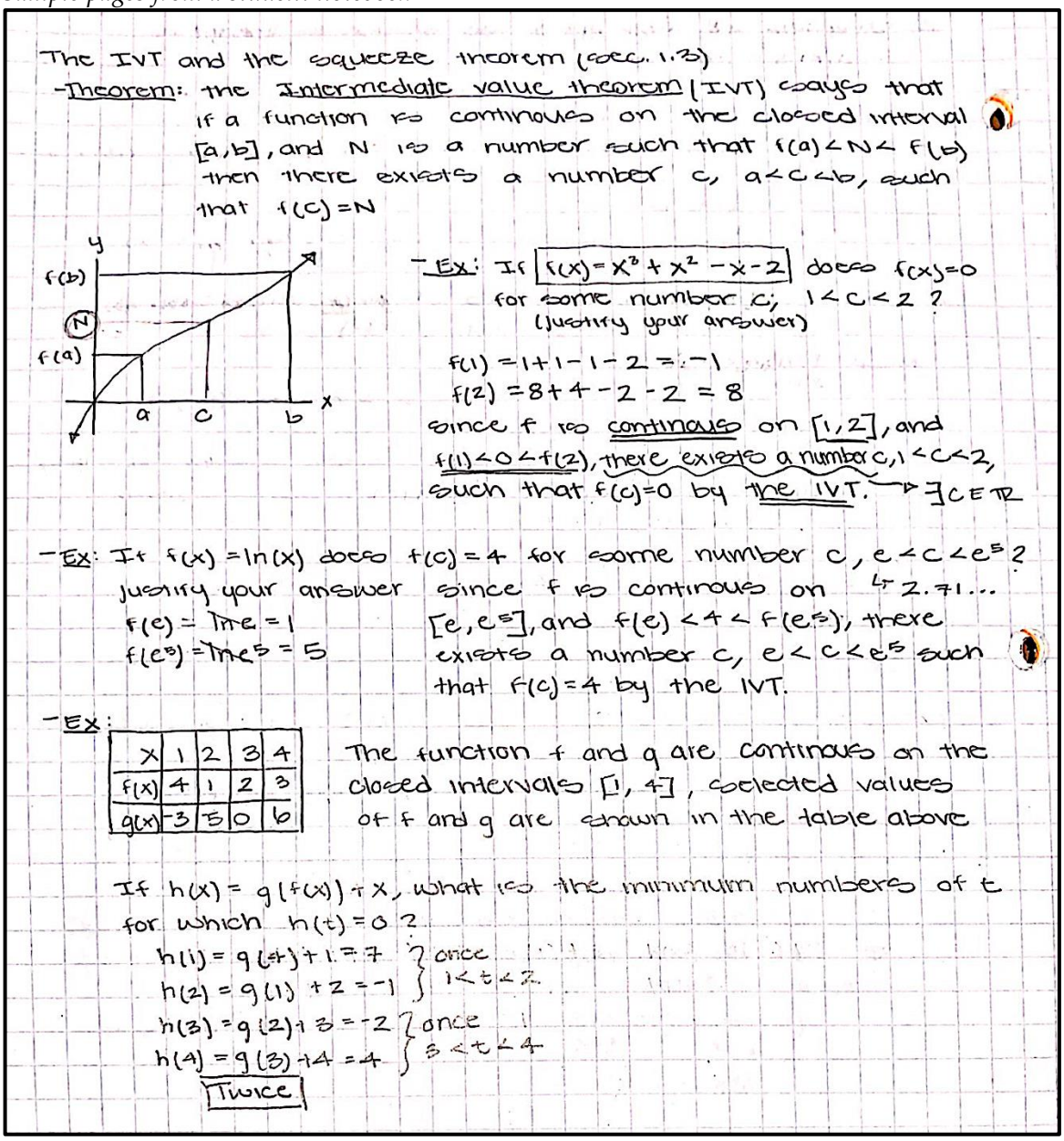
   (d) Let g be the function defined by $g(x) = xf(x)$. Find an equation of the line tangent to the graph of g at $x = 2$.

2

Note. The graphing calculator was only allowed on the Problem Sets during the second half of the second semester of the school year.

Figure A.6

Sample pages from a student notebook



Note. Graphs were often printed, precut, and handed to students to paste in their notebook.

Figure A.7
Sample student work from a Math Essay

THE INTERMEDIATE VALUE THEOREM – EXTREME VALUE THEOREM – MEAN VALUE THEOREM – ROLLE'S THEOREM

THERE ARE FOUR BASIC THEOREMS EVERY CALCULUS STUDENT SHOULD UNDERSTAND TO EXCEL THEIR ABILITY IN DECIPHERING A FUNCTIONS PROPERTIES. EACH ONE IS TO BE USED IN A CERTAIN STRATEGIC WAY TO IDENTIFY PROPERTIES OF CONTINUOUS FUNCTIONS. AND EVERY THEOREM NEEDS THE FUNCTION TO BE CONTINUOUS AND ON A CLOSED INTERVAL [A,B] IN ORDER TO BE APPLIED TO A FUNCTION.

BEGINNING WITH, WE HAVE THE INTERMEDIATE VALUE THEOREM (IVT), THIS IS A PRECISE MATHEMATICAL THEOREM THAT APPLIES TO THE PROPERTIES OF CONTINUOUS FUNCTIONS. THE (IVT) STATES THAT IF A FUNCTION IS CONTINUOUS ON A CLOSED INTERVAL [A,B], AND IF L IS ANY NUMBER BETWEEN F(A) AND F(B), THEN THERE MUST BE A VALUE, X = C, WHERE A < C < B, SUCH THAT F(C) = L. FOR EXAMPLE, IF WE TAKE THE FUNCTION WE CAN FOLLOW THIS PROCESS OF STEPS TO VERIFY IF OUR FUNCTION HAS A ZERO:

$$f(x) = 4x^3 - 6x^2 + 3x - 2 \text{ ON THE CLOSED INTERVAL } [1,2]$$

FIRST STEP: FIND F(A):

$$f(1) = -1$$

SECOND STEP: FIND F(B):

$$f(2) = 12$$

THIRD STEP: IDENTIFY IF THE (IVT) IS VALID:

$$-1 < 0 \ AND \ 12 > 0$$

FOURTH STEP: VERIFY WHETHER THE (IVT) IS APPLIED TO YOUR FUNCTION:

SINCE $f(1) < 0$ AND $f(2) < 0$ THERE EXIST A VALUE C SUCH THAT $f(C) = 0$

Note. Students were highly encouraged but not required to type their essays.