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Research Article

Bridging early mathematics and real-life applications in elementary classrooms

Mustafa Açıkgöz[1] and Zeynep Yıldız[2]

[1]*Yıldıztepe Primary School, İstanbul, Türkiye (ORCID: 0000-0001-8552-2251)*
[2]*Yildiz Technical University, İstanbul, Türkiye (ORCID: 0000-0002-1439-3592)*

The purpose of this study is to investigate the reflections that arise during the implementation of mathematical modeling activities with third-grade primary school students. To achieve this goal, a case study methodology was employed. The study consisted of 9 third-grade students (average age 8), selected through purposive sampling from a public primary school. In the 5-week preparatory phase preceding the main intervention, a distinct mathematical modeling activity was implemented each week to facilitate students' acquisition of foundational knowledge and experience on. Subsequently, a focus group consisting of three students was selected using criterion sampling. These students engaged with a mathematical modeling problem entitled 'The Healthy Money Saving Problem,' developed by the authors, under carefully structured problem-solving environments and processes. The entire process was video recorded, and the dialogues were transcribed verbatim. These transcriptions, along with the students' individual work sheets, were analyzed in alignment with the modeling cycle framework. The results indicate that students engaged in collaborative group work, where they generated and critically discussed mathematical ideas, interpreted and analyzed data, used relationships between data points, and applied mathematical concepts to real-world problems. Furthermore, they developed multiple solution strategies and were able to discard unsuitable approaches. The students also employed a range of mathematical skills such as problem solving, critical thinking, and reasoning throughout the process. Additionally, the students integrated informal knowledge (the knowledge students acquire outside of school) into the modeling process and made efforts to generalize their models to validate their applicability in real-life contexts.

Keywords: Cognitive processes; Healthy money saving; Mathematical modeling; Third-grade primary school

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1. Introduction

In the 21st century, where the need to access, utilize, and produce knowledge has been steadily increasing, countries have shifted from individualism to the concept of global citizenship. As a result, one of the fundamental goals for countries has become the cultivation of students with the characteristics required by the modern world, in the path to becoming global citizens (Ministry of National Education [MoNE], 2013). Therefore, fostering individuals with skills such as defining, explaining, analytical thinking, structuring, predicting, verifying, working in groups, and generating effective and creative solutions to the problems they encounter has become one of the

Address of Corresponding Author

Zeynep Yıldız, PhD., Yildiz Technical University, Faculty of Education, A-312, Esenler, İstanbul, Türkiye.

zeyildiz@yildiz.edu.tr , zeynepyildiz.2005@hotmail.com

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key objectives of contemporary education (English & Watters, 2004). When examining the objectives of mathematics education, understanding and explaining mathematical concepts and systems, testing assumptions, solving relationships, explaining, and learning to reconstruct have become essential for students (Thomas & Hart, 2010). From this perspective, mathematics education holds a crucial place within the education system for producing individuals who can solve problems, think critically, and reason effectively. In this regard, elementary education plays a significant role in developing these skills in children (English & Watters, 2004). In Turkiye, the curriculum focused on fostering such individuals was revised in 2005. The elementary school mathematics curriculum aims to cultivate individuals who can interpret what they have learned in the school environment and beyond, generate their own meanings, and apply these meanings to real-life situations (MoNE, 2018). Today, simply memorizing mathematical processes and applying them in the same way to similar problem situations is no longer sufficient. It is necessary to expose students to complex problem situations that promote the development of mathematical thinking and the creation of new concepts, allowing them to gain experience and prepare for their future lives beyond school (Lesh & Zawojewsky, 2007). Producing individuals who are aware of the connection between mathematics and the real world, and who enjoy and take pleasure in it, is one of the most important goals of mathematics education (Doruk, 2010). In this context, mathematics education has increasingly focused on practices where meaningful relationships between daily life and mathematics can be established (De Corte, 2004).

With the growing importance of the relationship between daily life and mathematics, mathematical modeling processes have emerged as a significant component of teaching and learning processes in mathematics education (Lesh et al., 2007). The implementation of model-eliciting activities involving mathematical modeling processes starting from elementary school years will be beneficial in fostering the teaching-learning processes mentioned in the curriculum (Carlson et al., 2016; English, 2006). As explicitly stated, mathematical modeling, which offers numerous benefits for students, is broadly defined as the process of representing a real-life problem mathematically (Ofem et al., 2024). Mathematical modeling does not merely aim to clarify a specific situation by selecting and examining a real-world scenario and performing simple calculations on relevant parameters. It goes beyond this, involving the observation of the real-world situation, the identification of relationships within the scenario, the application of mathematical analysis to the situation, the attainment of results, and the subsequent re-interpretation of the model (Swetz & Hartzler, 1991). According to Geiger et al. (2021), mathematical modeling consists of processes such as defining a problem encountered in real life, creating a mathematical representation related to the problem, determining the mathematical solution to the problem, interpreting the solution in relation to the original problem, and evaluating the appropriateness of the solution for the problem. According to Lesh and Doerr (2003), mathematical modeling refers to a phase or process occurring within the model-eliciting activities. In this context, model-eliciting activities are problems where students, working collaboratively in groups, are asked to generate solutions based on numerous possibilities to create a generalizable and prototypical model. Moreover, model-eliciting activities are non-routine, real-life-related problem types that may contain various potential solutions (Lesh & Doerr, 2003). Mathematical modeling activities require identifying the parameters, relationships, and hypotheses related to real-life problem situations, discovering mathematical structures, expressing these structures using mathematical language, employing various strategies for solutions, reviewing and interpreting the solution, and validating it again (Wess et al., 2021).

Problem-solving activities and traditional methods have been found inadequate in helping students transfer their mathematical knowledge to real-life situations and in fostering their problem-solving and mathematical thinking skills. As a result, mathematics educators have increasingly turned to mathematical modeling as a more effective alternative (Jablonski et al., 2023). Based on the results of the three-year cycles of PISA studies, researchers in many countries have begun to question the preparedness of their students to address real-world problems they will encounter outside of school and in their future professional lives (Blum, 2002; English, 2006; Mousoulides, 2007). When examining the performance rankings of participating countries in PISA assessments, it is evident that Turkiye's performance in mathematical literacy and mathematics lags behind many of the participating countries, placing Turkiye lower in the ranking (OECD, 2019a, 2019b). Additionally, when examining the achievements of students at the higher levels, specifically levels 5 and above, which involve higher-order thinking, it is observed that Turkiye's performance is generally below the OECD average. The fact that Turkiye predominantly resides at the 1st level, which involves more routine problem-solving steps, suggests that students have not sufficiently developed abilities to understand complex systems, plan, control results, and solve multi-faceted, multi-stage problems where communication is critical, as well as to quickly adapt to ever-evolving conceptual systems.

Studies have shown that engaging students in modeling activities enables them to solve complex, multi-component problems that reveal their thinking processes and enhance their understanding (English, 2006). Therefore, the importance of exposing students to mathematical modeling activities that involve real-life problem situations from an early age, starting in elementary school, has become increasingly significant (Carlson et al., 2016; Watters et al., 2004). A review of international literature on mathematical modeling activities reveals that research has increased and has primarily focused on the higher grades (Doruk, 2010; English & Watters, 2005; Leiss et al., 2010; Tekin, 2012; Tekin Dede & Bukova Güzel, 2013). When examining the national literature, however, it is apparent that there are very few studies on modeling processes at the elementary school level, with existing research largely focusing on fourth-grade students (Şahin, 2019; Şahin & Eraslan, 2017; Ulu, 2017).

Although the implementation of modeling activities, which are highly important for achieving the goals of Turkish national education and mathematics education, from the early years of primary school is clearly essential, the limited number of studies on this topic in the national literature is striking. Particularly, there is no existing study related to mathematical modeling activities at the third-grade level. From this point of view, the mathematical modeling activities to be developed within the scope of this study are intended to contribute to the literature and provide insights into mathematical modeling, especially for classroom teachers and educators in general. At the same time, these activities aim to reveal students' problem-solving approaches and performances during mathematical modeling processes conducted with younger age groups. Thus, it is expected that presenting data and analysis results regarding the development of mathematical literacy at early ages will also contribute to the literature from this perspective.

In this context, the research question of the study is: What are the reflections of mathematical modeling processes among third-grade primary school students? In line with this main question, the sub-questions of the study are as follows:

RQ 1) What are the reflections of the "Understanding the task" stage in the mathematical modeling process among third-grade primary school students?

RQ 2) What are the reflections of the "Establishing the model" stage in the mathematical modeling process among third-grade primary school students?

RQ 3) What are the reflections of the "Using mathematics" stage in the mathematical modeling process among third-grade primary school students?

RQ 4) What are the reflections of the "Explaining the results" stage in the mathematical modeling process among third-grade primary school students?

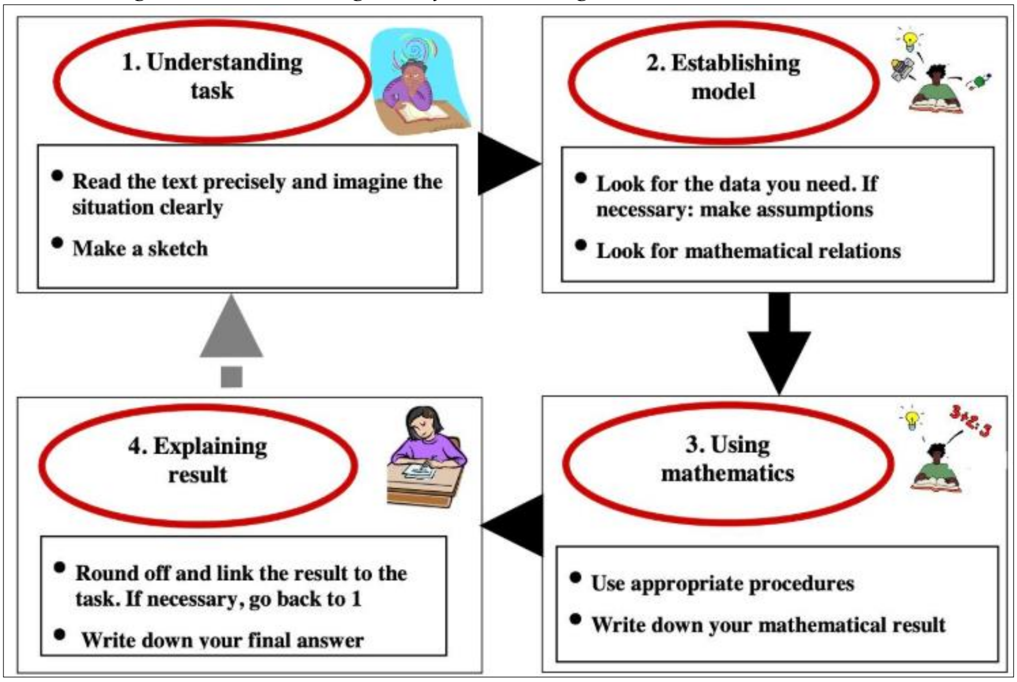
2. Theoretical Framework

Models are conceptual systems found in the minds of problem solvers or students, as well as in the equations they use, computer programs, diagrams, or other tangible representational media (Lesh & Doerr, 2003). In other words, models can be tools used to understand and interpret complex systems, and they may also arise from experiences, drawings, and spoken language (Lesh & Doerr, 2003). When examining the concept of modeling, it is understood that a situation can be modeled in different ways by different individuals. Therefore, models are representations that reflect the thinking of the individual modeling and cannot be considered universal systems (Dindyal, 2010).

In mathematical modeling, Blum and Niss (1991) argue that it is necessary to transfer concepts, data, conditions, relationships, and hypotheses into mathematics. Furthermore, it has been emphasized that modeling involves a process of mathematization, which spans from a real-life situation to a mathematical model. Mathematical modeling is a cyclical process in which real-world problems are abstracted and transferred into mathematical language, analyzed, and then tested for solutions (Haines & Crouch, 2010). In this process, everyday events, problem situations, and their relationships are represented mathematically, and mathematical patterns are derived. Rather than providing students with traditional or routine problems, mathematical modeling activities involve complex real-life situations with problem contexts that do not have a pre-determined solution path or a single result. In these problem situations, students have the opportunity to generate their own ideas and express them. Therefore, mathematical modeling activities contribute to the development of students' creativity skills (Blum, 2002; Blum & Ferri, 2009). Berry and Houston (1995) describe mathematical modeling as a method for solving mathematical problems, highlighting that a mathematical model presents the relationship between multiple variables of a problem or situation mathematically. Blum and Niss (1991) emphasize that mathematical modeling deepens students' insight into real-world phenomena, strengthens their mathematical learning—encompassing concept formation, understanding, motivation, and retention—cultivates a wide range of mathematical competencies and related skills, and ultimately promotes a more precise grasp of mathematics.

Modeling activities occur in a cyclical process. Blum and Ferri (2009) have developed a four-stage modeling cycle. Although the stages in this cycle are numbered, it should not be interpreted as a strict sequence that must be followed by those engaged in modeling work. As modelers pass through these stages, they reflect on the difficulties and thought processes they encounter, which they also share with practitioners. In this regard, the modeling cycle provides significant facilitation in identifying the challenges and thought processes encountered by modelers during the mathematical modeling process.

Figure 1
A Four-Stage Problem-Solving Plan for Modeling Problems (Blum & Ferri, 2009)



In the first stage, understanding the problem, students strive to comprehend the real-world problem by thoroughly reading the problem, visualizing it in their minds, and creating sketches.

Additionally, they examine relevant tables and graphs related to the problem. In the second stage, model-eliciting, relationships between data are explored, mathematical relationships are sought, and a model is constructed. In the third stage, application of mathematics, mathematical operations are applied to derive mathematical results. Finally, in the fourth stage, explaining the results, the obtained results are presented in the form of a report and evaluated from various perspectives. Upon examining the four-stage solution plan, it is evident that students undergo a multidimensional, interactive process in solving modeling problems. This process involves multiple cycles of trial and error, where students employ metacognitive thinking skills, engage in critical thinking, and provide feedback to one another to enhance their cognitive development.

3. Method

3.1. Research Design

This study employed the qualitative research method, specifically using the case study design. Case study involves an in-depth examination of one or more cases within their specific context (Merriam & Tisdell, 2016). The case examined in this study is to uncover the thinking processes of third-grade primary school students during their model creation processes through mathematical modeling activities. For this purpose, observations were conducted throughout the students' model creation process, and their worksheets were thoroughly examined using video and audio recordings.

3.2. Participants

The study group of this research was composed of students who were enrolled in the third grade of a public primary school in Istanbul during the 2022-2023 academic year. Prior to the formation of the study group, a five-week preparation phase was planned and implemented, where original mathematical modeling activities were conducted for a total of three hours each week, with one hour dedicated to preparation. During these activities, observations of the students, group interactions, and the reports presented by the groups were used to identify the students who would participate in the main study through purposive sampling. Purposive sampling is useful for discovering and explaining phenomena in many cases. The study of all cases meeting predetermined criteria is called criterion sampling. These criteria may be listed in advance or established by the researcher (Merriam & Tisdell, 2016). Therefore, the researcher identified specific criteria and aimed to gain in-depth insights based on these criteria. The researchers selected three students for the main study group based on specific criteria: their ability to collaborate effectively with peers over five weeks, their minimal difficulty during model creation activities in the preparation phase, and their high self-confidence in generating, expressing, and defending original ideas. To uphold ethical standards and protect participant identities, pseudonyms were used in place of real names.

3.3. Preparation Phase

In order to prepare the students for the main study to be conducted with the focus group, several preliminary activities were carried out. These activities aimed to help the students learn about mathematical modeling, understand at which stages it can be implemented, and gain knowledge regarding its purpose and importance. The literature reveals that there is limited research on mathematical modeling at the elementary school level (Şahin, 2019; Şahin & Eraslan, 2017; Ulu, 2017), indicating that elementary school students are inexperienced in this field. Therefore, it became necessary to conduct preliminary activities to provide the students in the study group with experience related to the topic. To achieve this, a different mathematical modeling activity was implemented each week over a five-week period. These activities were organized within the scope of the research, and feedback was collected from two mathematics education researchers and six graduate-level classroom teachers to finalize the activities with necessary additions and corrections. In this process, the content was selected to enable students to engage in mathematical modeling at an elementary school level. Furthermore, problem scenarios involving real-life situations were created to capture the students' interest. The activities and their content are as follows:

Activity 1: Choosing the Appropriate Winter Coat: In this problem, a table containing information about 10 different coats is provided. The variables in this table include thickness, number of pockets, waterproofness, whether it has a lining, whether it has a hood, warranty period, and price. Students are asked to develop a mathematical model to choose the most suitable coat in terms of price/performance.

Activity 2: Purchasing Books from Online Shopping Sites: In this problem, students are given information about seven different online shopping sites where two different books can be purchased. A data table comparing these shopping sites is provided, showing differences in the price of the first book, the price of the second book, shipping cost, shipping time, seller ratings, and sales volume. Students are required to develop a mathematical model to select the most suitable shopping site for purchasing both books.

Activity 3: Which Product Should We Plant? In this problem, a decision is made regarding which crop (potatoes, corn or garlic) should be planted on a farm, and the decision needs to be the most suitable for a farmer. Information about the crops for the years 2018-2022 is provided, including product quantity, profit, and storage time for each year. Based on this information, students are required to create a mathematical model to determine the most suitable crop for planting.

Activity 4: Selecting an Athlete: In this problem, a selection is made for the 100-meter hurdles race for the Olympics. Information about four different athletes who participated in various Olympics and championships is provided, including their rankings in these events, whether they were eliminated, and whether they participated in these events. Students are required to develop a mathematical model to determine the most suitable athlete to participate in the upcoming Olympics based on this data.

Activity 5: Race Horse Problem: In this problem, information is provided about the performance of three different types of race horses when fed two different types of feed. A table is provided for each type of feed, showing the distance covered by the horses at different times. Based on this data, students are expected to develop a mathematical model to determine the most suitable type of feed for each race horse type.

3.4. Data Collection

Ethical approval for conducting the research was obtained from the Social and Human Sciences Research Ethics Committee of Yıldız Technical University on March 27, 2023 (Approval No: 2023.03). To collect the research data, a mathematical modeling activity titled "Healthy Saving Money Problem" was applied. This activity is an original mathematical modeling problem developed by the researchers (see Appendix 1). During the development and finalization of the problem, adjustments were made based on the feedback and suggestions from two experts and a third-grade teacher. The final version of the problem was revised accordingly. The problem involves a primary school student who plans to buy a backpack. In order to buy the backpack, the student needs to accumulate enough money by saving a portion of their daily allowance from their family, and the participants were asked to assist in this process. The participants were required to develop a mathematical model to help the student save as much money as possible daily, while also maintaining a healthy and balanced diet, to purchase the backpack in the shortest time possible. In this context, participants used their skills to analyze the data, establish relationships, classify and rank information, make assumptions, and engage in group discussions to exchange ideas, all of which were crucial for developing the model.

In this study, focus group interviews were used as a data collection method. Focus group interviews are employed to understand how a topic is discussed by a group and how different perspectives emerge during the discussion process. In focus group interviews, participants listen to each other's responses and comments, often adding their own remarks to the discussion. The main purpose of this practice is to obtain more reliable data in situations where participants can evaluate events from the perspectives of other individuals (Merriam & Tisdell, 2016; Patton, 2015).

Following the five-week preparation phase, a modeling activity was conducted with three students selected through purposeful sampling. Since the activity was carried out during class hours, the information technology workshop was used to ensure that students could work more comfortably. The workshop was preferred because it was suitable for video recording and had work tables appropriate for the students' seating arrangements. The activity lasted approximately 75 minutes, and the entire process was recorded both visually and audibly. In addition, students' worksheets, which documented their individual work, operations, and personal comments throughout the activity, were collected as written documents.

3.5. Data Analysis

The ideas generated by third-grade primary school students and the written solutions they presented for the "Healthy Saving Money" problem were analyzed using descriptive analysis. In descriptive analysis, direct quotations are used to reflect the participants' views. The aim is to first organize the data collected throughout the study, and then interpret and present it to the readers. In line with the study's objective, the data analysis was conducted following the three-phase framework proposed by Miles and Huberman (1994): data reduction, data display, and conclusion drawing and verification. Initially, data reduction involved selecting and organizing raw data in accordance with the research questions and thematic priorities. Subsequently, the reduced data were displayed systematically through matrices and thematic summaries to allow for meaningful pattern recognition. Finally, conclusions were drawn by examining the data for recurring relationships, interpreting cause-and-effect links, and verifying the consistency of interpretations. In this context, the modeling cycle developed by Blum and Ferri (2009) was used to analyze the thinking processes of third-grade students during the model creation activities. This cycle includes four stages: understanding the problem, eliciting the model, using mathematics, and reporting and explaining the results.

3.6. Validity and Reliability

For qualitative research, trustworthiness is a key concept to ensure the study's quality. Four criteria enhance trustworthiness: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). Credibility ensures that the findings align with reality and accurately measure what was intended. To enhance credibility, several strategies were employed. A five-week preparation period involving weekly three-hour sessions allowed for long-term interaction, fostering trust and encouraging authentic responses. Data collection focused on depth, with responses being carefully evaluated and interpreted for authenticity. Voluntary participation and efforts to create an open and supportive environment further promoted honest expression. Additionally, the research was reviewed by two academic experts and one classroom teacher to ensure the trustworthiness of the findings. Transferability ensures that the findings can be applied to similar contexts. To support transferability, purposeful sampling was used to include diverse participants, enhancing the relevance of the results to comparable settings. Additionally, detailed descriptions and direct quotations from participants were provided to offer a rich, contextual understanding that allows readers to determine the applicability of the findings to their own contexts. Dependability ensures consistency and transparency throughout the research process. To achieve this, all stages of the study were systematically documented, providing a clear audit trail of the procedures followed. Furthermore, data were analyzed using Blum and Ferri's (2009) framework, ensuring a structured and replicable approach to interpretation. Confirmability ensures that the findings are grounded in participants' experiences rather than researcher bias. To support this, direct quotes from participants were included to authentically represent their perspectives, and the raw data were validated by experts to ensure accuracy and objectivity in interpretation.

4. Findings

This section presents an analysis of the cognitive processes and activities of third-grade elementary school students during the mathematical modeling process using Blum and Ferri's (2009) four-stage modeling cycle.

4.1. Findings regarding the "Understanding the task" Stage

Initially, the students worked individually by reading the problem and examining the table within it. Afterward, they shared their perceptions of the problem with their classmates. Immediately after reading the problem, the following conversations took place among the students:

> …
> Elif: "Now, I think having bread in the contents is a good thing, but having flour, oil, and sugar is bad."
> Beren: "Exactly, we shouldn't eat too much sugar."
> Elif: "And since we produce (actually consume) too many sugary foods, it would be bad. Flour or bread is a good thing because it fills us up, and if we put cheese in the bread, that would be good too. So, I think... Hatice, what do you think?"
> Hatice: "I think it would be good if it were like you said, but let me tell you something, the mixed toast is 17 lira, which I think is affordable, but the meatball is 20 lira, so it's a bit expensive."
> Elif: "Yes, and the meatball has a lot of fat, and bread is also fatty."
> Beren: "Exactly."
> Elif: "So, I think if the two are combined, we would consume too much fat and gain weight. Cheese is actually a good thing, sausage, salami, tomatoes... Beren, what do you think?"
> …

In mathematical modeling processes, it is expected that problem solvers initially focus on understanding the problem, attempting to express it in their own words, and addressing the given and desired elements. However, when examining the initial conversations among the students, it becomes apparent that instead of engaging in discussions aimed at understanding the problem, they immediately begin analyzing the data presented in the problem and attempting to solve it. These analysis and solution-oriented discussions are primarily centered around the variables of "healthy eating" and "price." Furthermore, by considering the data in the problem, the students identified both positive and negative attributes and opted to find solutions based on these considerations. For example, they made classifications of positive and negative situations using phrases such as "bread is a good thing," "having sugar is bad," "we'll be full," "the price is reasonable to me," and "we'll gain weight."

Looking at the following dialogue, it is evident that the students evaluate the food and drinks not only based on their beneficial or harmful qualities, but also in terms of how well they meet daily nutritional needs and their prices. Additionally, the students discuss what choice would be most suitable to both maintain a healthy diet at school and quickly obtain the bag they aim to buy.

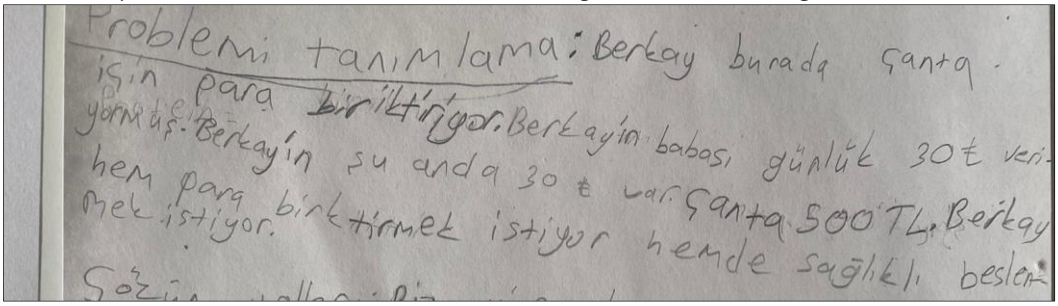
> …
> Beren: So now it would be better to look at the contents, just like you said. When we look at the sandwich, there's bread, cheese, sausage, tomato, and lettuce.
> Elif: Now, let's focus a little on the drinks. Ayran is 4 lira. For example, if we get a sandwich or a mixed toast, it would go well with ayran.
> Beren: Right.
> Elif: I think it would be great, but now with cheese, sausage, and ayran, it wouldn't be that great. It would be a bit oily, and ayran already has fat in it.
> Beren: Exactly.
> Elif: Water is 3 lira, fruit juice is 5 lira, milk is 5 lira, lemonade is 6 lira... What do you think, Hatice?
> Hatice: I think so. Right now, the most affordable is the breadsticks because they're 4 lira, but it wouldn't be a balanced diet menu.

Elif: That's true, but we wouldn't get full. After all, we're at school now, we'll get something from the canteen, and we won't be full. So, I think we should choose something that is both cheap and...
Beren: Exactly.
Elif: Because we're going to buy a bag for 500 TL, let's think about the cheaper drinks and share them later.
…

When the dialogue here is examined, the students' efforts to solve the problem in a manner similar to the previous examples can be observed. There is evidence of healthy verbal communication and social interaction among them. They exchange views on which foods are healthy or unhealthy, the nutrients present in the food, and the strategies that could be adopted to achieve their goal of acquiring the bag. However, as noted earlier, during this process, the students did not engage in conversations specifically aimed at understanding the problem, a step that is expected in mathematical modeling. Even though the students appear to focus on problem-solving in their conversations, it is understood that these dialogues also encompass the phase of problem understanding. This is because, from the conversations and video recordings, it is clear that the students read the problem, examined the table associated with it, and made comments to understand the problem. While analyzing the food contents to identify the foods suitable for "healthy, balanced eating," they simultaneously compared prices to determine the "most affordable product." The dialogue among the participants indicates that they compared the data in the table, analyzed it, and discussed the steps they would take to achieve the desired outcome by expressing the amount they aimed for. Additionally, the participants wrote down their understanding of the problem in their own words on the worksheets. In Figure 2, it is shown how one of the participants, Beren, wrote down her understanding of the problem on her worksheet.

Figure 2

Beren's expressions related to the "Understanding the Problem" stage.



[Translation: Defining the Problem: Berkay is saving money for a bag. Berkay's father gives him 30 TL daily. Berkay currently has 30 TL. The bag costs 500 TL. Berkay wants to save money and also eat healthily."]

In Figure 2, it can be seen that Beren correctly understood the problem, expressed it in writing using her own words, and also accurately identified the given information and what was required. A similar situation is observed for the other students as well. This result demonstrates that the first stage of mathematical modeling can sometimes be carried out simultaneously with other stages of the process, and at other times independently.

4.2. Findings regarding the "Establishing the Model" Stage

The research findings indicate that during this stage, students engaged in discussions and exchanged ideas to construct a model suitable for solving the problem. An analysis of the worksheets reveals that students employed various strategies, such as using plus and minus symbols to represent the nutritional content, creating menus and calculating their costs, and developing different methods to assign scores based on these methods. The following dialogue among the students provides concrete data related to the "model-eliciting" stage.

…
Elif: Now..., let's take a look. Actually, the mixed sandwich is good too, it has lettuce in it, I think.
Beren: Yeah, that could be filling as well.
Elif: Yes. For example, protein... Let's look at the mixed toast. Protein, dairy products, meat products... there's salami. I think the mixed toast has protein. Let's put a plus there. Then, what else... For example, there's bread.
Hatice: Carbohydrates.
Elif: Yes, carbohydrates, let's put a plus there too. There's cheese; dairy products.
Beren: Yeah, you're right.
Hatice: Tomatoes... in tomatoes
Elif: Tomatoes go under vitamins and green vegetables. Minerals are already in all foods, so there are minerals too.
Hatice: Bread already has a little bit of salt in it.
Elif: Lettuce also goes under green vegetables. I think it's great, and it's 18 lira too.
Hatice: I wonder if we should add a drink to it.
Elif: I think so too, but let's do all of this (she means "let's make a menu"), and give points to all of them. Now for drinks, ayran is 4 lira, so with our 18, plus 4, it's dairy products again. Water is 3 lira, but there's no water at all, did you notice?
Beren: I noticed.
Elif: There's everything except water. I think it would be better to get water with the sandwich.
Beren: I agree.
Hatice: Yeah.
Elif: Because there's no water at all.
…

The students ' attempted to identify variables and analyze the data accordingly. These variables were identified as the nutritional components of the products sold in the canteen, their prices, the students' daily nutritional needs, the taste of the products, and whether the foods are healthy or unhealthy. In this context, the students considered the nutritional components of the food and beverages sold in the canteen, analyzing their carbohydrate, protein, fat, vitamin, and mineral content. By analyzing the nutritional components, the students aimed to identify foods that are suitable for "healthy, balanced eating" as well as "affordable" options. At this stage, a student named Elif proposed the idea of "menu creation," which would play a significant role in the modeling process and subsequent stages. This idea was supported by the other students in the group. Consequently, the students shifted their focus to identifying "compatible" food and beverage options to create menus and aimed to solve the problem based on these menus. In the dialogue below, the students' initial attempts at menu creation are presented. It is evident that during these attempts, the students made a concerted effort to ensure that the menu content was both healthy and affordable.

…
Hatice: A sandwich and ayran combo... And I mean, there are dairy products. Ayran is also a dairy product, so it would be too much dairy.
Elif: Yes, I think it's too much dairy. If we chose this sandwich, we could add water for 3 lira. I think that would be a very balanced meal.
Hatice: And it would be 21 TL.
Elif: It would cost 21 TL.
Hatice: So, we'd still have quite a bit of money left in our pockets.
…

In the above conversation, it is observed that the sandwich and ayran menu is being discussed. Taking into account the variables of taste and health, the students concluded that this menu negatively impacts the price variable due to the presence of dairy products, specifically protein, in both the sandwich and ayran. For this reason, they determined that replacing the sandwich-ayran menu with a sandwich-water menu, which would be more affordable, would be a better option. Upon examining the continuation of the conversation, it becomes evident that the students classified positive and negative aspects, as they did in the previous stage, and proceeded with the process based on the menus they created. While creating these menus, they also considered how much money they could save, demonstrating an integrated approach to problem-solving.

> …
> Elif: There's no vitamin, but there's fat. There should also be vitamins. I think this is a bit poor (she means insufficient) nutrition for us because there are no vitamins here.
> Hatice: For the sandwich… well...
> Elif: I think we should do a scoring system.
> Hatice: Yeah, but first, I'm going to put the pluses and minuses.
> Elif: I gave this a minus because it doesn't have vitamins. Now, if we chose this mixed toast, what kind of drink would we get with it?
> Beren: I think not water.
> Hatice: But it could also be fruit juice.
> Elif: I think so too, fruit juice could work.
> Beren: I'm torn between water and fruit juice.
> Elif: Between water and fruit juice... Actually, getting water would be better, but getting fruit juice feels like, I don't know, it would be more satisfying. And besides, there's almost no difference between water and fruit juice.
> Hatice: I think they add a bit of water to fruit juice.
> Elif and Beren: Yes.
> Elif: Yeah, they add water anyway.
> Hatice: But not enough to meet what our body needs.
> Elif: So, for example, 17 lira, and if we add 5 lira for fruit juice, it would be 22.
> …

When examining the discussions in this section, it is observed that the students developed a method for analyzing the nutritional content while eliciting the model, using plus (+) and minus (−) symbols to indicate the presence or absence of components such as protein, fat, vitamins, etc., in the menu they created, and employing this as a scoring method. It is also noted that this method was suggested by Elif. Thus, the students matched the foods with plus and minus symbols based on their nutritional components, such as carbohydrates, protein, fat, minerals, salt, and water. Furthermore, they expanded their previous classifications to include categories such as "filling food" and "insufficient food" and exchanged ideas about which food and drink combinations would create a harmonious menu. These discussions reveal one of the most significant features of mathematical modeling processes: solving a mathematical problem involves recognizing that daily life is not independent of mathematics and building a bridge between real life and mathematics. Similarly, all three students emphasized the necessity of water for health and noted that the water content in fruit juice would not suffice for daily water consumption. This highlights that their knowledge of real-life contexts, such as how fruit juices are produced, their contents, or which foods contain specific nutrients (e.g., protein or fat), is crucial for solving mathematical problems accurately. Additionally, the importance of making real-life connections to solve the problem has become evident. Therefore, it can be seen that the mathematical modeling processes in this study progressed in alignment with the characteristics of mathematical modeling, as the following dialogue reflects the students' model-eliciting processes based on the variables they identified.

> …
> Hatice: For example, there's no protein in biscuits, but there's carbohydrate.
> Elif: There's carbohydrate, let me see... there's also vitamin. Because sugar is bad for us.
> Beren: Exactly, it's harmful.
> Elif: But it still needs to be there. Every food needs to have a bit of sugar, so let's put a plus for sugar, but since some things are missing, let's also add a minus. Now let's look at the ingredients of chocolate.
> Hatice: Sugar, cocoa, fat.
> Elif: Sugar and cocoa are very sugary and harmful, and so is fat.
> Hatice: But the fat in chocolate is usually low.

Elif: If it's low-fat chocolate, it's not a problem for us, but if it's very fatty, I think it's harmful because there's cocoa, sugar, and fat. Plus, consuming too much fat can lead to obesity.
Hatice: Too much fat is harmful for everything.
Elif: I think we should still put a plus here.
Hatice: I gave fat a half-plus.
Elif: Let's also give cocoa a half-plus. Sugar is something that has to be in every food. Cocoa, I don't know, it's in chocolate and sometimes in other things too. I think it shouldn't be consumed too much either. So let's give cocoa another half-plus, and the same for fat.
…

These conversations demonstrate that the students aimed to create a balanced meal plan while also selecting affordable products. Their efforts to develop a model based on assigning plus and minus symbols, scoring, and designing cost-effective menus to purchase the bag in a shorter time were also reflected in their worksheets. Although the students initially considered using a scoring system, they did not apply this method during the process. Additionally, they consistently utilized the classifications they had created in the first stage throughout this stage. While designing menus, they also classified these menus as "cheap-expensive," "harmful to health-not harmful to health," and "filling-not filling." In doing so, they attempted to establish the necessary data and identify a mathematical relationship between the time required to purchase the bag and daily savings.

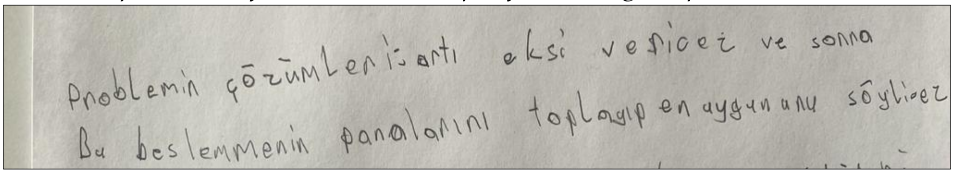
An analysis of the students' individual worksheets reveals that they not only used plus and minus symbols but also incorporated checkmarks. While the findings section examines the process in line with the four stages, it has become evident in nearly all dialogues that these stages often overlap and support one another. An analysis of the following dialogue shows that while the students were engaged in the model-eliciting process, they simultaneously made new discoveries about the problem's content. In other words, there were occasional transitions back to the problem-understanding stage from other stages.

…
Elif: There's fat in the bread. Now, we could reduce the fat in the meatball if we wanted to, but the price... How should I put it? For us...
Hatice: It's too much.
Elif: Yes, expensive, too much. And besides, we're going to get a drink with it, which would make it even more expensive. I think we'd need to put more money in the piggy bank.
Hatice: Yes, she'd have to wait more days to get the bag. Ayran could go with the meatball because there's no dairy product.
Elif: Yes, I think so too, there's no dairy product at all. But ayran has a lot of fat in it. And there's already fat in the bread. So, if we chose the meatball, ayran wouldn't work with it.
Beren: It would be too fatty.
Elif: If we chose the meatball, only water or lemonade would go with it.
Hatice: Milk could work too.
Elif: Dairy... But they add a little fat to milk too. Some fat, and there's already a lot of fat in the bread. So no, if we chose the meatball, it would have to be water or lemonade.
Hatice: Biscuits wouldn't be filling; the price is good, but they wouldn't fill you up.
Beren: Yeah, they wouldn't be filling.
Elif: There's flour, and there's sugar.
Beren: Sugar is harmful.
Elif: Yes. And you know, something like a sandwich would be better at school. I don't think we should start the day with sugar. Let's eliminate the biscuits.
…

In these conversations, it is observed that while the students were trying to solve the problem and build a model, they occasionally returned to the problem-understanding stage to recall the desired outcome. It is understood that they emphasized the necessity of saving as much money as possible daily to purchase the bag in the shortest time. Elif highlighted the need for greater daily savings, stating that more money should be put into the piggy bank and emphasizing that this should be considered when designing the menus. Additionally, Hatice underlined that if daily savings were insufficient, it would take longer to acquire the bag.

When the students' worksheets were analyzed in terms of the model-eliciting process, it was observed that Beren documented, in her own words, the method to be used for model construction, as shown in Figure 3.

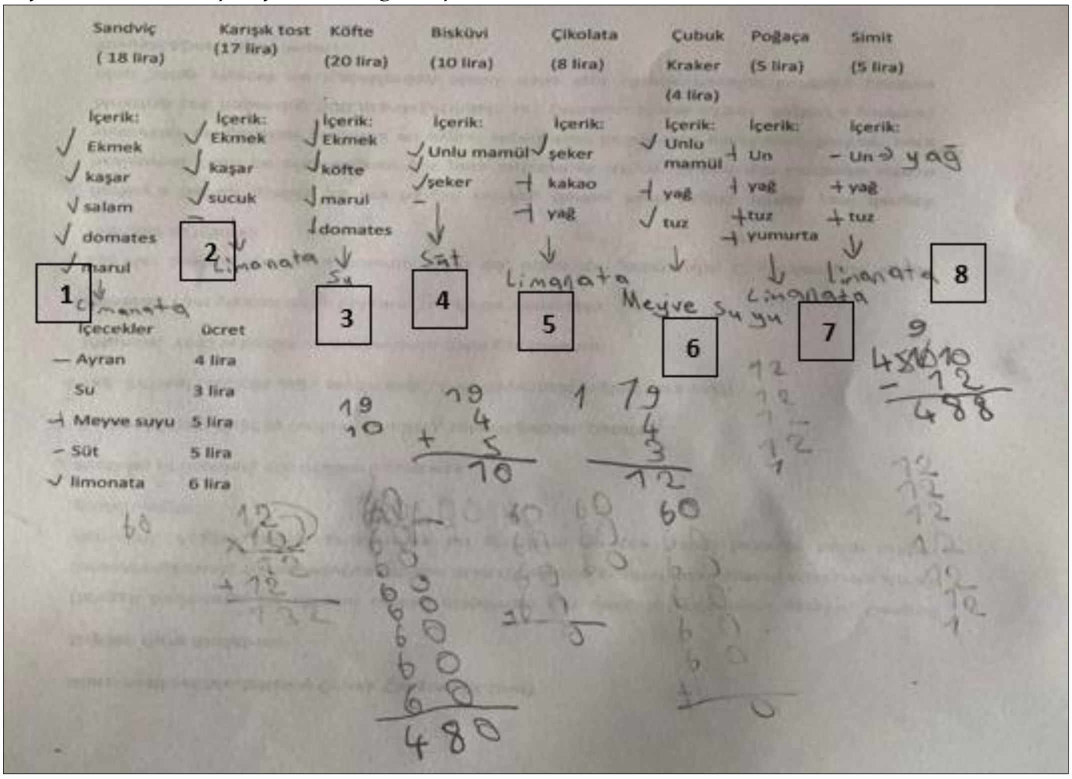
Figure 3
Beren's explanation of the model developed for solving the problem



[Translation: Solutions to the Problem: We will assign pluses and minuses, then add up the scores of these meals and determine the most suitable one.]

As stated in Figure 3, the students documented their thoughts for almost every stage of the process, which allowed them to approach the problem's solution in a more systematic manner. Although the students worked in groups, they also recorded their individual thoughts on their worksheets. Subsequently, they attempted to reach a common solution and model. For instance, the visual representation of Elif's worksheet is shown in Figure 4.

Figure 4
Elif's model developed for solving the problem



[Translation: 1: Lemonade, 2: Lemonade, 3: Water, 4: Milk, 5: Lemonade, 6: Fruit Juice, 7: Lemonade, 8: Lemonade]

In Figure 4, it is observed that Elif matched the food items with specific beverages to create menus. Elif paired each type of food with a beverage she deemed appropriate to form the menus. The numbers 1, 2, 3, 4, 5, 6, 7, and 8 in Figure 4 represent these beverages. Additionally, Elif used plus, half-plus, minus, and checkmark symbols to indicate whether the food items were beneficial or harmful. For example, it was previously noted in the dialogues that while creating menus, the daily nutritional needs for components such as carbohydrates, proteins, and fats were considered. Similarly, here Elif reviewed the nutritional content of the food items and used checkmarks or plus symbols if they met the student's daily nutritional needs, half-plus symbols if they partially met the needs, and minus symbols if they did not meet the needs. At the bottom of the worksheet, Elif calculated the prices of the menus she created and performed calculations related to reaching the bag's price.

Moreover, when examining the students' problem-solving and model-eliciting efforts, it is evident that their group discussions were reflected in their worksheets. However, each student added a unique perspective to their solutions based on their individual viewpoints, cognitive potential, and prior knowledge.

4.3. Findings regarding the "Using mathematics" Stage

Accordingly, students analyzed the nutritional content of food items, developed various menus comprising food and beverages, and utilized addition and subtraction to calculate the costs of these menus and the daily savings required to reach their goal. While applying the problem-solving method identified during the model-eliciting stage, students employed various mathematical skills. The following conversations illustrate their discussions during this process:

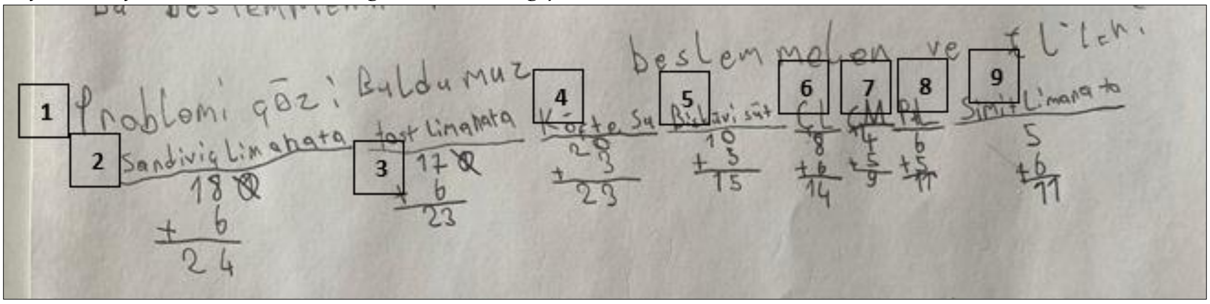
> …
> Hatice: I'll add up the money now.
> Elif: Let's divide the tasks. Hatice, you add up the money, and I'll do the scoring. Let's each write these down on our own papers and solve them.
> Hatice: I'll write the calculations here and then compare them.
> Elif: Has everyone done their totals?
> Hatice: For me, the menu that saves the most money right now is breadsticks with fruit juice. You save 21 lira, but it's not very filling or balanced. What do you think, Elif?
> Elif: Well… Breadsticks with fruit juice aren't very filling. Sandwich with lemonade comes to 24 lira, and toast with lemonade comes to 23 lira. We need something that's both filling and leaves more money behind. Biscuit with milk comes to 15 lira. Right now, we need the cheapest options. Choosing breadsticks with fruit juice wouldn't be right because it's not very filling. I think something like meatballs would be a better choice.
> Hatice: For example, if you take sandwich with lemonade, you save 6 lira, so I eliminated sandwich with lemonade because it doesn't leave much money. Mixed toast with lemonade leaves 7 lira, and meatballs with water also leave 7 lira. It's balanced, but the money isn't great. Biscuit and milk are affordable and might work for balanced nutrition. Pastry with lemonade could also be a great option.
> Elif: Actually, pastry with lemonade works. Look, for instance, chocolate with lemonade wouldn't work. Why? Because they're low in nutrients. They wouldn't fill us up. Breadsticks with lemonade wouldn't either. But a big pastry with lemonade would fill us up, and the price is 11 lira, so I think that would be great. There's also simit with lemonade.
> Hatice: Simit with lemonade is also great, filling, and balanced. Pastry with lemonade is balanced too. But with simit and lemonade, you save more money, and 19 lira stays in your pocket. That way, you can reach the bag you want in a shorter time.
> Elif: I think simit with lemonade would be the best choice. It saves more money and is more filling.
> Beren: I agree with you both.
> …

An analysis of the students' discussions during the modeling process reveals that they utilized mathematical knowledge, such as addition, subtraction, and comparing whole numbers, to identify the menu that would allow them to save the most money daily while maintaining a balanced diet and obtaining the bag in the shortest time. The calculations were performed not by a single student but collectively by all the students. Thus, the students engaged in both group work and individual efforts throughout the process. Additionally, the classifications introduced in the first stage, such as "cheap-expensive," "compatible food and drink," and "filling," were also employed in this stage. The students calculated the costs of the menus and compared these costs to determine the menu that would enable them to save money and purchase the bag more quickly.

It was previously stated that students utilized various mathematical skills during the mathematical application stage. An examination of both student dialogues and individual worksheets reveals that these mathematical skills include performing addition and subtraction operations, problem-solving, ordering, classifying, comparing, analyzing, synthesizing, and reasoning. This demonstrates that the students employed a wide range of skills throughout the process. However, it is evident that no single mathematical skill was exclusively used for a specific stage or operation. Instead, the mathematical skills were applied comprehensively across nearly the entire process, contributing holistically to problem-solving. The mathematical skills applied at each stage have been identified based on the prominent features observed in the following student solutions. Figures 5, 6, and 7 present the notes related to the use of mathematics during the modeling activity by Elif, Beren, and Hatice, respectively.

Figure 5

Elif's use of mathematics during the modeling process

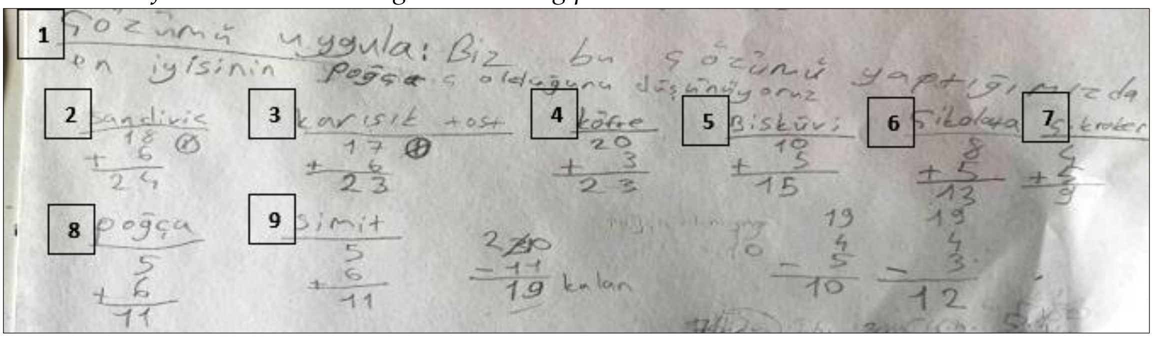


[Translation: 1: Solve the Problem: The meals (menus) we found and their TL (prices), 2: Sandwich-Lemonade, 3: Toast-Lemonade, 4: Meatball-Water, 5: Biscuit-Milk, 6: C L (Chocolate-Lemonade), 7: C F (Chocolate-Fruit Juice), 8: P+L (Pastry-Lemonade), 9: Bagel-Lemonade]

An examination of Figure 5 reveals that Elif recorded the menus she created in sections 2, 3, 4, 5, 6, 7, 8, and 9, along with the calculations of the total cost required to purchase each menu. The key mathematical skills Elif demonstrated during this stage include her ability to differentiate and classify, which she applied in determining the prices of individual food and beverage items in each menu, as well as her addition skills, which she utilized to calculate the total cost of the menus.

Figure 6

Beren's use of mathematics during the modeling process

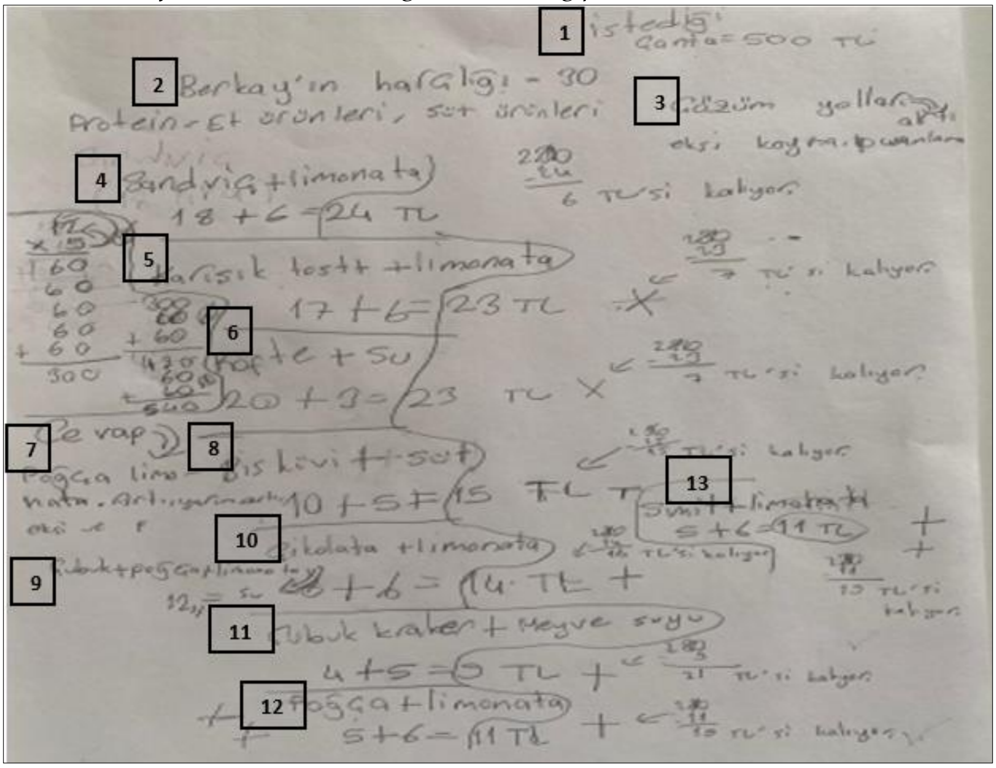


[Translation: 1: Apply the solution: When we applied this solution, we thought pastry was the best option. 2: Sandwich, 3: Mixed Toast, 4: Meatball, 5: Biscuit, 6: Chocolate, 7: Breadsticks, 8: Pastry, 9: Bagel]

An analysis of Figure 6 demonstrates that Beren constructed menus and calculated their costs in sections 2, 3, 4, 5, 6, 7, 8, and 9, employing processes and mathematical skills comparable to those described in Elif's solution (as detailed in Figure 5). Moreover, in section 1, Beren identified "Pastry" as the most suitable menu option, reflecting her judgment based on the criteria she applied. In this process, since there is a decision-making period by comparing with other menus, it can be stated that the comparison skill which is one of mathematical skills is involved. Additionally, since each menu's contents are evaluated separately in terms of being beneficial/harmful and cheap/expensive, it can be said that the analysis skill which is one of mathematical skills is activated. In processes such as creating a menu based on the student's hunger level, considering how much savings are needed for the bag they wish to buy, and deciding on suitable options for healthy eating, it can also be said that the reasoning skill which is one of mathematical skills is involved.

In addition, outside the numbered sections, Beren performed subtraction calculations to determine the daily savings she could allocate from her allowance to reach her goal of purchasing the bag. This highlights her application of subtraction as a fundamental mathematical skill integrated into the problem-solving process.

Figure 7
Hatice's use of mathematics during the modeling process



[Translation: 1: Desired bag: 500 TL, 2: Berkay's allowance = 30, Protein: Meat products, dairy products, 3: Solution methods: Assigning pluses and minuses, scoring, 4: Sandwich+lemonade, 5: Mixed toast+lemonade, 6: Meatball+water, 7: Pastry+lemonade, 8: Biscuit+milk, 9: Breadsticks+pastry+lemonade, 10: Chocolate+lemonade, 11: Breadsticks+fruit juice, 12: Pastry+lemonade, 13: Bagel+lemonade]

An examination of Hatice's solution in Figure 7 reveals that she approached the task with greater detail compared to her peers. In sections 1 and 2, she identified the problem data required for solving the task (demonstrating skills in analysis, reasoning, classification, and differentiation) and incorporated additional information beneficial for problem-solving, such as identifying which products contain protein (utilizing skills in differentiation, classification, and analysis). In section 3, she outlined her solution method, showcasing her analytical, reasoning, and differentiation skills.

In sections 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13, she determined the menu options and their respective prices using methods and mathematical skills similar to those employed by the other students (as described in Figure 5). However, unlike her peers, Hatice included subtraction calculations alongside each menu price to determine the remaining amount needed to reach her goal of purchasing the bag. This highlights her application of mathematics in planning and budgeting within the problem-solving process.

A broader analysis of Figures 5, 6, and 7 indicates that the prominent mathematical skills involved in the decision-making process for menu creation are problem-solving, analysis, synthesis, and reasoning. Additionally, the process also incorporated skills such as comparison, classification, ordering, and differentiation. The use of mathematics to determine which menus to select further required the application of addition and subtraction skills, complementing the previously mentioned competencies.

4.4. Findings regarding the "Explaining the results" Stage

Accordingly, at this stage, the students, guided by their analyses and mathematical operations, reached a conclusion through the model they developed, which allowed them to maintain a balanced diet while saving enough money to purchase the desired bag in the shortest time possible. As in other stages of the modeling cycle, they exchanged ideas and collaboratively arrived at the same conclusion, completing this stage by writing their reports.

As the students approached the conclusion and finalized the modeling process, the following conversation took place among them:

> …
> Elif: We found pastry and lemonade, but we're here for 6 hours. So we also need some snacks. We can get other things as snacks.
> Hatice: It has to be a snack, but it also has to be filling.
> Elif: And it needs to be affordable.
> Hatice: For a snack that's filling, there are breadsticks. Breadsticks are 4 lira, that's affordable.
> Elif: So we had 30 lira, and pastry with lemonade cost 11 lira. If we subtract that, we're left with 19 lira. We won't put all of this 19 lira in the piggy bank. We'll buy other things. We want to get filling things and still have some money left. Like if we get biscuits, it'll cost 10 lira.
> Hatice: I think breadsticks would work. If we subtract 4 lira from 19, we'll have 15 lira left.
> Elif: Breadsticks have flour, oil, and salt. So they're a good snack. Because we're here for 6 hours. If we get chocolate, it's just sugary, and it wouldn't be good for us as a snack.
> Hatice: If we have 15 lira left, that's good, I think.
> Elif: If we also get a drink with the breadsticks, that would be nice. If we get something we need but also affordable, that'd be good. Lemonade would cost 6 lira, so it's a bit expensive. If we choose breadsticks, we could get fruit juice for 5 lira or water for 3 lira.
> Hatice: If we get water, we'll have 12 lira left. Water would work because there's no water in the breadsticks, and our bodies need water.
> Elif: If we get fruit juice, we'll have 10 lira left. So if we get water, we'll have more money left. In the end, for the meal, we can choose pastry and lemonade, and for the snack, breadsticks with water.
> Hatice: We'll have 12 lira left, and we'll be able to get the bag we want faster.
> Hatice: If we save 12 lira a day, in a week—since we're in school for 5 days—that makes 60 lira. I can add another 60 lira to that and keep going.
> Elif: I calculated that in 7 weeks, we'd save 420 lira.
> Hatice: In 9 weeks, we'd have 540 lira, and we could buy the bag and still have 40 lira left.
> Elif: In 8 weeks, we'd have 480, and in 9 weeks, like Hatice said, 540. So if we save for eight and a half weeks, we'll have exactly enough for the bag.
> …

An analysis of the students' discussions during this stage reveals that they completed their mathematical operations and designed a daily menu consisting of two different food items and two different beverages, assuming the same menu would be maintained daily until enough money was saved to purchase the bag. Their conversations indicate that they employed fundamental mathematical skills such as classification, ordering, and comparison during this stage. When the researcher posed the question, "Will Berkay eat the same thing every day?" Elif responded, "If he wants to get the bag quickly and stay healthy, he'll have to endure it!" An examination of the students' individual worksheets shows that during the mathematical application phase, they first calculated weekly savings. Without using multiplication, they repeatedly added the weekly savings step by step until reaching the required amount to purchase the bag. Ultimately, the students solved the problem through mathematical calculations based on the model they developed and expressed their findings in their own words, presenting them in a report.

5. Discussion and Conclusion

In this study, a modeling activity conducted with third-grade primary school students following a five-week preparatory phase was analyzed using qualitative research methods, yielding significant findings.

The participants worked individually during the first stage of the modeling process, the problem-understanding stage, without engaging in any question-answer or discussion processes while reading and attempting to comprehend the problem. The students did not share their understanding of the problem or express their opinions to one another. However, they initiated group interactions during the model-eliciting stage by analyzing the nutritional content and expressing their prior knowledge from daily life. It was observed that students frequently utilized their everyday knowledge, such as "fat is harmful," "sugar is harmful," and "becoming obese," during this stage. Through this knowledge, the students analyzed and classified nutritional content to identify foods that align with the concept of "healthy food." This suggests that mathematical modeling processes involve the use of non-mathematical knowledge and skills, such as disciplinary knowledge, awareness of real-life topics, or familiarity with daily life problems. Naturally, accurate knowledge and skill levels in these areas positively influence students' modeling processes. Conversely, a lack of knowledge or inaccuracies can lead to the inability to develop a model or result in incorrect modeling. Similarly, English and Watters (2005) stated that informal learning used by students in problem-solving can sometimes facilitate solutions but may also hinder problem resolution in certain cases.

Throughout the modeling process, participants made various assumptions, exchanged ideas, and engaged in group discussions, eliminating results that did not align with the objective. While analyzing nutritional content to identify healthy food options, they also considered price differences and aimed to identify affordable food options. To this end, they created multiple menus, compared them, and excluded those that were not suitable for the goal. During the model-eliciting stage, participants used methods such as assigning pluses and minuses, marking with ticks and crosses, and applying scoring systems, reflecting these approaches in their solutions. However, it was observed that they eventually abandoned the idea of scoring menus. Similarly, in a study by English (2006), participants discussed various ideas, tested their solutions on different hypotheses, and evaluated their results against real-life situations to determine their appropriateness until reaching a conclusion.

During the mathematical application stage, participants utilized mathematical knowledge such as addition, subtraction, and comparing whole numbers. Instead of employing multiplication and division operations, they achieved results through repeated addition. Initially, participants calculated their weekly savings and then repeatedly added this amount to approach the target of 500 TL. However, challenges arose as 500 is not an exact multiple of 60. Throughout the process, various mathematical skills were observed to be employed at different stages. Skills such as analysis, synthesis, critical thinking, and reasoning were primarily utilized to establish accurate relationships between variables or situations and to identify appropriate solutions. Thus, these skills were not limited to the mathematical application stage but were integral to other stages as well. When examining mathematical skills such as classification, ordering, differentiation, and matching, it was evident that these were prominently used in tasks such as deciding on and creating menus, where solution implementation became more relevant. Skills related to calculation, such as performing addition and subtraction operations, were predominantly used to determine whether the created menus aligned with the goal and whether they were effective in achieving the bag's purchase. These findings suggest that the primary benefit of students utilizing these mathematical skills lies in their application to solving a real-life problem. Considering that the main goal of mathematical modeling is to bridge the gap between mathematics and real life (Berry & Houston, 1995), the use of these skills indicates that the purpose of mathematical modeling has been fulfilled.

During the results explanation stage, all participants reached their conclusions through discussions, exchanging ideas, reading and analyzing data from tables, and performing mathematical calculations. They concluded the modeling activity by presenting a generalizable model. Normally, when mathematical modeling is completed, a formulaic and generalizable product is expected (Pollak, 2007). One of the key findings of this study lies here: when conducting mathematical modeling activities with primary school students, it is unrealistic to expect products such as formulas, algebraic expressions, abstract concepts, or symbolic representations due to their age and grade levels (Lesh et al., 1987). However, it was observed that they were capable of producing outcomes with generalizable characteristics. While no algebraic equations emerged, it was evident that students in this age group could engage in modeling using alternative forms of representation. Similarly, in a study by Watters et al. (2004), it was found that third-grade students were prepared to perform math-focused tasks, explore multiple hypotheses, and interpret both acquired and provided data in the context of these tasks.

One of the prominent findings of the process analyzed through Blum and Ferri's (2009) modeling cycle is that participants did not follow the stages in a linear sequence. In other words, they did not progress sequentially through the four stages. It was observed that when participants first approached the problem, they carried out the problem-understanding stage in parallel with the second and even third stages. While engaging in cognitive processes such as reading, understanding, and identifying the given and required elements of the problem, they simultaneously focused on solving it. Similarly, in later moments of the process, during efforts to solve the problem and build a model, participants returned to the initial stage, reflecting on whether they had correctly understood the problem by asking questions such as, "What was being asked in the problem?" and "What was given in the problem?" Even during the final stage, explaining the results, participants exhibited statements reflecting a return to the problem-understanding stage. These observations suggest that while mathematical modeling processes are structured into distinct stages, these stages cannot be considered independent of one another. Transitions between previous and subsequent stages occur at every phase, enabling problem solvers to progress more confidently through the process. This interconnectedness positively influences problem-solving by allowing the stages to reinforce one another.

During the modeling activity, two participants were more active in generating and sharing ideas, while one participant remained more passive throughout the process. However, the more verbally passive participant filled out their worksheet more meticulously than the others, demonstrating superior written expression skills compared to their peers. This highlights another benefit of mathematical modeling processes: they reveal individual differences among students, providing insights into their strengths and weaknesses. These differences may be cognitive, as well as social, behavioral, physical, or spatial, as exemplified in this study (Borromeo Ferri, 2018; Stillman et al., 2009). Despite their individual differences, participants did not present opposing ideas; instead, they discussed, filled in gaps left by their peers, and used reasoning to arrive at similar outcomes. This collective effort highlights the benefit of group work, as participants successfully produced a shared outcome despite their differences (Borromeo Ferri, 2018).

The results discussed so far can be seen as specific reflections of one of the main reasons that motivated the researchers to conduct this study, which is the implementation of mathematical modeling with young children. This study demonstrates that mathematical modeling, typically conducted with older students and higher-grade levels, can also be implemented effectively with younger children. In the context of Turkiye, the lack of national studies focusing on third-grade students further underscores the significance of these findings. Many studies have emphasized the importance of mathematical modeling processes in international assessments such as PISA and TIMSS, which prioritize mathematical literacy and skills-based questions (Blum, 2002). In Turkiye, students take the High School Entrance Exam in the eighth grade, where skills-based questions constitute the majority. However, one major issue is the lack of a foundational preparation process for these exams starting from primary school (Çepni, 2016). The study findings reveal that students in mathematical modeling processes relate mathematical concepts to real-life situations. This connection highlights the importance of integrating mathematical modeling activities into primary school curricula, as such practices have the potential to improve students' mathematical literacy and enhance their ability to solve skills-based problems (Koç & Elçi, 2022).

Mathematical literacy is a crucial competency, as it is both a primary goal of mathematics education (MoNE, 2018) and a significant component of national and international assessments. Mathematical modeling, which plays a critical role in developing this skill (Lesh & Zawojewsky, 2007), has made valuable contributions to the national and international literature through its application to younger age groups. Furthermore, this study offers an example of how mathematical modeling processes should be conducted. Additionally, the activities developed and implemented during the preliminary and main phases of the study, tailored to the primary school level, are presented as ready-to-use resources.

6. Recommendations

This study demonstrated that third-grade students were able to engage in mathematical modeling following a preparatory phase. Therefore, future research could explore the applicability of mathematical modeling activities, modeling processes, and students' perceptions of mathematical modeling in other lower age groups within primary school. Given that mathematical modeling highlights the connection between real life and mathematics, incorporating modeling activities into mathematics lessons can help students understand the relevance of mathematics in their everyday lives.

Additionally, as mathematical modeling fosters higher-order thinking skills and 21st-century competencies expected of global citizens, it could be emphasized further in mathematics curricula and textbooks. Moreover, seminars and training sessions could be organized for educators to introduce them to the theoretical framework, benefits, and practical applications of mathematical modeling, equipping them with the necessary knowledge to implement such activities effectively.

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Data availability: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

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Appendix 1. Mathematical Modeling Problem

Problem: Saving Money for a Healthy Purchase

Berkay wants to buy a backpack that he really likes from his father. Berkay's father gives him a daily allowance of 30 lira, and if Berkay wants that backpack, his father tells him that he can save a certain amount of money from his daily allowance to achieve his goal. Berkay wants to stay healthy, meaning eat a balanced diet, and also save as much money as possible. To do this, he needs to determine the best food and drink options from those sold in the school canteen.
Since Berkay's goal is to save enough money to buy the 500 lira backpack he really likes, can you help Berkay by telling him which food and drink combinations will allow him to reach his goal and get the backpack as soon as possible?

Table: Foods and Drinks Sold in the Canteen

Food Item	Price	Ingredients
Sandwich	18 lira	Bread, Cheese, Salami, Tomato, Lettuce
Mixed Toast	17 lira	Bread, Cheese, Sausage
Meatballs	20 lira	Bread, Meatballs, Lettuce, Tomato
Biscuit	10 lira	Pastry, Sugar
Chocolate	8 lira	Sugar, Cocoa, Fat
Crackers	4 lira	Pastry, Fat, Salt
Pastry	5 lira	Flour, Fat, Salt, Egg
Bagel	5 lira	Flour, Fat, Salt

Drink	Price
Ayran	4 lira
Water	3 lira
Fruit juice	5 lira
Milk	5 lira
Lemonade	6 lira