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Research Article

Animations and depictions as a tool to improve pre-service elementary teachers' noticing of students' mathematical thinking

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The purpose of the study is to examine pre-service elementary teachers' noticing of students' mathematical thinking of division through animations and depictions. For this purpose, the data was collected from pre-service teachers' written explanations of important moments identified when watching a video clip, as well as pre-service teachers' animations and depictions of classroom scenarios. Findings indicated that what the pre-service teachers noticed in the video clip was more general, as they attended to the whole class environment, students' behaviours and learning in total, and teacher pedagogy. The pre-service teachers, however, were more focused on particular students' mathematical thinking in their animations and depictions. Furthermore, between animations and depictions, the pre-service elementary teachers included the most details about students and teachers in the latter one. Therefore, this study concludes that while both preparing animations and depictions have a positive effect on the pre-service elementary teachers' noticing levels, preparing depictions better facilitates their attention to students' thinking about mathematics.

Keywords: Noticing; Animations; Depictions; Mathematical thinking

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1. Introduction

The questions of "What is noticing?" or "Why is noticing important?" have become the focus of many studies in recent years. One of the answers for the first question given by Mason (2011), the acknowledged pioneer, is that "noticing is a collection of practices designed to sensitize oneself so as to notice opportunities in the future in which to act freshly rather than automatically out of habit" (p. 35). That is, noticing is an intentional act which enables teachers to realize moments throughout instruction in particular ways (Jacobs et al., 2010), and these realizations affect their ways of responding, and hence their students' understandings (Stahnke et al., 2016; van Es & Sherin, 2008). Having a significant impact on student understanding is precisely the answer to the second question. With an agreement that teacher noticing is important to build effective learning environments and promote students' understandings (Kersting et al., 2010), researchers have focused on how they can improve or contribute to noticing of teachers. In this respect, this study

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examines whether or not preparing animations and sketches improves pre-service elementary teachers' noticing of students' mathematical thinking.

2. Teacher Noticing

To assert the importance of teacher noticing, the National Council of Teachers of Mathematics [NCTM] (2014) states, "Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning" (p. 53). To state it differently, teachers need to listen to their students carefully and make necessary changes in the instruction while considering the ideas presented by the students (van Es & Sherin, 2002). With an increasing interest in teacher noticing and an agreement on the importance of it, different frameworks for teacher noticing have been proposed. One of these frameworks–Learning to Notice developed by van Es and Sherin (2002)–has three significant dimensions: "(1) identifying what is important or noteworthy about a classroom situation; (2) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (3) using what one knows about the context to reason about classroom interactions" (p. 573). While the first dimension is related to a teacher's ability to attend to noteworthy moments throughout the complex nature of instruction, the second one is related to the teacher's ability to make connections between the attended moment(s) and instructional principles, which is more than simply describe. The last dimension is about the teacher's ability to interpret the moment(s) using his/her subject matter and pedagogical content knowledge. As can be understood from this last dimension, although what teachers notice can be the same, the levels of what teachers notice might be different (Schifter, 2011; Schoenfeld, 2011). To emphasise these levels and to indicate the growth in their noticing over time, van Es (2011) developed a framework, learning to notice, and further organized what teachers notice across four levels: Baseline (Level 1), Mixed (Level 2), Focused (Level 3), and Extended (Level 4) (the details of these levels are given in Table 1 under the Methodology part).

What teachers notice includes teachers' decisions about where or whom to attend to in a particular moment. van Es (2011) further groups this dimension into two main categories: actor and topic. That is, teachers prefer to attend to actor (teacher, a group of students, a particular student, or other) or topic (classroom management, pedagogy, students' behaviours, or mathematical thinking). Teachers start to notice by attending to whole classroom environment, behaviour, as well as learning and teacher pedagogy, then to both teacher pedagogy and particular students' mathematical thinking and behaviours. It continues with attending to particular students' mathematical thinking and ends with attending to the relationship between particular students' mathematical thinking and teaching strategies. That is, there is a shift or growth in teacher noticing among the levels with Baseline being the lowest and Extended being the highest level of noticing.

It is not an easy task to adjust or improve instruction considering students' thinking for teachers. To accomplish this task, teachers should have the skill of noticing. Therefore, teacher educators include noticing activities or use different methods in their courses to improve pre-service teachers' noticing levels. Some of them are using written lesson excerpts or classroom scenarios (Ivars et al., 2018), as well as digital resources such as videos clips (Güler et al., 2020; Ozdemir Baki & Kilicoglu, 2021; Ulusoy & Çakıroğlu, 2018, 2021; Sherin & van Es, 2009; Star & Strickland 2008; van Es & Sherin, 2008), animations, and sketches (Amador & Earnest 2016; Chazan & Herbst, 2012). Based on the research cited, reviewing videos of classroom instruction, reflecting on lesson excerpts or classroom scenarios, as well as preparing animations or sketches all lead to improvements in teachers' noticing. Aside from the agreement that all of these methods contribute to teacher noticing, there is disagreement on which of these methods is the best. This disagreement leaves open the door to get insight into potential differences among the use of videos, animations, and sketches to improve teacher noticing. The findings of this study can be used to provide insight into this difference if exits. Furthermore, the majority of these studies was conducted with pre-service middle school mathematics teachers and secondary school mathematics teachers. Since only one study (van Es & Sherin, 2008) examined changes in fourth and fifth grade teachers' noticing, sufficient studies have yet to include elementary education level. Including teachers from elementary school level may allow us to see differences or similarities among teachers of different levels. Since this study is conducted with pre-service elementary teachers, the findings can give ideas for a discussion of reason for differences. Therefore, this study draws on the Learning to Notice Framework to examine how preparing both animations and sketches affect pre-service elementary teachers' noticing levels.

2.1. Using Animations and Sketches as Opportunities for Teacher Noticing

"Approximations of practice refer to opportunities to engage in practices that are more or less proximal to the practices of a profession" (Grossman et al., 2009, p. 2058). Similarly, Ghoussenini and Herbst (2016) explain that approximations of practice are "activities in which novice teachers engage in experiences akin to real practice that reproduce some of the complexity of teaching" (p. 83). Since animations and sketches presenting classroom scenarios provide pre-service teachers with opportunities to analyse the scenarios from different perspectives and make pedagogical decisions, they help pre-service teachers approximate teaching practice and develop their noticing (Amador & Earnest 2016; Chazan & Herbst, 2012). In these studies, animations and sketches were used both as an artefact to be analysed and a tool to improve noticing.

One of the programs that transforms classroom scenarios into animations is Vyond. By means of this program, it is possible to design a new scene or select a scene from outside of the program, add graphs, audio, and pictures from both the program and outside of the program, select characters among many options, and add facial expressions or body movements. For example, students raise their hands to ask questions or suggest ideas, as well as walk to board to solve a problem. One of the tools that helps users to visualize classroom scenarios in a comic format (hereafter "depictions") is Depict (Herbst & Chieu, 2011). Depict allows users to add characters, speech bubbles, and facial expressions to more thoroughly show how characters feel, as well as to create classroom environments. This study aims to examine how the points that pre-service elementary teachers emphasise in their animations and depictions differ from the points that they notice in their initial video-watching. That is, the programs, Vyond and Depict, are used to reveal whether–and if so in what levels– these programs affect pre-service elementary teachers' noticing of students' mathematical thinking. The findings of this study might provide insight for teacher educators about how they can design their courses to improve pre-service teachers' noticing of students' mathematical thinking. In the same way, this study might inform teacher educators about how these tools can be used to support their noticing levels. For this purpose, the following research questions guide this study:

RQ 1) What is the level of pre-service teachers' noticing of students' mathematical thinking in a video clip?

RQ 2) How and in what levels does preparing animations through Vyond affect pre-service elementary teachers' noticing of students' mathematical thinking?

RQ 3) How and in what levels does preparing depictions through Depict affect pre-service elementary teachers' noticing of students' mathematical thinking?

3. Method

To explore the research questions, I will be performing a case study, which is defined as "an in-depth description and analysis of bounded system" (Merriam, 2009, p. 40). The bounded system in this study is pre-service elementary teachers' noticing of students' mathematical thinking presented through animations and depictions designed by Vyond and Depict, respectively. Case study is also categorized into single-case holistic, multiple-case holistic designs and single-case embedded, multiple-case embedded designs (Yin, 2003). Specifically, the design of this study is single-case embedded design as the pre-service elementary teachers was the case; their noticing of students' mathematical thinking presented through animations by Vyond and their noticing of students' mathematical thinking presented through depictions by Depict were the units.

3.1. Participants and Course Setting

Participants of this study were 26 junior year pre-service elementary teachers enrolled in a 14-week mathematics methods course. Throughout their undergraduate program, these teachers took content courses, education courses, general method courses, method courses specific to mathematics, science, arts, literacy, etc., and teaching practice courses. At the time of the data collection, these pre-service teachers had already completed their content courses and were taking the second mathematics methods course. Two articles related to the topic of the week were uploaded to the course management system each week, except for the weeks in which micro-lessons were scheduled, and the pre-service teachers were asked to read and discuss these articles during the course hours. The pre-service teachers were also asked to plan a micro-lesson based on objective(s) given by the instructor of the course, then teach this planned lesson, and critique the taught lesson. For these micro-lessons, the pre-service teachers were divided into ten groups, each including three or four pre-service teachers. Since the pre-service teachers asked some questions to the pre-service teacher who was teaching the micro-lesson, it was a chance for him/her to see how he/she could attend and respond to the questions. In short, by means of these activities, the course aims to develop the pre-service teachers' knowledge and skills to help them prepare for their future professional teaching practices.

3.2. Data Collection and Analysis

In order to examine how the points that the pre-service elementary teachers emphasise in their animations and depictions differ from the points that they notice in their initial video-watching, the data were collected from multiple sources. The first source was a video clip of a lesson that involved students' mathematical thinking of division. In the video clip, after the teacher presented a problem, she gave some time to students to solve it. Specifically, the problem in the video clip was "If an eraser costs 5 liras, how many erasers can you buy with 62 liras?" While the students were trying to solve, the teacher monitored and asked questions like "Why did you group these lines in fives?" "Why did you draw 62 lines?" or "Why did you count in fives?" to understand how the students were solving the problem. Furthermore, in the video clip, there was a student who attempted to solve but could not correctly solve the problem. For this moment, the teacher did not tell the student that she wrongly solved it. Instead, the teacher, with the help of the student's friends, helped the student find her mistake by herself. As can be understood from the explanations given above, this video clip allowed the pre-service teachers to see how students solve a problem in different ways and how a teacher attends, interprets, and responds to students.

Before the display of the video clip, the pre-service teachers formed groups with 2 pre-service teachers in each group, 13 groups in total. Then, the pre-service teachers were informed about what they needed to do while watching the video clip. The video clip was shown two times. For the first time, the pre-service teachers were asked to write all points that they noticed in the video clip. Then, they were asked to decide together as a group one of the points written by themselves and take as much notes of this point while watching the video clip for the second time. This process of note-taking for the first time, discussion of the point that they will focus on, and detailing it for the second time took approximately 20 minutes.

After detailing the points, these notes were collected from the groups to be scanned and stored for the analysis. The analysis of these notes assisted the author in identifying what the pre-service teachers noticed. Therefore, it can be stated that these notes were used as a baseline to understand how the points that the pre-service teachers notice differ in different mediums, specifically, video clip, animations, and depictions. One week later, these notes were returned to the groups, and based on the points that they decided to focus on in the video clip, they were asked to prepare an animation via Vyond and a depiction via Depict, which were the second and third data sources of this study, respectively. The first one of these programs, Vyond, is one of the online animation creation program which allows the pre-service teachers to create web-based short videos. In these animations, pre-service teachers can add scene(s) and character(s), as well as add movement, action, and sound to these character(s). Although the pre-service teachers had already learned to use Vyond during the first year of the program, a brief overview of the program was provided to the pre-service teachers to remind its use by the instructor of the related course. In short, the animations prepared by the pre-service teachers were used as a material to see whether or not the points noticed in the video clip change. The second one of the above mentioned programs, Depict, is a storyboarding tool which allows the pre-service teachers focus on interactions between teacher and students in addition to among students. In that course, the pre-service teachers had also learnt to prepare depictions using different tools. However, since the instructor of the course preferred the pre-service teachers to prepare depictions through Depict, they were taught about the features of the program. Specifically, they were informed how they can register, create graphical narrations of instruction by adding characters, and speech bubbles, teaching materials, etc. Similar to the animations, the pre-service teachers prepared their depictions in groups of two. These depictions were used to examine how pre-service teachers notice complexity or details of instruction process. The pre-service teachers were also asked to submit a note about both their animations and depictions to show what they wanted to notice in their animations and depictions. The reason for this request is to prevent the author from misinterpreting the pre-service teachers' attentions. All the data collection process starting from watching the video clip, preparing the animations and depictions, and finally submitting them to the author took two months.

The data collected from the different mediums were examined in three steps using the deductive approach: an existing framework is used to analyse the data. Specifically, in this study, van Es' framework (2011), framework for learning to notice student mathematical thinking was used. As mentioned before, this framework examines what teachers notice across four levels: Baseline (Level 1), Mixed (Level 2), Focused (Level 3), and Extended (Level 4) as given in Table 1.

In the Baseline, pre-service teachers provide descriptive and general comments for their observations. They do not provide any evidence for their comments. In the Mixed, although the pre-service teachers still continue to provide descriptive and general comments, they start to notice some of the important moments. To support that they notice, they refer specific moments but with little details. Furthermore, they cannot explain why these moments are important. On the contrary, in the Focused, pre-service teachers not only notice important moments but also provide evidence why these moments are important. That is, they try to justify their reasons even if their reasons are still judgmental. Finally, in the Extended, pre-service teachers notice specific students by focusing on these students' explanations. They also interpret these explanations by providing evidence from their knowledge of teaching and learning principles. Moreover, they suggest some actions that can contribute to the students' understanding using their pedagogy.

Therefore, the pre-service teachers' noticing of students' mathematical thinking in different mediums were coded in four levels, and this coding process was performed in three steps. In the first step, the groups' scanned notes were analysed to determine what the groups noticed. Then, the same procedure was performed for the groups' designed animations by Vyond, which was the second step. Finally, in the last step, the depictions prepared using Depict were analysed. These steps allowed the author to examine the similarities and changes across the levels of pre-service teachers' noticings within video clip, animations, and depictions. To exemplify the coding process, some of the pre-service teachers' statements from these mediums and rationales for coding these statements under a particular level were given in Table 2.

To ensure the trustworthiness of the study, all the process throughout the data collection and analysis was explained in detail. Furthermore, a graduate student, who knew the framework for learning to notice student mathematical thinking, independently coded the pre-service teachers' written notes, animations, and depictions, resulting in 92% agreement.

Table 1
Framework for learning to notice student mathematical thinking (van Es, 2011, p. 139)

Baseline (Level 1)	*Mixed (Level 2)*	*Focused (Level 3)*	*Extended (Level 4)*
What Teachers Notice			
Attend to whole class environment, behaviour, and learning and to teacher pedagogy	Primarily attend to teacher pedagogy Begin to attend to particular students' mathematical thinking and behaviours	Attend to particular students' mathematical thinking	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking

Table 2
Examples of pre-service teachers' statements and rationale for coding level

Level	*Statement Example*	*Rationale*
Baseline	There are lots of teaching materials and students sit in groups. The class is also quite large.	With this statement, the pre-service teacher focuses on the general classroom environment.
Mixed	The teacher allowed her students to solve the problem on their own. The students solved the problem in different ways and the teacher walked around the classroom to observe how the students are solving the problem or how the solutions are different from each other.	With this statement, the pre-service teacher pays attention to the students' specific solutions and associated teacher pedagogy.
Focused	The second student used rhythmic counting in fives. That is, he used the information (each eraser costs 5 liras) given in the problem. He counted in fives like 5, 10, 15, …, 60. Furthermore, he wrote 1, 2, 3, …, 12 above 5, 10, 15, …, 12 to represent the number of erasers. When the teacher asked these numbers above, the student asked what those numbers meant.	With this statement, the pre-service teacher is able to pick up on specifics in the students' thinking process.
Extended	The boy (in the animation counts) 5, 10, 15, …, 60. => 12 eraser (writes) The girl (in the animation counts) 5, 10, 15, …, 60, 62. => 13 eraser (writes) The teacher (explains) let's check our friends' solutions. They look so similar. One of them counted until 60 and the other one counted 62. Why did you count until 60? And Why did you count until 62? So, while one of them found 12 erasers, the other one found 13 erasers. Why are they different from each other? How can we help these friends check if their solutions are correct?	With this statement, the pre-service teacher tries to emphasize the difference between particular students' solutions and make connections between the students' solutions and the teacher's behaviours.

4. Findings

4.1. Frequency of what the groups noticed within video clip, animations, and depictions

The findings are presented according to the order of the research questions. To be more specific, after providing the frequency of what the groups noticed in three different mediums—namely, video-clip, animations, and depictions—the findings were detailed by providing the quotations from their notes, animations, and depictions. The frequencies of these levels are given in Table 3.

Table 3
Frequency of what the groups noticed within video clip, animations, and depictions

	Video Clip	*Animation*	*Depictions*
Baseline (Level 1) Attend to whole class environment, behaviour, and learning and to teacher pedagogy	7(53.8%)	0	0
Mixed (Level 2) Primarily attend to teacher pedagogy Begin to attend to particular students' mathematical thinking and behaviours	4(30.8%)	2(15.4%)	0
Focused (Level 3) Attend to particular students' mathematical thinking	2(15.4%)	8(61.5%)	8(61.5%)
Extended (Level 4) Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking	0	3(23.1%)	5(38.5%)

As can be seen from Table 3, the levels of the pre-service teachers' noticings for the video clip were coded 3 or below, indicating that some of them were able to attend to particular students' mathematical thinking. Specifically, of the 13 noticed moments in the video clip, 7 of them were Baseline, 4 of them were Mixed, and 2 of them were Focused. Stated differently, the pre-service teachers most often attended to the whole class environment, behavior, and learning, as well as to the teacher pedagogy when they watched the video clip. Closer examination of these pre-service teachers' scanned notes indicated two main details: describing the physical features of the classroom environment and explaining what the teacher did or did not do. The first and eleventh group's explanations, two of the examples which noticed the physical features, can respectively be found below:

> Students sit in groups around tables rather than in desks which allows them to discuss the problem with each other. Sitting like this also enables students to move around the classroom easily. As we see there are also many materials in the classroom that the students can use while solving problems.
>
> The classroom where the students would not feel foreign and be comfortable was prepared. There is wide space for students. They can use the classroom according to their needs. There are students' pictures on the walls which makes them feel at home.

As can be seen from the above notes, these two groups included no details beyond the physical features of the classroom. Contrary to these groups, the remaining five groups also attended to teacher pedagogy in their notes. The third group's note is one of the five which mostly noticed the teacher pedagogy: "The teacher created an open environment in which the students freely discuss their ideas and show their solutions without fear of solving incorrect way. This may be because the teacher does not tell the student who found the wrong answer that "You are wrong" or "You solved it wrong way." Another example of the teacher pedagogy is "After the teacher gave students time to think about and solve the problem, she walked to the students who solved the problem and asked questions like "How did you solve it?" or "Why did you count in fives?" In these statements, it is clear that the groups referred to what occurred throughout the instruction or how the teacher managed the problem solving process, but did not attend to any of these students' mathematical thinking or ideas. Four groups did begin to attend to the students' mathematical thinking in their notes in addition to the teacher's pedagogy. As such, Mixed was assigned for the level of these groups' noticings. The following excerpt taken from the fourth group's note explains how the student solved the problem and discusses the possible reason for this student's confusion:

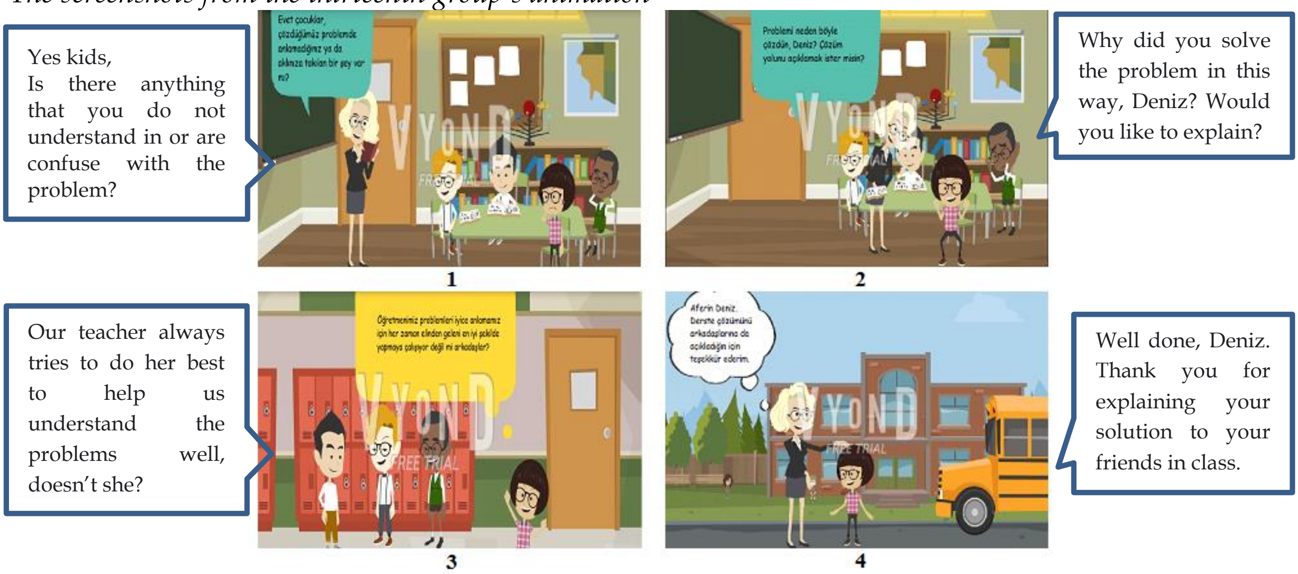
> The teacher asked the students to explain why they solved as they did or why they performed the steps as they did. Asking these kinds of questions is important for conceptual learning. For example, the teacher did not correct the student who could not solved the problem. The student tried to find the number of erasers by counting in fives. However, after writing 60, the student wrote 65 instead of 62 and hence found the number of erasers as 13. If the problem included 65 rather than 62, or another number divisible by 5, the student could have found the number of erasers correctly.

A few groups (2 of the 13) were able to attend to the particular students' mathematical thinking and thus illustrated the attributes of Level 3 as in the statement by the eighth group: "As understood from the first student's answers to the teacher's questions, he really knows why he drew 62 lines and then grouped these lines in five. In the same way, he knows the number of these groups of five was equal to the number of erasers. Otherwise, he could not say if I had 3 liras more, I could have formed another group of five and could have bought another eraser." Since the group both attended to and tried to interpret the student's thinking with respect to his answers or steps, the group's noticing level reaches a higher level, Focused.

When the groups' prepared animations were examined, it was found that no group noticed at Level 1 anymore. Contrary to this finding, there were groups who depicted a higher level of noticing. Specifically, the levels of the pre-service teachers' noticings for the animations were coded 2 or above (as shown in Table 3); 2 of them Mixed, 8 of them Focused, and 3 of them Extended. The groups whose levels were coded as Mixed primarily attended to teacher pedagogy without focusing on the mathematical details or ideas observed throughout the interactions between the teacher and the students. Some screenshots of an animation at this level, prepared by the thirteenth group, are given in Figure 1.

Figure 1

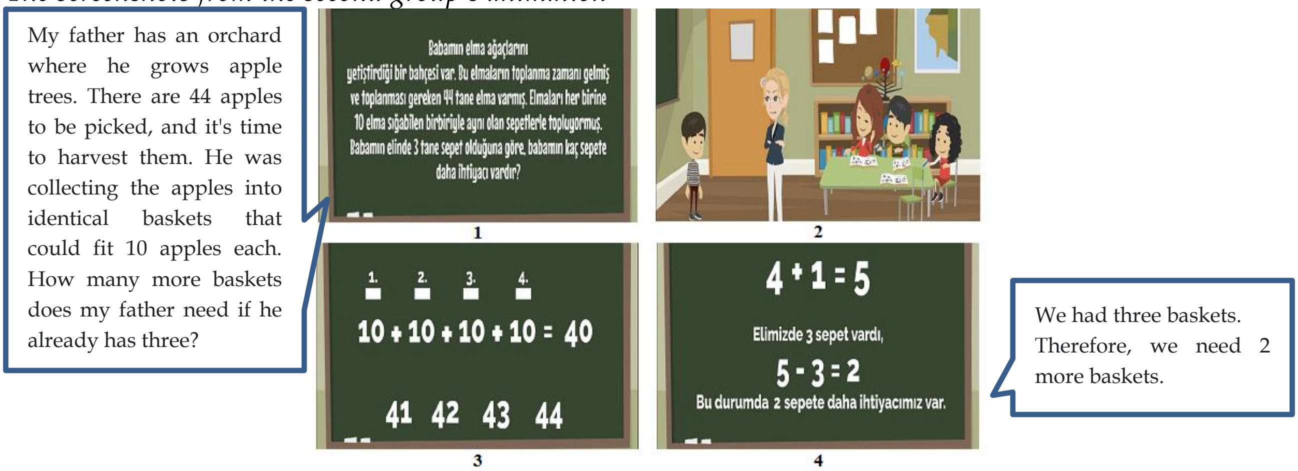
The screenshots from the thirteenth group's animation



In the animation, after the teacher started her lesson by reading a problem, the students tried to solve it (their solutions were not provided in the animation). Then, the teacher asked her students if there were any students who could not understand the problem. Since one of her students (the name of the student was not given in the animation) explained that he was having difficulty in solving it, the teacher asked Deniz to explain her way to her friends. The animation continued with the break time and the students telling each other "Our teacher always tries to do her best to help us understand the problems well, doesn't she?" The animation ended with the teacher thanking Deniz for sharing her solution. As it is clear from the summary of the animation, this group noticed how a teacher asks students their difficulties about a problem, questions a particular student's solution, and appreciates this student for sharing her solution. In this respect, the group's animation illustrates the characteristics of Level 2: Mixed. Furthermore, and more importantly, this group improved their noticing level, as the group's noticing level was Baseline for the video clip and it was Mixed for the animation.

The most frequent level of noticing within the animations was Focused, which means that the groups attended to the particular students' mathematical ideas. The following summary with some of the screenshots of the second group's animation (illustrated in Figure 2) reveals how the group primarily focused on the students' mathematical thinking and provided details from their thinking.

Figure 2
The screenshots from the second group's animation

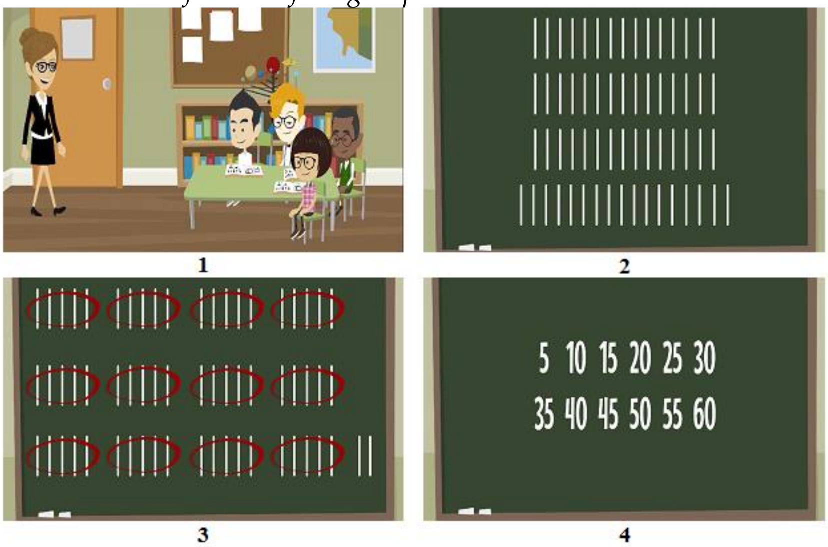


At the beginning of the animation, the teacher read and wrote a problem on the board. She walked around the classroom while the students were trying to solve the problem. Then, she invited Ali to the board to share and explain his solution to the whole class. While Ali was performing the steps shown above, the teacher asked some questions like "Why did you add these tens together?" "What does the number 1 mean in the equation of 4+1=5?" and "Why did you subtract 3 from 5?" to probe his thinking. After Ali finished his solution, Nihal came to the board to share her solution as well. The teacher asked similar kind of questions to prompt Nihal to explain her thinking. The animation ended with the teacher thanking these students for sharing their solutions. As summarized above and highlighted in their notes, the group primarily attended to these particular students' mathematical thinking about how they can find the number of necessary baskets which revealed an increase in their level of noticing of students' mathematical thinking from Level 2 to Level 3.

There are also three other groups who not only attended to the particular students' mathematical thinking, but also included connections between the students' mathematical thinking and teaching strategies in their prepared animations. These inclusions enabled their levels of noticing of students' mathematical thinking to be coded at Level 4. For example, Figure 3 provides the first group's animation including how two different students solved the same problem in different ways by providing visual representations of their solutions.

The teacher initiated the lesson by reading a problem, and then gave the students some time to work on it. Afterwards, the teachers asked one of her students how he solved it. While the student was explaining his solution, the teacher asked some questions to interpret his thinking herself as well as to enable other students to attend to his solution. Then, the student modelled his solution on the board and explained why he drew lines, grouped these lines by five, and found 12 as an answer. After the student concluded that he could buy 12 erasers with 62 liras, the teacher asked "How many more liras do you need to buy another eraser?" After the student stated that he

Figure 3
The screenshots from the first group's animation

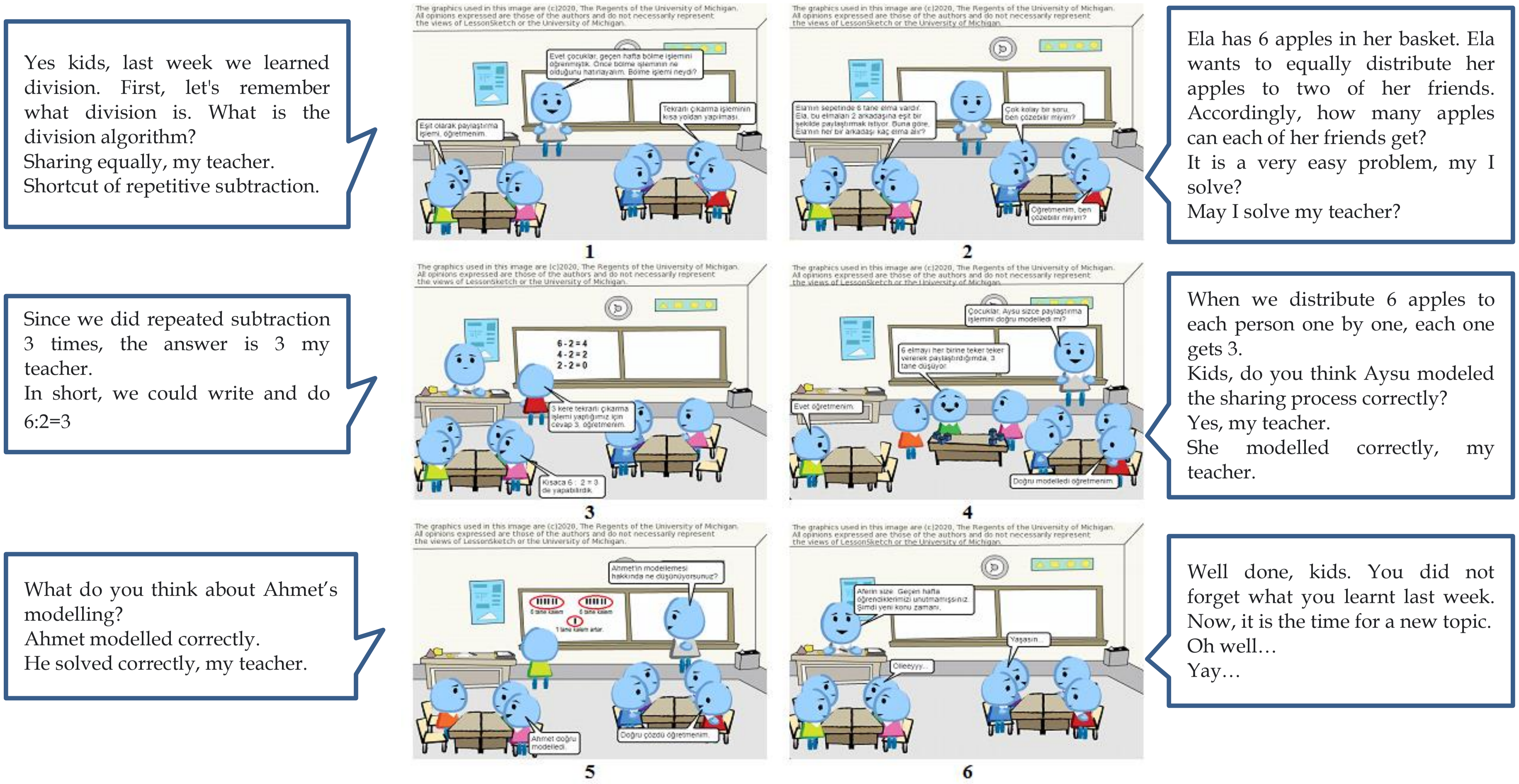


needed 3 liras to be able to buy another eraser, the animation continued with another student's solution. This student also came to the board and showed how she found the number of erasers by counting in fives. She continued this rhythmic counting in fives until she reached 60. Then, the teacher asked her why she did not count until 65. As an answer to this question, the student explained that she did not have 65 liras, but she had 62 liras, and hence she had to stop at 60. Then, the teacher summarized these solutions by emphasising their differences and similarities. Because of this emphasis, this group's noticing level was coded as Level 4 and this finding is important as the group's noticing level was Level 1 in their written notes for the video clip.

In the final part, the groups' prepared depictions were examined and it was found that in contrast to the groups' noticing levels for the animations, there were not any groups who noticed at Level 2. Similar to the animations, the most frequent level (8 of 13) observed in the depictions is Level 3. The sixth group, one of these eight groups, attended to the mathematical details by focusing on the interactions between the teacher and the particular students. Figure 4 provides some screenshots from their depictions.

The group's depictions start with reminding the students what they learned in the previous lesson and continues with asking the students to pose a division problem. After one of the students posed and read her problem: "Ela has 6 apples in her basket. Ela wants to equally distribute her apples to two of her friends. Accordingly, how many apples can each of her friends get?" the teacher invited another student (Aslı) to solve the problem on the board. Aslı explained that she could solve it using repeated subtraction and solved it on the board. While Aslı was returning to her desk, another student (Aysu) wanted to solve the problem by modelling. Afterwards, the teacher asked Aysu to explain and show her thinking. Aysu invited two other students to help her while modelling the problem. Aysu modelled the solution by giving blocks to those students one by one. When the blocks run out, Aysu concluded that she could give three apples to each friend. Afterwards, the teacher read a division problem with remainders: "Cetin has 13 pencils. How he can equally distribute these pencils to his two brothers so that each brother gets the maximum number of pencils?" and asked one of her students to solve it on the board. The student on the board (Ahmet), first of all, drew 13 lines to model the pencils in the problem and divided these lines into two groups by referring to two brothers in the problem. Then, he explained that "each brother can get maximum 6 pencils and 1 pencil will be left to Cetin" and the other students concluded that Ahmet solved the problem correctly. The depictions ended with the teacher appreciating the students not forgetting the topic that they learned last week. As can be understood from the summary of the depictions and the screenshots, more students were involved

Figure 4
The screenshots from the sixth group's depiction



in the teacher-student interaction and the group attended to the explanations of these individual students rather than that of the class as a whole. Throughout the depictions, the group also attended to the representations on the table or on the board to help other students understand the students' solutions. In this respect, the group's noticing level was identified as Level 3.

In addition to focusing on individual students' mathematical thinking, five groups in their depictions connected the students' solutions to the teaching strategies, which represented the characteristics of Level 4: Extended. One of these depictions, prepared by the tenth group, attended to how the teacher enabled their students to discover that they do not need to distribute one by one to be able to equally distribute the apples; instead, they can distribute them by twos or threes as shown in Figure 5.

In the first scene of the depiction, the teacher said that they learned the multiplication operation and would learn a new operation in that lesson. Then, to help the students be ready for this operation, the teacher gave an example: "Now, I will distribute the twelve apples on the table equally among you." After the teacher distributed the apples among the students one by one, she asked her students if she had to distribute them one by one. One of the students stated that the important thing is to give each person an equal number of apples each time and the teacher appreciated this student and restated what she said. Afterwards, the teacher asked a problem: "If I have 35 apples and want to distribute them equally among five people, how many apples can I give to each of them?" Later, two different students shared their solutions with the whole class. While the first student distributed the apples one by one, the second student gave each person two apples for three times and one apple for once. While the students were sharing their solutions, the teacher asked questions to make the students explain their thinking. Finally, the teacher summarized these students' solutions with further elaboration that they distributed the apples in different ways but found the same number as an answer. As it can be seen from this summary and the screenshots above, the group attended to the individual students' mathematical thinking and also used representations to model these students' solutions. In addition, they considered the teachers' questions in the depiction. Because of these considerations, their level of noticing was coded as Level 4. Like the sixth group who prepared the previous depiction (Figure 4), the tenth group who prepared this depiction (Figure 5) increased the noticing level. While the first one increased from Level 2 to Level 3, the latter one increased from Level 3 to Level 4.

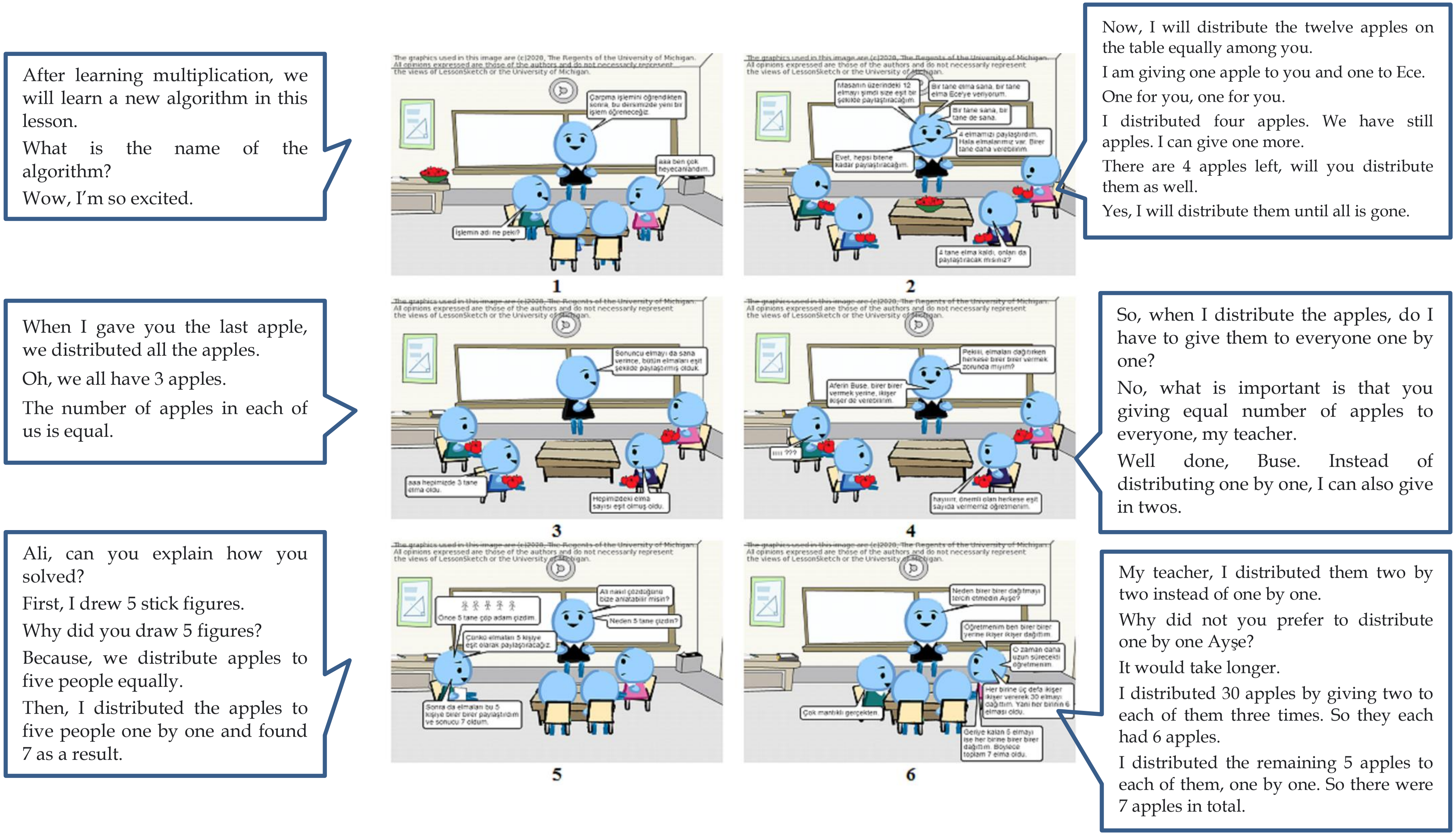
5. Discussion, Conclusions, and Suggestions

In this study, whether preparing animations and depictions affected the pre-service elementary teachers' noticing levels of students' mathematical thinking was examined. The findings suggest that the level of what the pre-service teachers notice is different among the video clip, animation, and sketch. Specifically, preparing animations and depictions increased the pre-service teachers' noticing levels as they started to attend to students' mathematical thinking, teaching strategies, and the connections between them throughout the animation and sketch design process.

More specifically, the pre-service teachers attended to the whole class environment, students' behaviours and learning in total, and teacher pedagogy in their written notes for the video clip similar to the findings in the literature (Star & Strickland, 2008; van Es & Sherin, 2002, van Es, 2011). They explained how the teacher in the video clip provided opportunities for students to express themselves, how she managed the problem solving process, or how she helped the students understand better. They also paid close attention to the classroom environment, including materials and pictures on the wall. However, particular students' mathematical thinking was missing in these written notes. Therefore, the level to what the pre-service elementary teachers was mostly found to be Level 2: Mixed. In teacher education programs, lessons in which pre-service teachers interact with students and experience to learn how to notice their mathematical thinking in real classroom environments are limited (Star & Strickland, 2008; Stockero et al., 2017). Hence, the reason for this deficiency can result from the pre-service teachers' lack of experience in real

Figure 5

The screenshots from the tenth group's depiction



classroom environments. Integrating video clips in teaching practice courses and guiding pre-service teachers to focus on and discuss students' mathematical thinking improve their noticing levels (Güler et al., 2020). Similarly, Ulusoy and Çakıroğlu (2018) also states that integrating micro-case videos in a course provides opportunities to consider students' mathematical thinking in detail. Similar to these courses, mathematics method courses can also involve activities which requires pre-service teachers anticipate and respond to students' mathematical ideas in more detail.

Regarding the animations, the pre-service teachers mostly focused on the particular students' mathematical thinking, which refers to the Level 3: Focused. The increase in their noticing levels might be attributed to different reasons. One of these reasons might be preparing new problems to use in the animations. Some of the groups did not use the problem provided in the video clip; instead, they wrote new problems and planned possible student solutions for them. Writing new problems or planning solutions for them might have enabled these groups to move from seeing the students as a whole to seeing them as individuals, another possible reason for the increase. In parallel to these findings, researchers emphasise that preparing animations enables pre-service teachers more carefully to consider different points which are not present in their written notes, such as focusing on students' solutions, connections among different solutions, and interactions between teacher and students (Amador & Earnest 2016; Amador et al., 2016). Smith and Stein (2018) assert that the lesson will be more effective "if the teacher has taken the time to anticipate ways in which students might solve a task and questions to ask students about their responses" (p. 14). As in this assertion, while these groups were preparing their animations, they did not only consider the students' answers, but they also included teacher-student(s) interactions. To be able to include these interactions, the pre-service teachers as a group might have discussed the teacher's alternative responses. That is, there might have been the exchange of ideas within the group about these interactions. Jacobs et al. (2010) emphasize that it is possible to learn to notice. In this respect, all these together might have enabled them to see these interactions from multiple perspectives and hence pre-service teachers learnt to notice. Apart from discussing these interactions as a group, participating in an intervention program like a video-club meeting enhances participant teachers' noticing levels (Star & Strickland, 2008; Sherin & van Es, 2009; van Es & Sherin, 2002, van Es, 2011). Although preparing animations in this study is not an intervention program and the increase in the pre-service teachers' noticing levels is not explicitly attributed to this preparation process, it provided the venue for pre-service teachers to consider students' mathematical thinking. By considering how to animate, pre-service teachers focus on particular students' ideas and what to do or how to respond to these particular ideas (Amador et al., 2016). In this sense, the pre-service teachers in this study had to think about the ways of responding to students which resulted in improvements in their noticing levels.

Although there were some similarities between what the pre-service teachers noticed in their animations and depictions by attending to the specifics of students' mathematical thinking, there were differences between them in terms of to which level they attended. They included more students who shared their solutions. Furthermore, the groups used materials like apples or drawings like stick men to model the students' solutions. As in the animations, some groups posed their own problems and included them in their depictions. Since these groups presented what the teacher would say or how the student would explain his/her thinking with the speech bubbles, they might have considered more carefully what they would write in these bubbles which resulted in individualizing these persons. Preparing depictions or comic-based lessons helps teachers see the importance of interactions or dialogues between teachers and their students to enhance students' understandings (Herbst & Chieu, 2011; Herbst et al., 2011; Rosebery, 2005). In the same way, reflecting on these interactions or dialogues provides teachers with multiple viewpoints and helps them consider alternative pedagogical strategies (Borko et al., 2008). Having these results in consideration, the pre-service teachers in this study might have returned and reflected on over and over again what they wrote in the speech bubbles. So having carefully considered these writings might have helped the groups attend to what the particular students thought. Preparing depictions allows teachers to "simulate the sorts of situations teachers confront in the midst of instructional practice and thus engage teachers in the ways of knowing involved in classroom teaching" (Kazemi & Hubbard, 2008, p. 438). In this study, likewise, the pre-service teachers more highlighted the connections among the students' solutions and were better able to respond to students' ideas by considering these connections as well. All of these highlights enabled these groups' noticing levels to be labelled as Level 4: Extended.

Returning to the purpose of this study, it can be concluded that preparing animations and depictions positively affects the pre-service elementary teachers' noticing levels as they attend to the particular students' mathematical thinking and instructional details that they did not previously attend to. Furthermore, between these two different designs, the pre-service elementary teachers included the most details about students and teachers in their depictions. To be more specific, while most of the pre-service teachers' noticings were more general in nature (Level 2: Mixed), some of them progressed to Level 3: Focused throughout the animation process, and five groups finally progressed to Level 4: Extended throughout the depiction process. Therefore, it can be concluded that preparing depictions better facilitated the pre-service elementary teachers' attention to student thinking. At this point, it should be noted that all the groups prepared their depictions after they prepared their animations. First of all, watching the video clip and noting what they noticed, then preparing an animation, and finally preparing a depiction might have allowed the pre-service teachers to become more aware and specific of the students' mathematical thinking or the importance of the interactions between the teacher and the students. Having these results in consideration, the mathematics method courses may not include theoretical definition and importance of noticing skill, but also may allow pre-service teachers to practice this skill by animating or depicting what might happen or what kind of interactions take place throughout the instruction.

In short, the increase in their noticing levels might be attributed to the pre-service teachers gaining experience and knowledge about noticing, which is a limitation of this study. Considering the above limitation, more research is need to focus on improving pre-service teachers' noticing levels is warranted. To overcome the above limitation, this study can be replicated by asking pre-service teachers to prepare depictions first and then animations. That is, conducting studies to answer the questions of "How and in what levels does preparing animations then depictions affect pre-service teachers noticing of students' mathematical thinking?" or vice versa "How and in what levels does preparing depictions then animations affect pre-service teachers' noticing of students' mathematical thinking?" might be helpful to see which one is more contributing to their noticing levels.

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