Journal of Pedagogical Research
Volume 9, Issue 1, 2025
https://doi.org/10.33902/JPR.202529564

Research Article

An analysis of algebra lesson: Can apples and pears be added?

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There is a strong relationship between the quality of education, teachers' decisions and instructional actions, and knowledge. All of them have an important role outcome of the lesson and students' learning. The focus of this study was to discover how the process of students' learning or incapability of learning was affected by teaching practices. Thus, the process of teaching and pre and post-lesson interviews conducted with a middle school mathematics teacher were analyzed. The data were collected through reflective interviews that were conducted with the teacher before and after the lesson as well as the video recording of the teacher's two-hour lesson. In light of the findings, it can possibly be argued that the teacher had actions that caused incorrect learning and misconceptions. This result cannot be explained by a single factor related to the teacher. It would not be wrong to claim that the instructional decisions (decision-making processes) in the teaching and teacher's content knowledge had decisive roles in the formation of this result, too. Therefore, the findings were discussed within the framework of instructional decisions and teachers' content knowledge.

Keywords: Analyzing teaching; Instructional actions; Teacher knowledge; Teaching algebra

Article History: Submitted 10 July 2024; Revised 20 September 2024; Published online 16 March 2025

1. Introduction

There is a strong relationship between the quality of education, teachers' qualifications, and knowledge. Especially, teachers' knowledge informs their practices and directs their actions in the classroom (Barnet & Hodson, 2001). Researchers highlight that teachers need to know how to teach mathematics (Ball et al., 2009). This knowledge should contain certain aspects of teaching such as the knowledge of the pedagogical content and the knowledge of the students' thinking. These knowledge categories should be both strong and as dynamic as teachers' content knowledge. The pedagogical content knowledge generally requires teachers to know how to teach mathematics. Meanwhile, the knowledge of students' thinking requires an understanding of the ways in which students can grasp concepts in an effective manner. In other words, teachers need to know which ideas students have produced for mathematical concepts, which ideas they have brought with them to the classroom, and finally which possible misconceptions they may have about the content. Such that, students can have differing learning styles, and they can also develop a variety of thinking strategies. In building suitable instruction in terms of students' thinking, teachers need

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How to cite: Bozkuş, F., Kalaycı, Ö., & Toluk Uçar, Z. (2025). An analysis of algebra lesson: Can apples and pears be added?. *Journal of Pedagogical Research*, *9*(1), 274-295. https://doi.org/10.33902/JPR.2029564

to know how students' mathematical thinking and understanding develop. However, some questions persist as to how teachers attend to students' mathematical thought processes and use this knowledge in their instructional decisions and classroom practices. For example, which content knowledge do students acquire or fail to learn at the end of the lesson? If students' learning is affected by teachers' decisions, how and why is it affected?

In addition, in the Programme for International Student Assessment [PISA] and Trends in International Mathematics and Science Study [TIMSS] applications carried out to determine the level of student knowledge and skills and to improve education systems accordingly, it is seen that the mathematical achievement of Turkish students in the age group of 15 remains below the average. In the PISA Turkey 2009, 2012, 2015, 2018 and 2022 reports, which measure students' literacy in mathematics, reading, and science, the average scores of the students are 445, 448, 420, 459, and 453, respectively, which are below the OECD average (OECD average is 496 for PISA 2009, 494 for 2012, 461 for 2015, 489 for 2018 and 472 for 2022). According to the TIMSS 2019 results, Turkey ranked 20th among 39 participating countries with 496 points, below the world average (500 points) in terms of scores obtained from the mathematics achievement test (International Association for the Evaluation of Educational Achievement [IEA], 2021). In this sense, Turkey ranks above the TIMSS proficiency levels (low level, intermediate level, upper level, advanced level) in mathematics achievement. In addition, when the rates of students who reached the TIMSS 2019 proficiency levels are examined, it is seen that the low level is 80%, the intermediate level is 56%, the high level is 32% and the advanced level is 12%. These results reveal that 12% of eighth-grade students in Turkey have advanced mathematics proficiency, while 20% do not reach the lower proficiency level (IEA, 2021). In addition, the low rate of students who can reach the advanced level indicates that students have low reasoning skills, including analysis and synthesis, evaluation, inference, generalization, and verification (Martin & Mullis, 2013). Developing these competencies in students brings with it the necessity of focusing more on the mathematical thinking of the student in the teaching process and shaping the teaching process in this direction. Indeed, it is noticeable that in the teaching practices of countries such as Hong Hong, Japan, the Netherlands, and the United States, which have achieved above-world average success in exams such as TIMSS and PISA, teachers focus on the mathematical thinking of their students and make inquiries that support the development of their mathematical thinking (Purdum-Cassidy et al., 2015).

In addition, policy interventions that have the greatest impact on student learning are related to in-class practices. The success of in-class factors also largely depends on the teacher (ERG, 2017). When we look at the PISA and TIMSS results, it is seen that teachers' experience and professional competence significantly affect student success. Therefore, it is thought that these results reveal the necessity of creating a framework for in-class teacher actions. In this context, it is thought that the findings to be obtained from this research, which focuses on the teacher's noticing skills and teacher actions, will contribute to the field by focusing on students' mathematical thinking and providing a framework for how these thoughts can be used and developed in the teaching process. Based on these considerations, this study focused on analyzing an algebra course. It was believed that such an analysis could yield some great insights into how teachers approach students' mathematical ideas, how much of teachers' decisions were based on students' mathematical ideas, and how students' learning or non-learning situations were affected by teachers' instructional actions.

1.1. Teacher Noticing

Effective mathematics teaching occurs when teachers encourage mathematical discussions that focus on students' mathematical thinking by providing students with opportunities to talk, recognize, and support themselves to develop a deeper level of mathematical thinking and understanding (Hiebert & Grouws, 2007; Van Zoest et al., 2016). Hence, teachers need to be familiar with students' mathematical thinking in order to build better teaching techniques in terms of what and how students think. Teacher noticing provides us with a good framework to assist an understanding of how teachers approach student thinking. Noticing can simply be considered to be a professional vision (Goodwin, 1994). van Es and Sherin (2008) defined teacher noticing as being composed of three skills: identifying related aspects of the teaching situation, using knowledge to understand the events, and making connections among specific events and general principles and ideas about teaching and learning.

Noticing, in terms of identifying the important moments of the teaching process, requires determining what is important and noteworthy in the classroom environment. van Es and Sherin (2002) stated that learning is a complex activity, therefore the teacher must be able to evaluate classroom situations well and determine which interactions are specifically important (Sherin & van Es, 2005). The second skill focuses on the teacher's awareness of details such as student thinking and student understanding in the classroom and how they use this in teaching. In this sense, in order for teachers to reason about the situations they analyze; the teacher's knowledge of how students think and field knowledge come into play (Dreyfus & Dreyfus, 1987). Therefore, in this dimension, the knowledge that the teacher has is used in the situations encountered and the knowledge is expanded. For example; a mathematics teacher can interpret the mathematical thinking of the student in any learning environment more effectively than a classroom teacher. In other words, each teacher can reason better about the situations that occur in their own field of expertise (van Es & Sherin, 2008). The third skill involves reflecting on the meaning of a special situation in the classroom in terms of education. In other words, the special situation that occurs is interpreted and discussed with broader learning-teaching principles. In this sense, in addition to realizing that a situation is important in the teaching process, it is also necessary to think about what that situation means (Sherin & van Es, 2005). For example; the teacher's statement "Here, the teacher is paying attention to the students' thoughts" or "It seems that all the students in the class have started to learn" regarding the video of a lesson he watched shows that the teacher associates a special situation he sees with a principle or concept related to learning and teaching.

More specifically, Jacobs et al. (2010) defined teacher noticing as compromised of three other skills: attending to children's mathematical thinking, interpreting this thinking, and then deciding what to do next based on this thinking. This definition implies that teachers need to pay attention to a particular student's mathematical thinking by observing the student's correct, and incorrect answers, procedural mistakes, justifications, and explanations (Lee & Francis, 2018). The details of students' answers and strategies can be complex, but they certainly give information about students' understanding (Carpenter et al., 2003; Lester, 2007). In the meantime, the way in which teachers interpret these details gains importance while teachers make instructional decisions because teachers' reasoning about the details of a specific student's thinking can affect the practice of teaching. In particular, such an insight into students' thinking can help teachers decide which mathematical activity should be planned in order to support students' mathematical understanding and which directions and questions should be used to enhance students' deeper understanding (Clarkson & Presmeg, 2008). In fact, the third component of teacher noticing is crucial in deciding how to respond on the basis of the students' understanding. Note that the third skill does not involve the execution of teachers' acts but rather the reasoning that comes before teachers' moves (Jacobs et al., 2015). Therefore, the decisions that teachers make in the moment of instruction affect the outcome of the lesson and students' learning (Jacobs et al., 2010). In the following section, the relationship between teachers' instructional decisions and students' learning is elaborated on.

1.2. Teacher's Instructional Decisions and Student Learning

It is known that teachers' instructional practices directly influence students' learning and achievement (Fennema et al., 1993). Besides, some evidence shows that attending to students' mathematical thinking or reasoning helps teachers improve their practices (Crespo, 2000; Sherin et al., 2011). Especially, an understanding of students' ways of thinking can help teachers to make better instructional decisions (Franke & Kazemi, 2001). When teachers are attentive to the students' thinking and students' alternative solutions to the problems, they are able to develop a richer understanding of students' thinking (Doerr, 2006). On the other hand, students' mathematical thinking might not always be mathematically correct and might include errors or misconceptions. However, students' mathematical thinking may be more powerful and productive than teachers realize (Empson & Jacobs, 2008). At this point, teachers should not only correct wrong answers and implement procedural steps, but they should also attend to what students say. Teachers should determine whether students' answers are meaningful for productive learning (Franke et al., 2007; Wilson et al., 2013). However, some studies (El Mouhayar & Jurdak 2015; Liu, 2014) reveal that teachers do not pay enough attention to student's thinking and benefit from their knowledge of students thinking in their practices. It is stated that teachers usually tend to enforce their own instructional decisions instead of building classroom instructions on students' thinking. For example, in Liu's (2014) study, one of the teachers focused entirely on her own instructional decisions in the classroom. In deciding how to progress during teaching, the teacher was informed by the students' understanding and ways of thinking only to a limited extent. Liu (2014) stated that the other teacher attended to the student's thinking, but s/he did not use this knowledge in her instructional decisions. What is important here is that a teacher should not only pay attention to the student's mathematical thinking but also build suitable teaching methods on the students' thinking. In light of all these, the question as to whether and how teachers should attend to students' thinking and use this insight while making instructional decisions needs to be responded to. Moreover, it is equally significant to examine how teachers approach students' learning. With this goal, the present study focused on an algebra lesson, and in the following section, some students' difficulties with algebra were discussed and some approaches to algebra teaching were presented in order to better understand teachers' instructional actions.

1.3. Student's Difficulties and Teachers Practices in Algebra

Algebra is an indispensable subject area of mathematics learning, and at the same time, it seems to be a difficult topic for students to learn because of its nature. Teaching algebra at school even complicates this problem further. Although it is pedagogically more appropriate to lay the foundations of algebraic thinking in primary school, what is generally performed in mathematics classrooms is a more result-oriented and calculation-focused approach, which falls short of achieving the aim of improving students' algebraic thinking skills (Toluk Uçar, 2018). This situation makes things much more challenging for students in terms of grasping algebraic relationships and concepts when they are in the upper grades (Berg, 2012; Kieran, 2004; Watkins, 2018). In this context, it is stated that the underlying reason behind the student's difficulties in relation to algebra is some poorly learned concepts such as variables and equations (Herscovics & Linchevski, 1994; Kaput, 1999). The term of variable is defined as "a letter or a series of letters representing one or more numbers" (Herscovics & Linchevski, 1994; Arcavi & Schoenfeld, 1998). Research reveals that students have trouble making sense of such letters and defining the relationship between the meaning of the letters, so they produce misconceptions (Kieran, 2007; MacGregor & Stacey, 1997; Usiskin, 1999). These misconceptions can be exemplified as such: "letters do not have any meaning in mathematics" (Kieran, 2007; Usiskin, 1999), "letters stand for numerical positions" (Perso, 1992), "seeing letters as object labels" (Booth, 1988), "letters do not act like numbers" (Perso, 1992).

One of the reasons for the challenges that are mentioned above and students' misconceptions is claimed to be deficiencies in teaching (Kieran, 1992). For instance, teachers often employ a fruit salad approach (addition/subtraction of apples with apples or pears with pears) to differentiate between like and unlike terms in algebraic expressions (Booth, 1988; Tirosh et al., 1998). In this approach, an algebraic expression such as "3a+2b" is defined as the adding of 3 apples and 2 bananas, which causes students to perceive the letters as object labels and not as variables that represent numerical quantities (Booth, 1998). Such an approach may lead students to think that the expression of 2a+5b as "2 apples + 5 bananas = 7 apples and bananas", which in turn makes way for them to produce an erroneous equality (Booth, 1988). However, students are required to know that an algebraic expression like "4a" represents "four times a number;" in other words, they should know the meaning of an algebraic expression in terms of corresponding to its structural feature (Pomerantsev & Korosteleva, 2003). If this structure and variable as a term are not appropriately conveyed to students, various learning difficulties are experienced (Herscovics & Linchevski, 1994). In relation to this, Pimm (1987) highlighted that an algebraic expression, such as "5a" does not correspond to "5 times an apple." Stating that "5 apples" was not a possible interpretation of "5a", she suggested that this approach was not useful. Moreover, she put forward that this approach might cause a student to develop the idea or belief that these two algebraic expressions "2a" and "5b" could not be multiplied. Indeed, with such an approach, it is impossible to multiply "2a" with "5b" by using such object labeling as apple and banana. Therefore, "the fruit salad" approach to the teaching of algebra is not only criticized for not being able to represent the exact meaning of algebraic expressions (Chalouh & Herscovic, 1988) but also is identified as the root of many difficulties that students experience (Booth, 1998). The offering of concrete models to symbols, on the one hand, promises success in the short term but on the other hand, makes way for students to create misconceptions in the long term, which is a widespread phenomenon among students (MacGrego, 1986).

2. Methods

The focus of this study was to discover how the process of students' learning or incapability of learning was affected by teachers' instructional decisions and actions. Thus, the process of teaching was analyzed. This study was part of a more extensive study aiming at developing middle school mathematics teachers' ability to attend to students' mathematical thinking skills. So, the teacher who is the focus of the present study was one of the two participating teachers in the pilot study. The teaching examined in this study was the first lesson of the teacher in the project, and the teacher did not participate in any training related to the project content. Therefore, the results presented in this study include the findings from a teaching process without any intervention.

We conducted a case study to examine teacher practices in mathematics lessons, focusing on a single bounded case (one participant-Lucy) to gain a more in-depth understanding of the topic (Creswell, 2007). During this process, we employed pre- and post-lesson interviews, classroom observations, and video recordings of the lessons.

2.1. Participants

This study involved a middle school mathematics teacher, Lucy, who was employed at a public school and taught 6th-grade students, along with her students from this class. Lucy had five years of teaching experience and had taught all middle school grades (from 5th to 8th grade). She expressed that she felt more comfortable teaching 6th graders, noting that students in this grade level tended to be more engaged and active participants in lessons compared to other grade levels. This factor was taken into consideration when selecting the class for the study. The classroom consisted of 35 students, including 18 female and 17 male students. The school had an average level of academic performance and socioeconomic status. The students exhibited a wide range of academic achievement, from low to high. After being informed about the study, both Lucy and her students voluntarily agreed to participate, with voluntary participation being a key criterion for this research. Furthermore, no participants expressed a desire to withdraw from the study at any point during the process.

Before the project began, interviews were carried out with the teacher, and some of her lessons (2 hours) were observed. In the interviews, it was observed that the teacher's perception of her own teaching included the following: "a teacher who gives enough time to the students to express themselves and takes the students' thinking into account". Along with this comment, it was determined that the teacher's self-confidence was high in terms of content and teaching knowledge. However, the observations made before the project demonstrated that the teacher's actions and approach to teaching did not support those personal beliefs. It can be argued that she was the one who spoke mostly and the one at the center of knowledge while the students remained in the shadows. So, it can be said that the teacher generally carried out the teaching through a teacher-centered approach. Therefore, it was thought that teacher Lucy could be a good sample in terms of revealing how the instructional actions of the teacher affected the learning situations of the students and revealed the difference between the teacher's approach in theory and her reflections in practice.

2.2. Data Sources

The data were collected through reflective interviews that were conducted with the teacher before and after the lesson as well as the video recording of the teacher's two-hour (40 + 40 minutes) lesson. In total, 80 minutes of lesson videos were recorded. Lucy and her students were informed that the lessons were being videotaped. However, they were not specifically told that the focus of the recordings was on classroom discourse, as the purpose of the study was to observe the teacher's actions and students' learning during mathematics instruction in a natural setting.

2.2.1. Reflective interviews

In order to reveal holistically how the teacher's actions in teaching affected students' learning, it is significant to reflect on what the teacher thinks and how she approaches students' learning in the process before and after these actions. To this end, pre- and post-lesson reflective interviews with the teacher were conducted. In the pre-lesson interview, some questions regarding the lesson were directed to the teacher, i.e., *What is the goal of the lesson? What are the objectives of the lesson with respect to the goal? Could you please describe the lesson plan in detail?* These questions not only revealed the teacher's instructional decisions but also reflected the overall structure of the lesson. The post-lesson interview included the teacher's self-evaluation of her teaching performance and student learning. In this context, questions as to whether the lesson mostly fulfilled the goal, what the students' reactions during the teaching and students' mathematical achievements at the end were, and whether she experienced any estimated or non-estimated occasion were asked of the teacher. This interview can be regarded as a reflection on the lesson from the perspective of the teacher. Both interviews were carried out in approximately 20 or 30 minutes and were recorded.

2.2.2. Lessons

The lesson that was observed for the study was a video recording of a two-hour lesson (80 minutes) that focused on the achievement of a specific goal, which was "the student is able to do the operations like addition and subtraction using algebraic expressions." Table 1 presents a brief synopsis of the lesson.

Table 1
Information about the lesson

Seg.	Length	Description
Lesson 1		
1	0-16 min.	Questioning about like terms
2	16-30 min.	Lecturing on the addition of algebraic expressions
3	30-40. min.	Doing exercises about the addition of algebraic expressions
Lesson 2		
1	0-20 min.	Doing exercises about the addition of algebraic expressions
2	20- 29 min.	Lecturing on the subtraction of algebraic expressions
3	29- 40 min.	Doing exercises about the subtraction of algebraic expressions

The teacher began the lesson by reminding students about like terms. To do this, she wrote 10 different terms on the board, asked the students to identify which terms were like and which were unlike, and encouraged them to elaborate on their reasoning. In the second part of the lesson, the teacher gave a lecture on the addition of algebraic expressions. She started by asking simple questions involving the addition of natural numbers (e.g., adding 4 apples and 6 apples) and then explained how to represent the objects in the question using symbols. Following this, she demonstrated how to perform the operations related to the addition of algebraic expressions. Then, in the final part of the first session and the beginning of the second session, the teacher wrote exercises involving the addition of simple algebraic expressions on the board and asked the students to solve them. Once the exercises were completed, she explained how to perform subtraction with algebraic expressions. In the last part of the lesson, she provided a sample solution that involved subtraction with algebraic expressions containing like terms.

2.2.3. Analyses

The data that was obtained from the interviews with the teacher and the video recording were first transcribed and then analyzed. In order to analyze the data, a framework for instructional analysis that was developed by Hiebert et al. (2007) was utilized. In this framework, four skills used in planning, implementing, and reflecting on the teaching are at the center of the attention. These skills seek answers to questions, such as *"What should students learn?"; What did they learn?"; "What is the effect of teaching on students' learning?" How can teaching help students to learn more efficiently?"* Informed by this framework for analyzing and developing teaching, the decision was made to scrutinize the data in terms of four categories: "the teacher's plans for teaching", "teacher's instructional actions", "students' approach to learning algebra and the teacher's role", and "evaluation of teaching from the perspective of the teacher".

Consistent with these categories, the interviews, and video recordings were divided into meaningful units, and certain codes were formulated in accordance with the teacher's discourse. In order to analyze the video recording, the teacher's actions were associated with the students' learning and divided into meaningful units. These units sometimes contained the teacher's expression, a question, or relatable sequential expressions of the teacher and, sometimes included a dialogue between the teacher and students. Therefore, some units indicated a process of interaction and bilateral conversations involving a student's reaction to the teacher's action. To establish coding reliability, two researchers coded randomly selected transcripts, which accounted for 25% of the data. They then created codes for the units, and the consistency of their codes was found to be 92%. The researchers then conferred and reached a consensus regarding the discrepancies in their codes. After coding the entire data, the percentage and frequency of codes for each category were calculated and the results of the analysis were represented in the findings.

3. Results

The findings were presented under three sub-headings: planning, implementation (the teacher's actions and students' learning), and a reflection process.

3.1. Findings on Teachers' Planning

The determined codes vis-à-vis the teacher's planning for the teaching and the teacher's explanations in the pre-lesson interview are described in Table 2.

In Table 2, it is seen that the teacher superficially defined the objective of the lesson and was not able to determine which idea was important for the lesson. Therefore, she was not able to define the mathematical thinking that she wanted the students to develop and the preliminary knowledge necessary for the development of this thinking. In a similar vein, it can be argued that the teacher's predictions about the students' thoughts were restricted to her experiences, along with her poor description of what difficulties the students might experience. On the other hand, it was deduced from the explanations of the teacher that she had no planned activity regarding the teaching and that she would improvise the lesson. Not only did she not define the mathematical ideas related to the objective, but she also defined the knowledge that the students should have attained as a result of the addition and subtraction of algebraic expressions. Therefore, in the

Table 2
Teacher's planning for the teaching

Codes	*Teacher's explanations*
Superficial definition of the goal	It is sometimes necessary to add or distinguish different objects or units in our daily life. Therefore, in our lesson, we will learn how to perform operations like addition and subtraction by linking like terms that we have learned in algebraic expressions.
Not being able to elaborate on the activity or assignment	We will do a revision for like terms. I will then let the students question them. I will try to get the students to find like terms without interfering with them as much as possible. We will talk about how we can add and subtract. For algebraic expressions, I will give more concrete examples, such as apple and pear, and I will try to teach through them.
Not being able to define what is significant.	The students are required to multiply a natural number with an algebraic expression. They are required to know like terms. They are also required to know how to add and subtract integers.
Not being able to estimate students' mathematical thinking	As I have explained the like terms, some of the students can associate like terms while some of them cannot establish a connection at all. They are able to do addition and subtraction.
Being able to predict some difficulties	
Different exponents to like terms	The unknown values of like terms must be the same. They may sometimes not notice the dissimilarity among the exponents.
The use of the distributive property	The students will be confused in the cases where there are three or four like terms and the constant term is involved in removing this word. We will use the distributive feature when adding and subtracting. If there is a minus in front of the parenthesis, if we can get there, the students may have difficulty at that point.
Being able to detect student's difficulties	I will do a question-and-answer activity.

planning interview with the teacher, it was clearly realized that the teacher did not have a goal of paying attention to the students' thoughts, making decisions according to those thoughts, and putting these decisions into action; rather, she had a goal of providing procedural knowledge to the students in accordance with her own plan.

3.2. Findings on Implementation Process

The findings regarding the implementation process were structured on two main frameworks. The first relates to the findings from the teacher's removing quotation marks which referred to the teacher's approach to students' thinking in response to her teaching and her pedagogical approach based on content knowledge. The second concerns the findings in terms of perceptual or misconception that was developed by the students as a result of the teacher's actions, the difficulties that they experienced, and the findings in terms of the teacher's role in this process. These findings were presented with examples from the teaching.

3.3. Teacher's Instructional Actions

The teacher's instructional actions were analyzed under two categories. One of these categories included the teacher's defined actions regarding what she focused on in the implementation process and how she approached the students' thinking. The other category contained the teacher's actions regarding the pedagogical approaches that affected the students' learning depending on the teacher's content knowledge. The findings as to the teacher's actions and the frequency of these actions are presented in Table 3.

Table 3
Findings on the teacher's instructional actions

Category		*Codes*	*f*
Pedagogical approach	Not attend students' thinking	Not giving voice to students	42
		Not being able to notice the student's thinking	15
		Not paying attention to the students' thinking	12
	Teacher response to student	Trying to elicit the correct answer	35
		Listening to the student and explaining herself	18
		Postponing students questioning	7
		Having judgmental attitudes toward the student	6
		Asking questions	
		• Using general questions	19
		• Using leading questions	19
		• Using special questions	12
		• Questioning for approval	
The pedagogical approach based on content knowledge		Explaining focused on procedural knowledge	27
		Deficiency in content knowledge	
		• Sticking to similar examples	9
		• Not being able to determine important mathematical ideas	5
		• Not encouraging the use of mathematical language	4
		• Not being able to suggest alternative solutions	2
		• Not being able to simplify the topic	2
		Creating or supporting misconception	
		• Seeing variables as object labels	5
		• Supporting the idea that unknown values and variables are the same	5
		• Matching letters from left to right	2
		• The position of the letters in algebra	1

The findings in Table 4 indicated that the teacher's general approach to students' thinking did not give enough opportunities for the students to think and express their ideas (f=42). The teacher tended to make explanations constantly during the teaching, so the student's ideas remained in the background. For this reason, the teacher sometimes did not notice the students' ideas (f=12) and sometimes did not pay enough interest to what the students expressed although she heard the students' ideas (f=12). Because she looked for the correct answer or because she was looking to care too much about implementing her own pedagogical plan. The instances in which the teacher did not notice the students' thoughts were mostly the moments in which the students mislearned and developed misconceptions. Therefore, the teacher missed the opportunities that could prevent the students from mislearning, because she did not notice those cases. The examples that related to such instances were given under the following sub-title.

Besides, in the cases in which the teacher noticed the students' thoughts, she did not pay much attention to those ideas as she did not try to understand them. In the teachers' responses to students, the teacher sometimes listened to the students but did not give a reaction and continued the lesson. Sometimes, the teacher listened to the students and explained herself by related statements. Such instances can be defined as cases in which the teacher struggled to get the answer that she wanted from the students and did not find out what the students really thought. Such, when students said wrong, she tried to elicit correct answers with leading questions. For example, a dialogue between the teacher and the students in relation to this finding is reflected in the following:

> Teacher: How can we reorder 3x + 2x = 5x? Phoebe?
> Phoebe: Distributive property.
> Teacher: It's the opposite? (Trying to elicit the correct answer)
> Students: Reverse distribution property.
> Teacher: Ok, you are getting closer. Frank? [Trying to elicit the correct answer]
> Frank: Associative property.
> Teacher: Oh, no! Okay, I will remind you. 5 (2 + 3), we would distribute it: 5.2 + 5.3. What is the name of this property? (Judgmental attitude; trying to elicit the correct answer)
> Students: Common factor parenthesis (Meanwhile, answers such as sharing property come from the class). (Not paying attention to the student' answer).
> Teacher: Can you see the common terms here? (Pointing out the expression 3x + 2x = 5x). If they are under multiplication, both terms have x. Can we say x in parenthesis 3 plus 2 (writing x (3+2) on the board)? (Not giving an opportunity to students because she is explaining)
> Stds: Yes.
> Teacher: x (3+2). Here, the first thing is to do the operations in parenthesis. You know that. Three plus two makes five. We can say X times 5. Which property does multiplication have? [explaining herself- asking general question]
> Students: Commutative property.
> Teacher: We can reorder x(3+2) as x.5, which can also be reordered as 5x. We can think it like this, can't we? [Asking questions to get approval from the students]
> Student: Teacher, can I ask a question? When we do addition in natural numbers, we will subtract the number if the smaller one is in the front. Is it similar, right?
> Teacher: Do you mean differences in symbols?
> Students: Yes, teacher.
> Teacher: We will come to that soon. From the simpler to the more complicated. Of course, symbols may differ, and terms may remain the same. Any problem? Have we learned something in total? [Postponding student questioning - explaining herself]

As the dialogue above demonstrated, the teacher asked the students a question, but she immediately gave the explanation without giving the students enough time to think. Although the answer to the common factor bracket came from the students, the teacher explained the procedures for disregarding those answers. When the teacher explained how to express the expression 3x + 2x = 5x in different ways, she also related to the students how to think. On the one hand, such an approach could be regarded as directing the students to think in a uniform way. On the other, when a student-directed a question to the teacher, she dismissed the student's question and stated "*We will come to that later*", even though she understood what the student asked. So, it can be discussed that the teacher did not pay attention to the student's question, and she was looking proceeded to sticking to her own plan.

As to the teacher's questioning attitude, she generally benefited from general (f=19), leading (f=19), and special (f=12) questions. The general questions directed by the teacher examined the students' knowledge but did not elaborate on the students' answers, such as "What are the like terms?", "*Can we add these terms?*", "*Can we bracket these terms?*", "*Would* $17b^2a$ *and* $17b^3a$ *be the same?*", "*How many of you agree?*" Such questions were not effective in terms of elaborating students' thinking and reasoning. Another question type that the teacher utilized was the leading question, which is defined as questions that draw students' attention to a certain point and lead the students to an answer (Franke et al., 2009). The teacher frequently used guiding questions (f=19) during the teaching. Previous sections noted that the teacher's focus was on her own plan rather than the students' ideas. The teacher actually used such questions in order to implement her own plan. On the other hand, the teacher used special questions only 12 times. A special question refers to a certain or special part of the student's explanation and relates to the missing, unclear, or incorrect part of it (Franke et al., 2009). Therefore, it is an efficient type of question to elaborate on students' ideas. In a dialogue, the teacher utilized a special question in relation to the correct equation of the expression "*2 (12x + 1y)*", and said, "How did you get there?" However, the teacher did not give the student an opportunity to express her or himself and did not have the role of facilitator. At another moment, the teacher wanted to question the students' correct answer but she gave the answer without offering the students a chance to answer. The following dialogue illustrates such a condition.

> John: 14x + 12x + 7x = 35x
> Teacher: It says 35x. Do you agree? Any objection? (Asking for the first time). How did we decide on the similarities among the terms? According to the variables next to them, isn't it? We added the factors (not allowing the students to make inferences). Come here, Tony. Not paying attention to the students' thinking- Not giving voice to students (class) because she is explaining]
> Tony: 3x+3+8x. Teacher, there is no x here?
> (Teacher did not give a reaction and continued the teaching)

The dialogue clearly reveals that the teacher answered the question she asked the students and did not allow the students to make inferences about the addition. So, the teacher did not genuinely care about how the students thought. For example, facing the unfamiliar expression "3x+3+8x", the students were buffed and did not know what to do. In order to delve deeper into the student's remark, the student created memorized knowledge or developed such an understanding as "x must be in every term" and "the necessity of adding the numbers next to x" while doing the addition by referring to the explanation that was given by the teacher in the previous example. Therefore, it can be claimed that the teacher's approach that did not offer a chance to the students caused them to memorize the knowledge and develop misconceptions.

When the pedagogical approach based on content knowledge was examined, the moments when the teacher affected the students' learning process. One of them, teachers' explanations included procedural knowledge and could not make sufficient conceptual explanations. The teacher focused on like terms, and the steps of addition and subtraction in algebraic expressions, and used similar examples for these processes. Another one, it was observed that due to insufficient content knowledge, it could not facilitate student learning. On the other hand, with respect to the teacher's pedagogical content knowledge, she displayed actions that negatively affected the students' learning due to her poor content knowledge. Within this respect, the teacher could not able to determine important mathematical ideas (5), encourage the use of mathematical language (4), suggest alternative solutions (2), simplify her explanations (2), and generally teacher stuck to similar examples with repeated explanations (9). As a result, the teacher caused students to develop non-mathematical understanding, created misconceptions and caused the students to experience difficulties at times. Related instances were described under the sub-title of findings on students' learning.

3.4. Students' Approach to Learning Algebra and the Role of the Teacher

The situations given in Table 4 regarding the learning statement of the students in this lesson were determined.

Table 4
Students' attitudes to the learning of algebra

Codes	*f*
Developing non-conceptual knowledge	13
Forming misconceptions	
• *Assuming that +, -, and = produces results*	3
• Perceiving variables as *object labels*	3
• Using *unknown* and *variable* interchangeably	2
• Leaving, unlike terms in the end.	1
Not being able to use suitable *mathematical language*	7

As demonstrated in Table 5, the instances in which the students developed non-conceptual knowledge (f=13) were mostly defined. Lucy utilized procedural knowledge in her explanations of addition and subtraction with algebraic expressions. This situation caused the students to develop solely procedural knowledge but at the same time caused them to form misconceptions from time to time. A sample dialogue that demonstrates this situation was portrayed in the following:

> Teacher: If we add 2 more apples to the apples of Mary, who has 3 apples, how many apples will he have? (Writing on the board). How do we solve this problem? [Using leading question]
> Students: We add.
> Teacher: Very simple, we add 2 to 3. Let us specify the units. 3 apples plus 2 apples. 3 apples +2 apples = 5 apples. How can we relate this to the title? We know the definition of algebraic expression. [Supporting the notion that +, -, and = produce results]
> Students: We substitute x and y for apples. [Assuming letters as object labels]
> Teacher: So, what does apple actually represent?
> Students: The unknowns. [Unable to distinguish between unknown and variable]
> Teacher: Good. Now, if we adapt each of the apples to our lesson, what would 3x + 2x = 5x be? Can we make analogies? What could be the result of 9x + 12x? Hans? [Not giving voice to students and explaining herself and Letting students assume letters as object labels]
> Hans: The total? 21x.
> Teacher: Let us consider this. There are 3 apples. We added 2 more to them. We counted 5 apples. Nice (drawing apples on the board). Can we do this in a different way? (Pointing out algebraic addition operation). Look, under which operation 3 and x were? ([etting students assume letters as object labels- trying to elicit the right answer]
> Students: Multiplication. (Dialogue continues).

The teacher wanted to activate the students' prior knowledge by starting from the operation 3 apples + 2 apples = 5 apples which is expressed in primary education in order to step into algebraic expressions. So, the teacher employed a process-oriented approach by drawing the students' attention to the operation of addition. Following this, she wanted to adapt that expression to the algebraic expression by associating the expression of 3 apples + 2 apples = 5 apples with algebra, but she shared mathematically incorrect information in return. One of the deductions that students could make out of the teacher's explanation is that an algebraic expression defines an operation. However, this analogy is not mathematically correct. Later in the dialogue, the students say that they could replace the apples with the letters x and y, and add that the apples represented the unknown. Approving such statements, the teacher causes the students to acquire false information because one of the basic features of algebra is that numbers are represented by symbols, and the symbols that represent the numbers are variable, although these symbols do not have to be letters always. In this case, however, the students think that symbols are constituted by letters. Since the use of letters in algebra is not the same as the use of regular letters, the assumption that symbols consist only of letters may lead students to think that letters are abbreviations of objects or that letters do not represent a numerical value. For this reason, it is necessary to use different symbols other than letters. To illustrate, the teacher's constant use of the letter x attracted the attention of a student, and the student asked, "Teacher, why is it always x?" As a response, the teacher stated that "Because x is the most frequently used one, different things can also be used", and then she wrote 20a + 6x – 12a as an example on the board. This explanation is partially correct, but it is not sufficient to make sense of the concept of a variable fully, and besides it is an approach that may cause students to hold the belief that symbols consist only of letters. Validating this, the teacher constantly exploited similar examples and did not use such symbols as the box or a triangle in structuring the concept of a variable with the student's prior knowledge.

Another situation that was observed in relation to the false knowledge that was developed by the students is that the students said that the letters represented the unknown. The teacher did not pay attention to this statement which may lead the students to fail to distinguish between the concepts of unknown and variable and to think that a specific symbol represents a single value.

Supporting this, it was observed that the students defined letters as unknown and variables. The teacher did not notice the students' confusion in this process. In line with this, the teacher's expression of 3 apples + 2 apples as 3x +2x= 5x is an explanation that may cause the students to perceive the variables as object labels. So, the teacher's misdirection of the students reinforces some existing misconceptions about algebraic concepts while creating new misconceptions.

Another misconception of the students is to assume that the +, -, and = symbols produce results In one example, it was observed that the students focused on the operational process and tried to find the result of the algebraic expression due to the teacher's previous remarks as demonstrated in the following dialogue.

The teacher asked the students to define the example of 5 apples + 8 pears with algebraic expressions.

> Dany: Teacher, is the result 13x? Or, is it 13y?
>
> Teacher: But, that is a pear (pointing to 8y). Why did you change it to an apple? Well, I tell you there are 5 apples and 8 pears. Tell me about their varieties. How many fruits do I have altogether? [Not being able to suggest alternative solutions]
>
> Dany: 13.
>
> Teacher: Can you change Xs to Ys? You cannot. You cannot transform an apple into a pear. However, if you say let us add, and I ask how many variables there are in total, you may say 13 variables. It is similar to fruit, isn't it? Are x and y types of variables? Are apples and pears different types of fruit? Then we can say that there are 13 fruits in total or 13 variables. Yet, if we want to specify them separately, we cannot perform any other operation. Yet, how can we do it only? We can attribute values. Okay? [Mathematically inaccurate association and explanation, explanations that do not help the students to understand, and removing this word are repetitive]
>
> Lewis: Teacher, is the result 13xy? [Continuing the misconception that the +, -, and = always produce results]
>
> Teacher: That is the result (pointing 5x + 8y). I have just explained Lewis, are you listening? (Judgmental attitude)
>
> Lewis: Yes, you have explained, but I tell you the result.
>
> Teacher: I am not asking you how many variables there are in total. Why are you changing x to y, do not you see that they are different? Do you understand me? [Not paying attention to students' thinking, not trying to understand, and an erroneous mathematical explanation]
>
> Students: Yes.
>
> Teacher: Lewis, let us remember. We learned how to read these algebraic expressions, remember. Let Lewis's age be x and Lucy's be y. 5 times Lewis's age, 8 times Lucy's age. If we knew their ages, we would not say x and y. We cannot add unknowns. Roy? [Focusing on procedural knowledge]
>
> Roy: It is complicated.
>
> Teacher: We learn step by step. Ask what you do not understand. [Unable to offer alternative solutions]

The teacher employed the fruit-salad approach. Apart from that, it is clear that the students did not understand = symbol as an equivalence, but as a transaction symbol in the examples. The tendency of the students to always equate the expression 5x + 8y to a result shows that they were inclined to think that the +, -, and = symbols always produced results. Therefore, the teacher repeatedly explained to the students that the expression was not equal to 13x, 13y, or 13xy. In addition to this, the students' insistence on giving similar answers and the teacher's constant remarks as "I have just explained Lewis, are you listening?", and "Do you understand me?" demonstrate that the teacher had difficulty in explaining the situation and was not satisfied with the situation that she was involved in. The thoughts that were held by the students can be interpreted as resulting both from the teacher's failure to teach the conceptual meaning of the = and from her constant employment of similar examples that focus on addition or subtraction. The students continued to think that the = produced results, and they had difficulties at this point throughout the lesson. As represented in the following dialogue, the students thought that the expression 5x + 5y was equal to 10xy.

Teacher: Can you underline the like terms?
Laura: $5x + 5y + 2x = 5x + 5y$ (the student writes this expression, and she is silent)
Teacher: Laura, remember the coefficients, the ones in front of the variables are the coefficients. For example, $5x + 5y$ is not available in the options. If you need reordering, how do you do it? Daniel? (Not giving the student an opportunity, the teacher focuses on the operation process)
Daniel: It is $5x + 5y = 10xy$. [Believing = produces result = misconception]
Teacher: Yes, good luck with it (ironically). Are these the like terms, İbrahim? If they are equal, will you calculate the values of the algebraic expressions for $x = 1$ and $y = 2$? [Lack of adequate conceptual explanation]
Daniel: It is $5+ 10 = 15$.
Teacher: Since it says equal, both sides must be the same, right? Write the other one?
Daniel: It is 20.
Teacher: Look, it is $15 = 20$. So, is it possible? İbrahim, now clean it and do it again. Did you understand why it was wrong? Lando? [Drawing attention to the operational aspect of algebraic expressions]
Lando: It is equal to $x (3+2) + 5y$.
Teacher: It is not wrong, but it becomes complicated. Is there any comment? Come here, Jordan. [Ignoring students' thinking without deepening it]
Daniel: It is equal to $5(x+ y)$.
Teacher: Look, is 5 the common term? Can we take 5 out of them both and put $x + y$ in parenthesis? Let's not be surprised then. [Drawing attention to the operational aspect Judgmental attitude]

The dialogue illustrates that the teacher focused on how to write the expression "5x + 5y" rather than what it meant, but the students approached the algebraic expression as the addition of natural numbers, so tended to apply the idea of adding natural numbers, which they brought from their previous learning about arithmetic. Because of the teacher's approach to the student's response, the teacher clearly did not question it is seen that the teacher did not question why the student's thought process and, as a result, could not analyze the student's thought process. Nonetheless, the teacher wanted to show that this statement was not true and asked the student to assign values to the variables x and y and make calculations this time. Although it is possible to think that the teacher tended to eliminate false knowledge by giving different values as $x = 1$ and $y = 2$ and showing that the equality of $5 + 10 = 15$ was not achieved, she actually ignored the case of assigning the value 1 to both variables. Therefore, it is seen that the teacher, on the one hand, did not realize the students' conceptual confusion and did not try to understand the difficulties that were being experienced by the students, and, on the other hand, the solution offered by the teacher did not eliminate the basic problem that was experienced by the student. Also, emphasis on like terms likely caused the students to think that they should focus on like terms first and perform necessary operations and leave, unlike terms until the end. Thus, by removing this phrase the teacher could not define what was mathematically significant in the algebraic expressions. In addition, as this example reveals, the teacher did not give the student the opportunity to explain her or her own opinion once she or he suggested an incorrect or incomplete solution but rather gave the right to speak to another student until the correct answer was found. In this process, it is a striking finding that the teacher exhibited a judgmental attitude, saying "Yes, good luck with it" (ironically) and "Let's not be surprised then" in response to the students' missing or wrong solutions removing this phrase. This situation can also be shown as an illustration of the teacher's instructional approach.

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3.5. Findings on the Teacher's Reflection

The codes regarding the evaluation of the teaching and the teacher's explanations in the post-lesson interview are represented in Table 5.

Table 5

Findings on the teacher's reflection

Codes	*Teacher's explanation*
Self-centered approach	Recognizing the moments when the student did not understand but defining what she did rather than the student's thoughts Holding the student responsible for their learning failures, such as she or she did not revise or she or he does not listen to me.
Imprecise definitions of students' outcomes.	They learned how to do simple operations like addition and subtraction in algebraic expressions at the end of this course. They learned to do operations between like terms. They learned to add and subtract by making associations with integers.
Considering students' thinking carefully during instruction	In general, my plan did not change, but when I realized that the students did not understand, I made new additions (calculating the result by giving value, explaining the distributive property, etc.).
Identifying the observations on students' learning	Some students tried to find results. They did not understand that the exponents of the variables were different. The students expected terms to be next to numbers. The students had difficulties when faced with a new type of question. They had difficulty with performing operations that involved more than one operation and like terms.
Trying to understand what the students think	Giving examples from specific situations, and making inferences about how the students think.
Assuming that she attained the objectives	To a large extent, the students achieved the learning outcomes that I envisioned. I tried to ask questions to all students, but there were some students who, I think, did not understand totally because there was a problem with the background knowledge of some of the students.

The course evaluation revealed that the teacher held the opinion that the students had achieved the desired objectives. However, the teacher was not able to define which mathematical ideas in particular, the students developed, and she made the superficial assessment as "the students learned addition and subtraction". Along with this, the teacher claimed that a few students could not attain the outcomes. She expressed the reasons behind the inability of these students as the lack of background information and the students not doing enough revising, and she added that the students were responsible for this situation. To illustrate, a student did not know what the coefficient and variable were, and the teacher expressed the reason why she or he had difficulty as follows:

> In the subtraction example, like terms were a bit more complex, for there was more than one similar term. When subtracting them, she or he did not know what to subtract from what. She/He could not show us that she or he understood similar terms very well. When I asked what the coefficient was, she or he could not respond, so she or he made a mistake.

Two points are noteworthy in the statement that the teacher made above. First, the teacher assessed the students' learning outcomes according to whether the student could solve the question correctly so the student either knew or did not know. Since the teacher assessed the student's learning with such a method, the teacher acted with the idea that. Secondly, the teacher stated that the students did not know the concept of variables. However, the teacher did not make sufficient conceptual explanations about the variable in the lesson, and at the same time, she used the concepts of unknown and variable interchangeably, which made the situation even more complicated for the students. Nevertheless, the teacher put all the responsibility on the students, which indicates that the teacher only focused on her own teaching plan and did not pay attention to how the students thought.

One of the points that the teacher reflected on in response to her own teaching is that she did not make significant changes to her teaching plan, but she made new decisions according to the students. One of these decisions related to the student's equation of the expression 5x + 5y as 10xy. By giving numerical values to the letters and thus making the students see that both sides of equality were not the same, she wanted to show that these terms could not be added. In this example, the teacher considered the difficulty that was experienced by the student but did not question why the student thought so and did not try to understand her or her ideas. In the post-lesson interview, she realized that the student was trying to find a result there and that this idea came from the four operations with natural numbers as her previous experience supported this view. Therefore, the teacher noticed the difficulties and struggles with learning that were experienced by the students during the teaching and took them into account from time to time, but she took her own previous experiences as a reference point in her actions the point of eliminating these problems and not the student's thoughts.

As a result, the teacher placed too much emphasis on her teaching while reflecting on the lesson and defined her instructional actions rather than focusing on how students think. The teacher thought that she paid attention to the students' thinking in the lesson and guided the students by questioning them enough, but her approach to teaching did not support these goals. The teacher suppressed the voices of the students. When she asked questions of the students, she either explained or tried assertively to get the answer that she wanted from the students without giving the students the opportunity to speak most of the time. Therefore, the teacher's decisions and instructional actions were possibly only shaped by her teaching plan.

4. Discussion

This present study revealed how students' learning or non-learning situations were affected by teachers' instructional decisions and actions. To this end, teaching (40 + 40 min.) was analyzed, and pre and post-lesson interviews were conducted with the teacher. In light of the findings, Lucy presented actions that caused incorrect learning and misconceptions. This result cannot be explained by a single factor in relation to the teacher. The instructional decisions (decision-making processes) in the teaching and the teacher's content knowledge also played decisive roles in the formation of this result. Therefore, the findings were discussed within the framework of instructional decisions and the teacher's content knowledge.

4.1. Instructional Decisions

The teachers' decisions affect the course of the teaching and the learning situations of the students (Jacobs et al., 2010). In spite of the fact that teachers often make a course outline before the lesson, they usually make instant decisions during their teaching (Sherin et al., 2011). This study demonstrates that a teacher can take her plan as a reference point while making instructional decisions and may leave the students' thoughts in the background. The teacher could not specify the mathematical ideas that she wanted to teach the students in her planning, and her goal was to explain operations, including how to add and subtract in algebraic expressions, to the student. The focal point of the teacher's instructional decisions was to implement her plan and to transfer procedural knowledge to the students according to this plan. The teacher paid attention to the difficulties that the students experienced and the mistakes that they made during the process, but she did not make an effort to question the source of these difficulties and mistakes and to understand what the students were thinking. Therefore, she could not build her instructional decisions upon the students' thinking, and she thought that these mistakes could be corrected with repetitive explanations and similar, sample solutions. However, these repetitions did not help the students to understand, and both the teacher's instructions and the students' learning were negatively affected as a result of this process. This finding reveals once again that teachers should build their instructional decisions on students' thinking. Similarly, Hawkins (2016) claimed that teachers had a "manager perspective" in such a teaching approach, while Liu (2014) stated that it was caused by the teacher's having a teacher-centered approach. Liu (2014) added further that a teacher who has such an approach that focuses only on her or her own agenda in the lesson does not pay enough attention to students' thoughts and possible misconceptions. In light of the findings of the study, Liu (2014) stressed not only how and what teachers should pay attention to but also what their instructional decisions were in the teaching was affected by teachers' personal beliefs and knowledge.

> If the teacher believes in helping students see mathematics connections through capitalizing on what students bring with them to the learning situation, …[if this belief is] Supported by the teacher's pedagogical content knowledge about the mathematics topics, the teacher attends to mathematically significant details in student thinking that could inform instruction interprets student understandings using these details, decides how to respond based on … [students' mathematical development]. (Liu, 2014, p.121)

Otherwise, Lui stated that students' thoughts, strategies, and misconceptions would not be among the sources of the teacher's instructional decisions.

Another significant finding of the study is that the teacher had the opinion that she paid attention to the students' thoughts and questioned them in her lesson. However, the data obtained reveal that there was a dilemma between the teacher's thoughts and her actions during the teaching. In their study, Lee and Francis (2018) pointed out that there was a great difference between the statements of teachers and their actions in practice, and they related the discrepancy to teachers' knowledge, orientations, beliefs, and perceptions of students' thinking. Among these factors, they emphasized that the teacher's perceptions of students' thinking affected the detail to which the teacher paid attention to students' thinking and her or her approach to interpreting these details, which affects the overall decision-making process. If we look at Lucy's approach to students' thinking in terms of her teaching, the teacher attached more importance to the wrong answers and the misconceptions that were developed by the students but did not try to understand these details and did not question the correct answers. For effective learning, correct answers should be questioned as well as wrong erroneous, or missing answers (Caram & Davis, 2005). Therefore, the action of the teacher here brings the following idea to mind: the teacher wants to see concrete results that show that the knowledge that she wants to impart to the students has been learned, and she readily pays attention to the students who do not demonstrate these behaviors. So, to correct the students' mistakes, the teacher directed questions to the students and enabled them to reach the correct result. The teacher interpreted these actions as though she paid enough attention to the student's thinking and made new decisions according to them. However, she did not focus on and try to understand how the students developed erroneous thoughts while taking those actions. This finding can be interpreted as the teacher needs some support in terms of what it means to benefit from the student's thinking and how to utilize the student's thinking in her teaching. In order to utilize students' thinking, it is first necessary to understand a student's thoughts and identify the source of their problems. Along the same line of thinking, Lee and Francis (2018) stated that for a teaching environment in which a student's ideas are supported, teachers should first of all have accepted students' ideas as valuable learning resources. Such an acknowledgment can be seen as a threshold or stage that can help teachers shape their instructional decisions according to their students' thinking.

4.2. Teacher's Content Knowledge

The findings show that the teaching caused the students to learn the place of letters in algebra, have such faulty knowledge as Letters refer to objects, +, -, and = symbols in algebraic expressions always produce results, and caused them to be unable to distinguish between the concepts of unknown and variable. Besides, it has been observed that the teacher's approach to the mistakes tended to show how the result was achieved with repetitive explanations rather than trying to understand how the students thought, and she was not aware of the students' misconceptions. The explanations made by the teacher in such cases were rule- and operation-oriented. Considering all these, the teacher's content knowledge, the teacher's pedagogical content knowledge, and the students' thought processes were not sufficient. While Schoenfeld (2010) stated that teachers' knowledge affected teachers' decision-making and actions in teaching, Mason (2011) emphasized that teachers must have had strong pedagogical and content knowledge to make instructional decisions based on students' thinking and reasoning. Considering teacher Lucy's approach in the process, it has been observed that she could not define in detail the mathematical ideas that the students should have attained while planning the lesson. In the teaching, it has been found that the teacher was not able to interpret the students' mistakes and confusions and so she was not able to grasp the reasons why they produced such ideas. Since some misconceptions about algebra may stem from the nature of the subject and students' previous experiences (believing that the symbols "+", "-", "and "=" always produce results), it is a natural process for the students to develop such misconceptions. Therefore, teachers should be aware that these possible misconceptions are not caused by students themselves and they should take appropriate instructional decisions depending on the situation. However, it has been observed that teacher Lucy constantly repeated similar explanations, thinking that too much repetition would correct mistakes, and in the case where students repeated their mistakes, she held them responsible and had a blaming attitude. Problematizing such an approach, Confrey (1993) drew attention to the need for teachers to be stripped away from their points of view and to take students into account in the teaching. That is, teachers need to be aware that what is easy and simple for them will not easily apply to students. Regarding this situation, Cofrey (1993) stated that the content knowledge that teachers had learned in traditional ways was not sufficient to see and interpret the multifaceted thoughts of students. For this reason, previous research has underscored that teachers' content knowledge is important but not sufficient (Barton & Shreyn, 2009; Goos, 2013), that is, teachers should also have a good grasp of students' prior knowledge, how they think about the subject, possible difficulties or misconceptions they may experience (McGowen & Tall, 2010; Stein et al., 2008). Being familiar with the ways of student thinking can help teachers look at the thoughts coming from the students from their points of view and better understand them in the teaching. Thus, teachers can make their instructional decisions following students' mathematical thinking. Converging with some previous research, this study claims that teachers' student thinking knowledge is as important as her/his content knowledge and teaching knowledge, for it shapes her/his instructional decisions and so it has a significant effect on students' achievement (Meschede et al., 2017).

5. Implications and Suggestions for Further Research

In summary, this study has revealed how teachers' instructional decisions and actions shape students' learning and lack of understanding. As a result, a significant dilemma has emerged between the teacher's perception of their teaching and their actual actions in the classroom. While the teacher believes they have planned and implemented student-centered instruction, it becomes apparent that student thinking has been sidelined in practice. This result contributes to the literature in several significant ways by revealing the inconsistency between teachers' perceptions of their instructional practices and their actual actions in teaching. Firstly, it emphasizes the need to assess whether teachers' instructional actions are genuinely based on student thinking, as their perceptions may be misleading. This finding highlights the necessity of supporting teachers in objectively evaluating their instructional strategies. Moreover, it underscores the importance of raising deeper awareness of how much teachers consider student thinking in their pedagogical decision-making processes. While the literature acknowledges that instruction based on student thinking is critical for effective learning, this study offers a new perspective on how the gap between teachers' perceptions and actions can impact the learning process. Consequently, it suggests the need for teacher education and professional development programs to focus on strategies that encourage teachers to critically reflect on their pedagogical approaches and more effectively adopt a student-centered perspective. This need can be presented as a recommendation for future work.

This research highlights that teachers not only need content knowledge but also awareness of students' thinking patterns, conceptual difficulties, and potential misconceptions, which are critical in the teaching process. It was observed that the inability of teachers to detect students' misconceptions about algebraic concepts and the use of repetitive strategies to correct these misconceptions negatively impacted students' understanding. Therefore, this finding emphasizes the need to develop pedagogical content knowledge related to students' thought processes to understand their mistakes and turn these errors into opportunities for learning during instruction. In this regard, future research could explore how teachers can better understand students' thinking and how pedagogical content knowledge regarding student thought processes can be effectively utilized in teaching practices.

Acknowledgements: This study was presented as a presentation at the 4th International Congress of Turkish Computer and Mathematics Education, Turkey: Izmir.

Author contributions: All authors were involved in concept, design, collection of data, interpretation, writing, and critically revising the article. All authors approve final version of the article.

Declaration of interest: The authors declare that no competing interests exist.

Data availability: Data generated or analyzed during this study are available from the authors on request.

Ethical declaration: Authors declared that the study was approved by the Ministry of Education on 12 March 2018 with approval number: 99332087/605.01/5137432.

Funding: The authors stated that they received no financial support for their study.

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