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Research Article

A preservice middle school mathematics teacher's knowledge of student thinking about line graphs

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Interpreting statistical graphs and making inferences based on the graphs are a precursor for formal statistical inferences. To support student inferences, both teachers and future teachers should have adequate knowledge regarding students' thinking on graphs as well as their potential misinterpretations and difficulties in interpreting graphs. In this qualitative case study, I examined a preservice middle school mathematics teacher's knowledge of student thinking about interpretation of line graphs. During her practicum course, preservice teacher planned a lesson to teach *creating and interpreting line graphs*. I first interviewed with her on the lesson plan and then observed her teaching the subject. The data indicated that the preservice mathematics teacher's actions regarding knowledge of student thinking before and during the lesson supported the students in interpreting graphs beyond reading. Preservice and in-service teachers with more knowledge of student thinking would provide teaching practices to support students' learning of statistical graphs. I thus suggest that a curriculum may be designed for improving preservice mathematics teachers' knowledge of student thinking about statistical concepts and informal statistical inferences.

Keywords: Graphical interpretation; Knowledge of student thinking; Line graphs; Teacher education

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1. Introduction

Schools are increasingly asked to prepare students to be flexible thinkers, lifelong learners, and to manage complexities of an uncertain world (Makar & Rubin, 2009). Mathematics lessons including the skills such as reasoning, problem solving, communication have important roles in fulfilling community expectations from schools. In particular, statistics learning area differently from numbers, algebra and geometry in the school mathematics is an important field supporting students' skills related to real life because there are many events requiring statistical knowledge in the real-world contexts. The National Council of Teachers of Mathematics [NCTM] (1989) emphasizes that statistical knowledge is necessary for students to be able to make critical and informed decisions and become intelligent consumers. Being able to provide reliable and persuasive evidence-based arguments and critically evaluate data-based inferences are crucial skills in the everyday lives of citizens worldwide through statistical skills (Ben-Zvi & Makar, 2016).

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For example, when people watch TV or read a newspaper including statistical information related to political decisions or discount rates, participate in community activities, attend a civic or political event, read workplace materials or listen to reports at work (Gal, 2004; Guler et al., 2016), they need to understand and interpret many dirty data to be able to make decisions about their lives. These skills point up the importance of having statistical literacy. Statistical literacy is the ability to understand and critically evaluate statistical information and results experienced in daily life and this ability affects people's decisions and arguments about important events based on their statistical knowledge (Gal, 2002; Wallman, 1993).

These explanations point out that statistical literacy is an important skill for people to bridge between their statistical knowledge and their daily lives. People especially encounter statistical graphs in their real lives and interpret these graphs in terms of their trends. The increasing use of visual presentation of data in the media and everyday life is based on the assumption that graphs and diagrams are transparent to the viewer, meaning that the reader will gain an immediate understanding of the visual message (Chick & Pierce, 2013; Glazer, 2011). Graphs often provide people with important information shaping their choices and help them improve understanding of the risks and benefits associated with many areas (Galesic & Retamero-Garcia, 2011). So, reading, interpreting and analyzing graphs allow people to have a global view of many phenomena and the direct visualization of numerical data in problems (Bruno & Espinel, 2009). In other words, graph interpretation is an information processing skill essential for employment in the twenty-first century and thus, it is important to develop students' graphic interpretation skills (Patahuddin & Lowrie, 2019). Especially, in middle schools, interpreting statistical graphs and making inferences based on these graphs are a precursor for formal statistical inferences. Because students first encounter statistical graphs at the middle grade in their school lives, students' understanding about the graphs would affect their inferences in the future. Thus, it is a requirement in middle schools that representing the data with the graphs such as bar plot, line plot, dot plot, and making inferences from the graphs by interpreting them (Common Core State Standards, 2010). Among these plots, line plots are representations of phenomena changing over time and are frequently used in presenting real-life data. These graphs show the relationship between two or more variables, each represented on an axis.

In order that middle school students interpret the line graphs appropriately to the data which the graphs represent, teachers should give importance in the process of interpreting the graphs and orchestrate classroom discussions about this process. Beside that improving the instructions to assist students resolve their misconceptions about graph interpretation is critical, teachers themselves need a robust knowledge of graph comprehension to address this instructional implication (Patahuddin & Lowrie, 2019). Knowing line graphs and their characteristics, interpreting them by referring to different statistical concepts and relating different graphs are a precursor for designing effective learning processes. However, only having content knowledge is not satisfying. Teachers need to consider students' possible thinking about graphs and real-life context, their potential misinterpretations and difficulties in interpreting graphs, etc. For example, if a middle school teacher can anticipate which contexts are motivating for students while selecting examples related to a graph, the context of the graph will be meaningful for students and they can reason about it and engage in classroom discussion about the graph. Similarly, during the teaching process, if teachers overcome the unanticipated students' responses or interpretations, students can improve their understanding of the graphs. Thus, teachers' effective approaches before and during teaching are important factors in supporting students to interpret statistical graphs effectively and to develop their statistical literacy for their lives (e.g. Çakıroğlu & Güler, 2021). Teachers' knowledge related to these practices is explained by the knowledge of content and students which combines knowing about students and knowing about mathematics (Ball et al., 2008). In this study, I specifically focus on a preservice middle school teacher's *knowledge of student thinking* (KoST) embedded in the *knowledge of content and student* (KCS) in the framework of mathematical knowledge for teaching. Accordingly, this study aims to investigate a preservice middle school mathematics teacher's KoST about interpretation of line graphs. In the regard, I seek a response to the following research question: What is the evidence of a preservice mathematics teacher's knowledge of student thinking in teaching of the line graphs?

1.1. The Need for the Study

Teacher education is an essential element for quality teaching in any subject, including statistics and is often seen as the major key to improve education (da Ponte, 2011). It is important that teacher education curriculums focus on and develop preservice teachers' knowledge, beliefs and practices because they are able to conduct effective mathematics teaching in the schools after graduation (Hiebert et al., 2019). However, mathematics teacher education programs are critiqued in terms of many components such as their perspectives, focus, and their teaching methods. It is still continuing to have limitations in supporting preservice teachers to improve their content and pedagogical content knowledge. Especially, in the report of The Mathematical Education of Teachers II (CBMS, 2012), it is emphasized that improving middle school pre-teachers has a critical importance:

> Because middle grades teachers receive their students from elementary school and prepare them for high school, college courses and professional development opportunities for middle grades teachers should also attend to how the mathematical ideas of the middle grades connect with ideas and topics of elementary school and high school (p. 40).

When we considered that interpreting line graphs is of significance and that many countries PMTs have generally had procedural understanding of statistical concepts consisting of having a collection of isolated rules rather than an appropriate conceptual schema (Leavy, 2010), teacher education programs should support PMTs in having necessary knowledge and skills for teaching statistical graphs.

Results from the studies on statistical graphs will give important insights to teacher educators. These studies have focused on PMTs' interpretations and knowledge of the statistical graphs (Bruno & Espinel, 2009; Monteiro & Ainley, 2004), students' reasoning and interpretations about the different graphs (Catman Aksoy & Isiksal Bostan, 2020; Chaphalkar & Wu, 2020; delMas et al., 2007; Watson & Moritz, 1998; Whitaker & Jacobbe, 2017); adults and teachers interpretations of graphs (Chick & Pierce, 2011). However, there is still a gap in the literature in terms of PMTs' KoST related to the statistical graphs. In this context, this study would support the literature in filling this gap by giving insights to teaching method courses to improve students' statistical literacy by line graphs. Additionally, case studies in teacher education programs point to the need to look beyond the surface structural features of teacher education programs in order to understand the key elements of program effectiveness (Zeichner & Conklin, 2008). With this regard, this study aims to investigate a preservice middle school mathematics teacher's KoST about interpretation of line graphs.

2. Theoretical Framework

Throughout the past three decades, researchers within the field of mathematics teacher education have been expanding the models and categories of teacher knowledge (Silverman & Thompson, 2008; Tchoshanov, 2010). Shulman (1986) coined the term pedagogical content knowledge (PCK) combining general teaching knowledge and specific content knowledge and referring to transferring the content to others. Ball and colleagues develop a practice-based model "mathematical knowledge for teaching (MKT)" with the aim of determining what else teachers need to know about mathematics and how and where teachers might use such mathematical knowledge in practice (Ball et al., 2008). In this model, Hill et al. (2008) have defined KCS "as content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (p. 375) and emphasized that this knowledge is a primary element in Shulman' s (1986) pedagogical content knowledge. Considering the definition of KCS, knowing students' thinking about a specific concept, KoST, comes into prominence for an effective teaching process.

Takker and Subramaniam (2012) have explained that KoST includes knowing students' understanding, conceptual difficulties and possible learning ways and developing awareness of students' thinking. Researchers (An et al., 2004; Brendefur et al., 2013; Lee, 2006; Ozaltun, 2014) elaborated the content of this knowledge. An et al. (2004) articulated KoST with four main components which are addressing students' misconceptions, building on students' mathematical ideas, engaging students in mathematical learning, and promoting students' mathematical thinking. Based on An et al.' s (2004) explanations, Lee (2006) expanded KoST by defining the components of questioning triggering divergent ideas, motivating students' learning, evaluating students' understanding and using prior knowledge to this knowledge. Brendefur et al. (2013) have explained that KoST allows teachers to predict possible student solution strategies, anticipate likely misconceptions, and to interpret students' ideas. Ozaltun (2014) presented a framework including the teacher actions related to KoST by considering the existing literature on KoST and by analyzing high school teachers' teaching for different mathematical concepts. There are mainly nine components related to KoST (see Table 1). Building on students' mathematical ideas includes knowing students' prior knowledge, interests and understanding. Teachers need to consider students' all ideas and use representations, the related rules and procedures while supporting students to develop important understanding and concepts. Promoting students' thinking mathematically includes planning and using several in-class actions such as questioning, using learning tasks, different representations and relating real life examples. Triggering and considering divergent thoughts includes knowing different solution ways and contradictory examples and, encouraging students to listen and question the others or teachers. Engaging students in mathematical learning included designing tasks, using different representations, connecting students' prior knowledge, giving examples of mathematical ideas, providing students to understand their difficulties. Evaluating students' understanding is related to teachers' evaluating students' understanding of instructions, their learning of a topic, concept or procedure and their performance in the classroom. Motivating students' learning includes the actions of praising students, giving them motivational advice when they struggle or fail, relating mathematics to real life and using motivational contents such as historical development of concept and the importance and necessity of concept. Considering students' misconceptions, errors and difficulties includes knowing, estimating and noticing possible students' misconceptions, errors and difficulties both before and during the teaching process and having actions to overcome them. Estimating students' possible ideas and approaches is related to the actions considering students' thinking while planning lessons.

When examining teachers' actions related to their KoST, we can say that this knowledge is important for teachers to be able to use their content knowledge in improving students' understanding and directly affects students' learning. In this study, I examined a preservice mathematics teacher's (PMT) KoST in the teaching process of line graphs.

3. Method

I conducted this study by a qualitative case study. Creswell (2013) defined case study as a research design in which a researcher deeply analyzes cases such as a program, an event, an action, a process or individuals. According to Yin (2014), case study research is useful in responding to a "how" or "why" question. As this study sought how a preservice teacher's KoST came up while teaching line graphs, the case was the preservice mathematics teacher's teaching process. By this holistic single case approach, I aimed to provide a possible best descriptive picture of the teaching process related to line graphs and to present a detailed way to bring into a view about preservice mathematics teachers' KoST.

3.1. Participant

There were seven PMTs, who enrolled in the Teaching Practice Course in a college of education from Turkey. They completed the teaching method courses in their education program. In the context of the Teaching Practice Course, the PMTs required to teach in two-hour lessons in a week for twelve weeks at a middle school. In this study, I focused on one PMT, Sophie (pseudonym). Before I determined Sophie as a participant, I observed seven PMTs' teaching process at two times in terms of KoST with the aim of determining the participant tending to consider students' thinking more than the others. The purposeful sampling in this process would provide "selecting information-rich cases" (Patton, 2002) by examining KoST in terms of different components and determining the necessities regarding teaching the statistical graphs. Also, I informed the preservice student, and the mentor teacher and the students about the process of the research in terms of ethical consideration. The preservice teacher was willing to participate in the study.

3.2. Data Collection and Analysis

I mainly collected the data from Sophie's teaching of −*students create and interpret line graphs*− learning objective in Grade 7. This lesson was her last teaching in middle school in the context of Teaching Practice Course. Sophie first prepared a lesson plan by using a guideline developed by Smith et al. (2008). This lesson plan guideline encourages teachers to prepare lessons by considering students' thinking. After she planned the lesson, I did a semi-structured interview with her on the lesson plan and videotaped this interview. My focus in the interview was to understand how Sophie was approaching students' thinking before the teaching process. Then, she taught the topic in a two-hour lesson in which there were about forty students. I observed her teaching and videotaped it to examine it later in a detailed way. The data consisted of the lesson plan, the interview and the classroom observation. This triangulation process related to the data collection was important for the trustworthiness of the research design (Guest et al., 2014).

Before analyzing the data, I initially transcribed video recordings of whole classroom interaction and the interview. In the analysis process, I first examined the lesson plan and transcription of the interview. Since the interview elaborated the lesson plan, I focused on the interview and used the lesson plan as supporting data. I then analyzed the transcription of two-hour classroom teaching. While examining the documents, I determined the parts in which she referred to students' thinking. This stage helped me in the coding process by providing a general view about Sophie's actions. Then, I used open coding in order to identify her KoST while analyzing her talks and gestures. After this stage, I related the codes to the KoST framework (Table 1) and determined the frequency of evidence of the KoST components. I had a descriptive and precisive codebook presenting a detailed definition of each component. This codebook was important to enhance the credibility of the analyzing process (Guest et al., 2014). This analysis process was the first round.

By the second round after three months from the first round, I recoded the all documents based on the codebook. I also aimed to enhance trustworthiness and credibility for the data analysis by providing intercoder agreement. After I completed to recode the data, I compared the codes generated in two rounds. Since the codes were consistent with each other, I completed the analysis process. In both data collection and data analysis process, it was ensured confidentiality of all research subjects, including data, information about the preservice teaches and the students. Also, the data were carefully organized and stored to ensure that no unauthorized use is made of them.

Table 1
KoST Framework (Ozaltun, 2014)

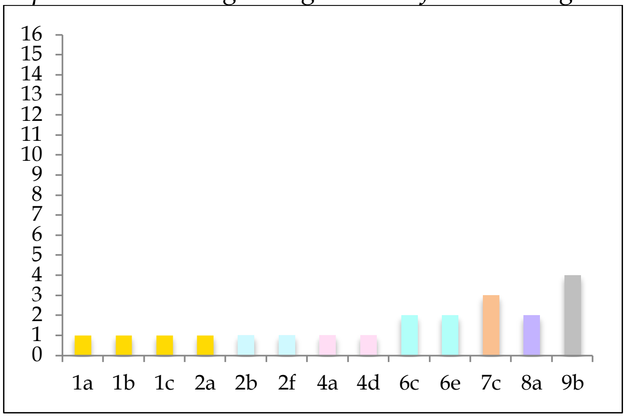
Components / *Sub-components*
1. Building on students' mathematical ideas
(1a) Knowing prior knowledge related to concept and connect them to new knowledge
(1b) Determining students' prior knowledge and consider their deficiencies
(1c) Using concepts or definitions to provide understanding
(1d) Focusing on rules and procedures to support/reinforce/improve the mathematical knowledge
(1e) Attracting students' interests to subject/concept
(1f) Using representations/analogies/concrete models defining concepts explicitly
2-Promoting students thinking mathematics
(2a) Asking questions and design tasks/examples for students to think
(2b) Having students estimate about questions/problems
(2c) Asking questions and design tasks to develop students' existing understanding
(2d) Asking students to product mathematical thoughts by representations such as figural/tabular/graphical
(2e) Providing students opportunities to think and respond questions
(2f) Relating examples/questions/problems to real life
3-Triggering and considering divergent thoughts
(3a) Asking questions to elicit students' ideas
(3b) Creating class discussion about a student's idea/solution/question or any thoughts
(3c) Asking students to produce thoughts or to explain about teacher's expressions
(3d) Asking students to explain/expand/interpret about ideas proposed by them
(3e) Asking students to express each other's explanations in different ways
(3f) Asking students to give contradictory examples
(3g) Encouraging students to produce different solutions
(3h) Explaining/expanding students' ideas
4-Engaging students in mathematical learning
(4a) Arranging activities to activate students
(4b) Using different representations of concepts
(4c) Giving example of mathematical ideas
(4d) Knowing prior knowledge related to concept and connect them to new knowledge
(4e) Allowing students to understand their difficulties/obstacles/failures while reflecting on instructions and strategies
5-Evaluating students' understanding
(5a) Evaluating how students understand the instructions, how they learn and how they perform during teaching
6-Motivating students learning
(6a) Praising students when they provide appropriate thoughts
(6b) Giving students motivational advice when they struggle or fail
(6c) Relating examples/questions/problems to real life
(6d) Giving the historical development of concept
(6e) Addressing the importance and necessity of concept
7-Considering students' misconceptions and errors
(7a) Knowing students' misconceptions and errors
(7b) Determining students' misconceptions and errors
(7c) Focusing on concepts/rules/procedures to prevent misconceptions and errors
(7d) Using different representations to prevent misconceptions and errors
(7e) Focusing on concepts/rules/procedures to remove misconceptions and errors
(7f) Using different representations to remove misconceptions and errors
(7g) Giving students clues to realize their misconceptions/errors
(7h) Ensuring students' understanding of the problems/question
8-Considering students' difficulties
(8a) Estimating students' difficulties
(8b) Simplifying/Explaining step by step what students have difficulties
(8c) Recognizing students' difficulties
(8d) Asking questions to determine the reasons of students' difficulties
(8e) Giving students clues to overcome difficulties
(8f) Focusing on concepts/rules/procedures to overcome difficulties
(8g) Using different representations to overcome difficulties
9-Estimating students' possible ideas and approaches
(9a) Estimating possible thoughts to be produced by students
(9b) Estimating students' solutions related to questions/problems

4. Results

In this section, I first present the evidence of Sophie's KoST from the planning stage, and then the evidence from the teaching stage.

In the planning stage, Sophie referred to the students' thinking in different aspects. Figure 1 shows her actions regarding the KoST and in this figure the horizontal axis shows codes and the vertical axis shows frequency. She related line charts to the students' knowledge about bar chart and focused on the ideas and procedures regarding the line chart to prevent students' possible misunderstanding and difficulties.

Figure 1
Sophie's actions regarding KoST before teaching



Sophie initially referred to the significance of learning the statistical graphs and stated, "This topic is very important because students encounter a lot of data in daily life, especially in the news. They need to notice misleading graphs to reach the correct information." By this explanation, she emphasized that the purpose of mathematics teachers should be to prepare their students for the real world and that students needed to learn statistical graphs as a part of interpretation of real-life data so that they would make inferences based on graphs appropriately.

While talking about the beginning of the lesson, she focused on the students' prior knowledge about representing data with graphs and planned to build the lesson in the students' understanding of bar charts. She had ideas about the students' prior knowledge and planned to use them for learning a new graph.

Sophie:	I plan to start the lesson with the bar charts which they knew and then to shift to the line charts. I will also present examples of bar charts leading the students to make incorrect interpretations.
Researcher:	Why do you plan to start with the bar charts?
Sophie:	They are more understandable for students because they taught them at Grade 6.
Researcher:	What is the context you planned?
Sophie:	It is about air temperature. I researched to provide an interesting context, but I couldn't find it. I thus decided to use this context. The data shows the air temperature values of the all week in which I will teach the concept. They may be interested in these values.

She assumed that the students would reflect their knowledge about the bar charts on the new kind of graph, line charts (1a). Since the students learned bar charts at grade 6, these graphs were meaningful for them. The concepts, which were familiar to the students, could lead them to engage in the lesson actively (4d). This approach might trigger the students to think about characteristics of the line chart and thus, they could make inferences based on the data when they could read the graph correctly. The data context included students' living district's air temperatures during the week in which the lesson would be conducted. She selected this context because she considered it to attract the students' interest. She gave importance to the context in which the students would be interested. Sophie planned to motivate students' learning by relating this topic to the students' daily lives (6c).

Sophie assumed that the students would not have difficulties about bar charts and could interpret these chars. She made the explanations about this: "This topic is, in my sight, easy, it may be difficult for students, but it may be easily comprehended based on its appearance." Additionally, she planned the lesson for the students to especially interpret different line charts representing the same data with the aim of preventing students' possible incorrect interpretations (7c).

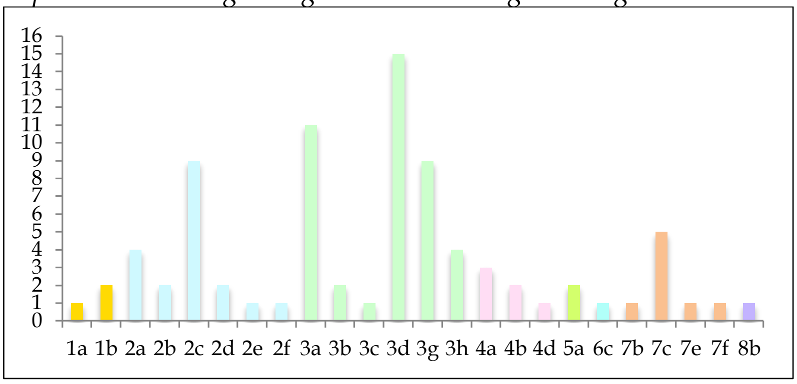
Sophie: The focus point.. The images of graphs are quite different, but the data are the same for each graph. If you want to manipulate, you manage data and graphs, as you want. My purpose is to provide students with awareness about these issues. They will read news in the future. To some extent, I want to support them to examine and question data or a graph.

Sophie estimated students' possible difficulties in their daily lives, and she considered these obstacles in the planning stage. She emphasized that the students should have statistical literacy and make inferences from graphs they would encounter in their real lives.

Sophie also conducted the lesson by considering the students' thinking (Figure 2) and did not hesitate in doing some changes in the lesson plan when she found them necessary. She was flexible for implementing the lesson plan and did not view it as a stable tool for teaching. Also, she asked the students to justify their ideas and responses during the lesson and to make inferences based on the data.

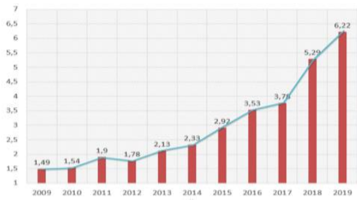
Figure 2

Sophie's actions regarding the KoST during teaching



She discussed how to draw a line chart with the students based on their ideas about bar charts, and then presented a graph representing the exchange rate of dollars depending on years in Turkey.

Sophie:



This graph represents the exchange rate of the dollar by 2009 and 2019. What do you think about the economics in this country?

Student 1: In a bad way!

Student 2: It changes by years.

Student 3: It doesn't make production.

Sophie: You say that it doesn't make production. It will lead to monetary depreciation.

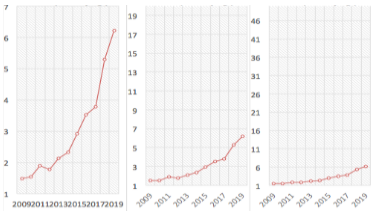
Student 4: It imports.

Student 5: Economic situation in the country is going from bad to worse.

She asked the students to make inferences about the economic situation in Turkey by considering this graph. The students did not focus on the data and interpreted the graph based on their informal experiences and the context. Sophie's question was good to prompt the students to make inferences and think mathematically (2a). However, she did not effectively manage the process. In this stage, she could not respond to the students' thinking and did not thus have effective actions related to the KoST. The students examined the graph descriptively and did not make inferences by thinking beyond the data.

After this graph, she indicated different graphs representing the same data and asked students to interpret the differences.

Sophie:



Examine these graphs. What properties of a graph affect its view?

Students: Starting point, endpoint and interval.

Sophie: When you encounter the graph on the far right, what can you say about the economy in the country?

Students: Not bad!

Sophie: Based on the graph on the far left?

Students: Too bad.

Sophie: What can you say about it?

Student 1: It affects our interpretations on the data.

Since Sophie paid attention to the students' statistical inferences they made in their lives, she had planned to discuss different graph views by considering their possible errors (7a). Discussing different representations and examples (3a) supported the students to have ideas about the characteristics of the line graphs and thus to notice possible misinterpretations (7c). In accordance with the lesson plan, she presented three graphs in different views but representing the same data. The purpose was to notice that the data values were important, and the view led people to make incorrect inferences and decisions. She actually provided the students with having strong meanings for the line graphs to make inferences. In this process, she did not ask them to interpret the graph beyond the data. Similarly, she discussed the rate of unemployment in Turkey with the students. They generally described the data and compared the different graphs.

After all, she asked the students to make inferences about the economic situation in Turkey in 2030.

Sophie: How do you estimate the economic situation in 2030 by considering these graphs representing rate of unemployment and rate exchange of dollars?

Student 1: Dollar will be equal to 10 Turkish Liras.

Sophie: Why? He says that a dollar will be equal to 10 Turkish Liras.

Student 2: 7

Sophie: Why do you think so?

Student 3: It can be more also.

Student 4: I think 20.

Student 5: I think that it is not 20 or 7, it will be the middle value between them.

Student 6: 13,5

Student 7: It can be 17.

Sophie: Explain your ideas. What properties of the graph lead yours to think so?

Student 8: It continuously increases; thus, I think it will continue to increase in the future.

Sophie: This means a trend in terms of the economy. What can you say about it in 2030?

Student 8: I think, 15.

The question she asked was evidence of her KoST. She aimed to encourage her students to think statistically (2a) and to develop their existing knowledge about the graphs (2c). In the process, she encouraged them to explain their ideas (3a), listened to her students' different solutions (3g) and asked them to justify responses (3d).

5. Discussion

In this study, the PMT's actions regarding the KoST before and during the lesson served to support the students' interpretations of the line graphs. Planning the lesson by estimating the students' thinking and considering their possible approaches was provoking for students' inferences based on the graphs in some respects. She planned the lesson for the students to engage in the lesson actively and relate the concepts to their prior knowledge. The context was important in improving students' inferences. If she did not consider her students' thinking about the context, she could have determined any context without real data and only exemplified the line chart. This action showed that she planned the lesson according to her students' ideas and knowledge. Selecting real data from daily life in the teaching process is important and students may improve statistical literacy skills when they examine these real contexts. Therefore, she tended to determine the context that would be effective for students' inferences and interpretations about the real-life situations. Paparistodemou and Meletiou-Mavrotheris (2008) explained that engaging context might influence students' inclination to look beyond the data. Therefore, the PMT's approach was consistent with the actions that would support the students in interpreting the graphs beyond reading them.

The PMT also considered the students' possible difficulties about the statistical graphs while planning the lesson. This approach made her ready for possible challenges in the classroom and helped her to make decisions quickly when she encountered the students' difficulties (Brendefur et al., 2013). Learning the line graphs and their characteristics is important for making inferences in real life because people generally encounter tables and graphs, especially line graphs, representing data. People can effectively interpret a line graph when they know the structure of this type of graph. If teachers plan the lessons by assuming students' possible difficulties and misunderstanding, students can reason about the graphical data and make inferences in their real lives. The PMT supported the students to evaluate the graphs based on the values of the data and emphasized that views of graphs could be misleading. She did not plan the lesson based on her own ideas and the mathematical concepts. If she did so, she would not have considered her students' difficulties and possible understandings and could not have determined the contexts appropriate to the students. This also could have led for the students to have procedural understanding of the graphs and not have engaged in making informal inferences. Lesson planning is a precursor stage for effective teaching practices. In the study, the PMT referred to students' thinking in the planning stage. The lesson plan guide (Smith et al., 2008) might support preservice and in-service teachers to consider students' possible ideas about the graphs. In the teacher education programs, discussing different lesson plan guides focusing on students' thinking is significant in order to improve PMTs' lesson plans and teaching. Because, focusing on students' possible mistakes, misconceptions, and difficulties, their reasoning about concepts, and different representations to be used for helping students to comprehend concepts is significant during planning stages (Ball et al., 2009). When we consider that these expectations are directly related to possessing the KoST, such guidelines may help educators to implement teacher education programs in a way to prepare PMTs to be qualified teachers.

However, in the study, the lesson did not continue in line with the lesson plan because sometimes unexpected moments for the PMT came up. The students focused on only context by ignoring the graphical data during the lesson. In this situation, it was important for the PMT to draw the students' attention to the graphical data for making inferences. But the students made inferences about the situation based on their ideas about the context because the PMT could not respond to the students' interpretations and did not have effective actions for the students to extend their interpretation beyond the data. Even if she aimed to prompt the students to think mathematically with her questions, she had to have them justify their explanations based on the data. Pfannkuch (2011) stated that some students seemed to be constantly making sense of the data by proposing possible stories rather than entering into an abstract world of images that represent underlying concepts such as sampling variability. For overcoming this problem and supporting students in focusing on the data and in considering variability, teachers practices related to KoST are a significant factor. They should orchestrate the discussions in line with students' thinking (An et al., 2004) As well as KoST, teachers should know how they conduct a teaching process. Chapman (2004) referred to practice knowledge by describing it as "experiential, procedural, situational, particularistic, and implicit." (p. 192) and emphasized teachers' positions in the classroom. In this study, since the PMT had no experience about teaching the statistical graphs and had lack practice knowledge, it might be possible that she could not drive this process to encourage the students to focus on the representativeness of the data. If the PMT had responded to the students' thinking, this process of making inferences would have been more effective. Another reason for this approach could be the PMT's lack of content knowledge about making inferences based on the data represented with the graphs and this insufficient knowledge might lead to not notice students' thinking. de Vetten et al. (2018) also concluded that many PMTs have a limited understanding of sampling methods, sample size, representativeness and sampling variability which were important statistical concepts in making inferences.

The PMT frequently had the actions of prompting the students to think mathematically and considering students' different thoughts. For example, when she asked, "What do you estimate by considering this graph? What properties of the graph lead you to think so?", the students focused on the data and had aggregate thinking on the data. Also, Sophie's questions and expectations of different ideas were effective for the students in using probabilistic language while interpreting the graph. It was seen that the transcription parts comprising these actions were related to the students' further interpreting the graphs. However, she addressed the students' possible difficulties based on her own experiences.

6. Conclusion

This case study exemplifies KoST and gives ideas to researchers for the future studies and to mathematics educators for revising their programs by presenting the episodes from a PMT's real-classroom practices. The PMT had actions reflecting her KoST both before and during the lesson. Having KoST and reflecting her knowledge on the teaching process supported her students to make inferences about the data. Thus, it is apparent that considering students' thinking is important to develop statistical thinking. On the one hand, this study showed that teachers' KoST is significant to support students in making inferences on statistical graphs and teachers having KoST can design effective learning tasks to develop students' statistical literacy, on the other hand, it indicated that it is still a necessity to support mathematics preservice teachers in order to improve their KoST.

In this study, some KoST components were more apparent during planning the lesson while some were observed during the lesson in prompting the students to interpret the line graphs. Besides, the actions in the lesson planning and during the lessons should be consistent and complementary. Thus, preservice and in-service teachers' KoST may convert to teaching practices to support students' learning of statistical graphs. In teacher education programs, reflections of KoST on PMTs' practices are also needed to discuss for developing their effective statistical teaching prompting inferences. Additionally, the opportunities related to examining the students' different thinking, difficulties and understandings from real classroom should be given to PMTs. Van Zoest and Stockero (2008) emphasize that using video-case curriculum in teacher education programs provide valuable opportunities for in-service and preservice teachers to access students' thinking and thus, their understanding of the mathematics needed for teaching may be improved.

The interactions between teacher and students in real classrooms such as the evidence from this study may also be used in mathematics teaching method courses in order to improve PMTs' KoST.

This study contributed to the literature by presenting a case indicating how students' statistical inferences and literacy can be enhanced while teaching line graphs. However, it included the data from one preservice teacher with the potential of having and using KoST. In the future studies, a different variety of preservice teachers can be selected and their KoST can be analyzed. I did not analyze the data with the lens of statistical inference. The relationship between preservice teachers' KoST and statistical inference can be revealed based on different data sets including quantitative and qualitative data. That is, the effect of the actions regarding the KoST on students' interpretation of the graphs can be examined in a more detailed way. Also, a curriculum can be designed for improving PMTs' KoST about the statistical concepts and informal statistical inferences. In addition to these, it will also be significant if the studies include in-service teachers.

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