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Review Article

A bibliometric analysis of Scopus-indexed research on Toulmin's argumentation model in mathematics education

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Toulmin's argumentation pattern (TAP) provides a conceptual framework for structuring and analysing the development of arguments and has been increasingly adopted in mathematics education research. This study conducts a bibliometric analysis to provide an overview of the development of TAP-related research in mathematics education and to identify prominent authors, publication venues, and thematic trends within the field. Bibliographic data were retrieved from the Scopus database using search terms related to *Toulmin*, *mathematics*, and *education*, covering publications up to the end of 2025. Following the screening process, 113 publications were included and analysed using the R environment in conjunction with VOSviewer. The findings indicate a marked increase in scholarly interest in TAP within mathematics education in recent years. Mejía-Ramos, J. P. and Conner, A. emerged as prominent contributors, while *Educational Studies in Mathematics* was among the publication venues with a substantial presence in the analysed dataset. Thematic analysis suggests that the scope of TAP-related research has gradually broadened from a primary focus on the structure of mathematical argumentation to a wider range of contexts, including collective argumentation, classroom argumentation practices, teacher education, and technology-supported learning environments. In particular, recent studies indicate growing interest in integrating TAP with interactive pedagogical approaches and digital technologies to support the construction and development of mathematical argumentation.

Keywords: Bibliometric analysis; Mathematical argumentation; Mathematical reasoning; Mathematics education; Proof; Toulmin's argumentation pattern

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1. Introduction

Argumentation has become increasingly prominent in mathematics education, reflecting broader curricular and assessment priorities that emphasize students' capacity to construct viable arguments, justify mathematical results, and critically evaluate reasoning rather than merely produce correct answers (Common Core State Standards Initiative, 2010; OECD, 2019). This orientation is also consistent with recent didactical perspectives that foreground principled reasoning and structured justification over the reproduction of procedures or memorized results (Kludze, 2026). Mathematical argumentation is closely intertwined with reasoning and proving, as

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learners formulate claims, develop justifications, examine the adequacy of supporting reasons, and progressively coordinate informal argumentation with more formal forms of proof (Urhan & Zengin, 2024; Zengin & Tapan Broutin, 2025). At the classroom level, this emphasis builds on a longstanding body of research showing how mathematical claims and justifications are collectively constructed and negotiated through classroom interaction (Forman et al., 1998; Krummheuer, 1995). More recent work similarly demonstrates that mathematical discourse develops through teacher–student and student–student interactions around the explanation and development of mathematical ideas (Çelik Demirci & Baki, 2023). Teachers play a particularly important role in creating and sustaining such environments by eliciting students' reasoning, orchestrating productive discussion, and supporting the development of collective arguments (Bağdat & Yanık, 2023; Conner et al., 2014). Realising the potential of argumentation, however, requires learners to coordinate claims with the evidence and reasoning used to justify them in coherent argumentative structures (Slavit et al., 2022). This requirement has sustained theoretical interest in frameworks that make the internal structure of mathematical arguments visible and analyzable, particularly in relation to reasoning and proof.

Among the frameworks developed to analyse the structure of such arguments, Toulmin's argumentation model [TAP] - introduced in *The Uses of Argument* (Toulmin, 2003), and also referred to as Toulmin's argument pattern - remains one of the most widely adopted in both science and mathematics education research (Lazarou & Erduran, 2021; Yang, 2022). The model decomposes an argument into six interrelated components. The *data* are the evidence, facts, or observations from which a claim is derived, answering the question of what the claim is based on; the *warrant* is the underlying principle or rule that licenses the inferential step from data to claim, functioning as the logical bridge between the two; and *backing* supplies further justification for the warrant when it is not, on its own, sufficiently persuasive. Two further components qualify the strength and scope of the claim: the *qualifier*, which indicates the degree of certainty attached to the claim (e.g., "certainly," "most likely," "possibly"), and the *rebuttal*, which acknowledges the conditions or counterarguments under which the claim might not hold; together, they enhance the rigour and precision of an argument by clarifying its scope and limitations. The *claim* itself is the conclusion the argument is intended to establish. By distinguishing the evidential, inferential, and conditional components of an argument, TAP offers researchers a common analytic vocabulary for describing how students move from mathematical evidence to justified conclusions, which has made it a durable reference point for studying mathematical argumentation across grade levels and instructional contexts (Inglis et al., 2007). The model has not gone without criticism, however: scholars have noted that, in its original monological form, TAP gives limited attention to argumentation as a social, dialogic process of persuasion between participants (Jiménez-Aleixandre & Erduran, 2007), and that the warrant component can be ambiguous when the underlying rule connecting data to claim is left implicit. These critiques have, in turn, motivated extensions of the model - for instance, to incorporate modal qualifiers as a substantive marker of argumentative sophistication (Inglis et al., 2007) and to embed TAP within broader sociocultural accounts of classroom argumentation (Krummheuer, 1995) - underscoring that TAP continues to function as an evolving reference framework rather than a fixed taxonomy.

The application of TAP in mathematics education has itself become the subject of synthesis. In a systematic review of 55 studies on mathematical argumentative writing in K-12 education, Lehmann and Friend (2025) found that more than half (56.52%) drew on TAP to analyse students' arguments, underscoring its centrality as an analytic reference in this literature; their review further pointed to systematic instruction and feedback as key levers for improving the quality of students' written arguments. Closer to the present focus, Badjeber et al. (2024) synthesised 19 Scopus-indexed studies (2020–2024) on the use of TAP in mathematics and science teaching, concluding that the model functions flexibly both as an analytic tool and as a design component of argumentation-based instruction, with teachers playing a decisive role in scaffolding students' arguments. Two existing bibliometric studies bear on this literature without directly addressing it:

Kartika et al. (2023) mapped 173 Scopus-indexed publications on argumentation in mathematics education broadly (1995–2021), identifying proof, reasoning, teacher education, and geometry as dominant themes and the United States as the leading contributor, while Ekawati et al. (2025) conducted a narrower bibliometric analysis of 34 studies on collective argumentation in mathematics education (2013–2024), again identifying Conner and the University of Georgia as central nodes in this sub-literature. Taken together, these four studies establish that TAP occupies a prominent and growing position in mathematics education research, yet none offers a bibliometric mapping of TAP specifically: the two systematic reviews (Badjeber et al., 2024; Lehmann & Friend, 2025) synthesise substantive findings rather than the field's structural and bibliometric features, while the two existing bibliometric studies (Ekawati et al., 2025; Kartika et al., 2023) examine argumentation in mathematics education at a broader or differently-scoped level- the former spanning all forms of mathematical argumentation, the latter restricted to collective argumentation- without isolating TAP as the unit of analysis. This distinction matters: systematic reviews answer what TAP-based research has found, while bibliometric analysis answers how the field producing those findings is organised and evolving (Donthu et al., 2021; Gutiérrez-Salcedo et al., 2018)- a question that, for TAP in mathematics education specifically, remains unanswered.

Bibliometric analysis addresses this gap by applying quantitative methods to publication and citation data in order to map a field's developmental trajectory, identify its most influential contributors and venues, and surface emerging thematic trends (Donthu et al., 2021; Gutiérrez-Salcedo et al., 2018). Beyond merely counting outputs, the approach can reveal latent knowledge structures- co-authorship and co-citation networks, thematic clusters, and research fronts- that are difficult to discern through narrative or systematic review alone (Chain et al., 2019), and it has recently been applied productively across several strands of mathematics education research, including studies of mathematics education broadly (Julius et al., 2021), innovations in science education (Kondrashev et al., 2024), and mathematical modelling instruction (Fajri et al., 2025). The present study adopts the four-step bibliometric procedure proposed by Donthu et al. (2021), detailed in the Methods section, as its methodological framework.

Despite this potential, bibliometric studies remain comparatively rare within mathematics education research as a whole (Drijvers et al., 2020; Julius et al., 2021), and a search of the Scopus, Web of Science, and Google Scholar databases conducted in preparation for this study did not return any prior bibliometric analysis focused specifically on TAP in mathematics education. The present study addresses this gap by mapping the structure and evolution of Scopus-indexed research on TAP in mathematics education through the end of 2025. In doing so, it aims to complement the existing systematic reviews (Badjeber et al., 2024; Lehmann & Friend, 2025) and the broader-scope bibliometric studies (Ekawati et al., 2025; Kartika et al., 2023) by offering researchers, journal editors, and doctoral students entering this specific area a structural map of the field-its growth trajectory, its leading authors, institutions, and outlets, and its evolving thematic agenda-that can inform literature selection, identify potential collaborators, and surface underexplored directions for future research. Specifically, the study addresses the following research questions:

RQ 1) What are the publication volume and temporal growth trends of research on TAP in mathematics education indexed in Scopus through the end of 2025?

RQ 2) Which researchers have been the most influential in research on TAP in mathematics education through the end of 2025?

RQ 3) Which publication sources have had the greatest impact on publications concerning TAP in mathematics education through the end of 2025?

RQ 4) What are the dominant and emerging thematic trends in research on TAP in mathematics education through the end of 2025?

2. Methods

The method used in this research is bibliometric analysis. In this study, we adopt the bibliometric analysis procedure proposed by Donthu et al. (2021), which consists of four main steps: (i) defining the aims and scope of the study, (ii) selecting appropriate bibliometric techniques, (iii) collecting and preprocessing the data, and (iv) conducting the analysis and reporting the findings.

2.1. Aims and Scope

This study aims to provide an overview of the development of research on TAP in mathematics education through the end of 2025 and to identify the most influential authors, countries, publication sources, and research themes in the field. The analysis was limited to English-language publications indexed in the Scopus database, published up to the end of 2025, and directly concerned with the application or investigation of TAP in mathematics teaching and learning.

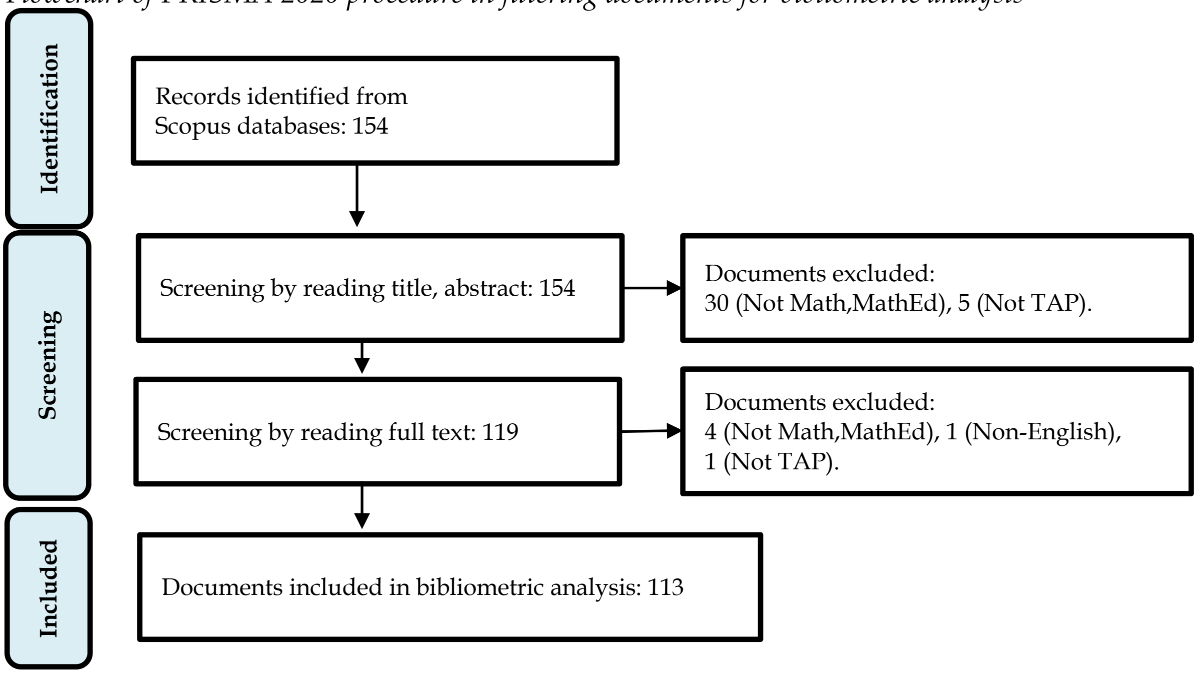
2.2. Bibliometric Techniques

The bibliometric analysis comprised two complementary components: performance analysis and science mapping. Performance analysis was used to assess publication output and citation impact at the author and source levels using the number of documents, total Scopus citations, citations per document, local citations, h-index, and the publication year of the first document in the dataset. For publication sources, SJR 2025 (SCImago Journal Rank for 2025) was additionally included as a journal-level indicator to provide contextual information on their broader scholarly standing. Science mapping was employed to examine the conceptual structure and thematic development of the field through co-word analysis, using standardised Author Keywords as the primary data field.

2.3. Data Collection and Cleaning

To ensure rigour and transparency throughout the literature identification and screening process, this study followed the PRISMA 2020 guidelines (Page et al., 2021), as illustrated in Figure 1.

Figure 1
Flowchart of PRISMA 2020 procedure in filtering documents for bibliometric analysis



2.3.1. Identification

Scopus (http://www.scopus.com) was selected as the sole bibliographic data source for this study. This decision was informed by recent evidence indicating that Scopus provides broad multidisciplinary coverage of peer-reviewed journals, conference proceedings, and books, together with structured metadata on citations, authors, affiliations, and keywords that are well suited to bibliometric analysis and science mapping (Baas et al., 2020; Visser et al., 2021). Singh et al. (2021) further reported that 99.11% of the journals indexed in Web of Science were also covered by Scopus, although database coverage may vary across disciplines. The suitability of Scopus for the present topic was additionally assessed using a benchmark set: both the baseline search strategy (Query A) and the modified strategy (Query B) retrieved all 10 predefined core publications on TAP-based argumentation in mathematics education, yielding a core-publication retrieval rate of 100%. Scopus was therefore retained to ensure a single and internally consistent source of bibliographic and citation metadata. Nevertheless, the use of a single database does not imply exhaustive coverage, and relevant publications indexed exclusively in other databases may have been omitted.

To enhance the comprehensiveness and methodological rigour of the literature search, an initial set of keywords was developed through a preliminary review of 10 core publications. Insights from this review informed the construction of the Scopus search strings, which were subsequently refined in consultation with a subject-matter expert to incorporate additional synonyms and related terms. Two search strategies were implemented. Query A served as the baseline search and used "Toulmin" as the central term, whereas Query B was a modified, more specific search based on full-form expressions referring to the Toulmin argumentation model and pattern. On July 28, 2026, literature searches were conducted in the Scopus database using the title, abstract, and keyword fields. Two search strategies, Query A and Query B, were applied as follows:

Query A

TITLE-ABS-KEY (Toulmin AND (math* OR geometr* OR algebra* OR calcul* OR statistic* OR proof* OR numerac*) AND (educat* OR teach* OR learn* OR instruction* OR pedagog* OR classroom* OR school* OR student* OR teacher*)) AND PUBYEAR < 2026 AND (DOCTYPE (ar) OR DOCTYPE (re) OR DOCTYPE (cp) OR DOCTYPE (ch)) AND (LIMIT-TO (LANGUAGE , "English"))

Query B

TITLE-ABS-KEY (("Toulmin argumentation model" OR "Toulmin argument model" OR "Toulmin argumentation pattern" OR "Toulmin argument pattern" OR "Toulmin scheme" OR "Toulmin analysis" OR "Toulmin theory" OR (Toulmin W/3 argument*) OR (Toulmin W/3 model*) OR (Toulmin W/3 pattern*) OR (Toulmin W/3 framework*)) AND (math* OR geometr* OR algebra* OR calcul* OR statistic* OR proof* OR numerac*) AND (educat* OR teach* OR learn* OR instruction* OR pedagog* OR classroom* OR school* OR student* OR teacher*)) AND PUBYEAR < 2026 AND (DOCTYPE (ar) OR DOCTYPE (re) OR DOCTYPE (cp) OR DOCTYPE (ch)) AND (LIMIT-TO (LANGUAGE , "English"))

To assess the robustness of the search strategy, a sensitivity analysis was conducted using percentage change and overlap ratio metrics, informed by Lagisz et al. (2025), together with an assessment of whether each query retrieved the 10 predefined core publications. Percentage change and the overlap ratio were calculated as follows:

$$\text{Percent Change} = \frac{N_{\text{modified}} - N_{\text{baseline}}}{N_{\text{baseline}}} \times 100, \text{Overlap} = \frac{N_{\text{overlap}}}{N_{\text{baseline}}} \times 100$$

where N_{baseline} is the total number of baseline documents, N_{modified} is the total number of modified documents, and N_{overlap} is the number of overlapping documents between the two searches.

The results of the search robustness and sensitivity analysis are presented in Table 1. Query A retrieved 154 records, whereas Query B retrieved 135 records, representing a 12.34% reduction and an overlap ratio of 87.66%. Both strategies successfully retrieved all 10 predefined core publications. However, Query B failed to retrieve 19 records included in the Query A dataset, eight of which were identified as relevant to the research topic. These findings indicate that the narrower formulation of Query B reduced search coverage and increased the risk of omitting eligible studies. Query A was therefore retained as the primary search strategy, while Query B was used to assess the sensitivity and stability of the retrieval process.

Table 1
Robustness and sensitivity analysis of the Scopus search strategy

Search variant	*Records retrieved*	*Shared with baseline*	*Unique records*	*Overlap with baseline (%)*	*% Change from baseline*	*Core publications retrieved*
Query A	154	–	19	–	–	10/10
Query B	135	135	0	87.66%	-12.34%	10/10

2.3.2. Screening

Query A retrieved 154 records. All records were independently screened by three reviewers based on their titles and abstracts, using the inclusion and exclusion criteria presented in Table 2. Records that met the preliminary eligibility criteria, or for which eligibility could not be determined from the title and abstract alone, were retained for full-text assessment. Inter-rater agreement was evaluated using Fleiss' kappa (κ) (Fleiss, 1971). Following the conventional benchmarks proposed by Landis and Koch (1977), κ values of .81 or higher were interpreted as indicating almost perfect agreement. Any discrepancies in classification were resolved through discussion until consensus was reached.

Table 2
Eligibility criteria for inclusion and exclusion review

Category	*Inclusion Criteria*	*Exclusion Criteria*
Language	IC1: Publications written in English.	EC1: Publications not written in English.
Research field in Scopus	IC2: Publications in mathematics education, understood as the practice of teaching, learning, and conducting scholarly research on the transmission, acquisition, and development of mathematical knowledge.	EC2: Publications outside the field of mathematics education.
Document type	IC3: Peer-reviewed journal articles, books, and conference proceedings papers.	EC3: Non-peer-reviewed documents.
Research focus	IC4: Studies that apply, adapt, or analyse TAP.	EC4: Studies applying TAP in other scientific disciplines, such as biology, chemistry, or physics, or studies addressing argumentation in general without explicitly referring to TAP.
Publication year	IC5: Publications issued up to and including 2025.	EC5: Publications published after 2025.
Database	IC6: Publications indexed in the Scopus database.	EC6: Publications not indexed in the Scopus database.

During title-and-abstract screening (Figure 2), 35 records were excluded. Of these, five did not address or apply the Toulmin Argument Pattern (TAP; EC4), while 30 were outside the fields of mathematics or mathematics education (EC2), primarily representing communication, computer science, physics, chemistry, the social sciences, and linguistics. The three reviewers demonstrated almost perfect agreement, with $\kappa = .929, Z = 20.00, p < .001$, and an overall percentage agreement of 96.1%. The remaining 119 records were subsequently assessed in full text.

At the full-text assessment stage (Figure 2), six additional publications were excluded. One Spanish-language publication did not meet the language criterion (EC1), one did not substantively address or apply TAP (EC4), and four were outside the fields of mathematics or mathematics education (EC2), primarily focusing on chemistry, philosophy, and integrated STEM education. The three reviewers achieved perfect agreement at this stage, with $\kappa = 1.00, Z = 18.90, p < .001$, and an overall percentage agreement of 100%. Following completion of the screening process, 113 publications met all eligibility criteria and were included in the bibliometric analysis.

2.3.3. Included

After consensus was reached, a total of 113 relevant publications remained. The bibliographic information from the selected studies was subsequently organised and standardised into a unified dataset. This dataset was then exported in '.csv' format to facilitate its use with bibliometric analysis tools (Aria & Cuccurullo, 2017).

2.4. Analysis and Reporting

The primary software used in this study included the R environment with the bibliometrix package (version 5.2) and VOSviewer (version 1.6.20). Microsoft Excel (version 2510) was used to support the initial inspection and preprocessing of the bibliographic data. Prior to analysis, bibliographic fields directly relevant to performance analysis and network analysis-including author names and Scopus Author IDs, affiliations, addresses, countries, source titles and abbreviations, keywords, DOIs, EIDs, and cited references - were standardised following the recommendations of Nowakowska (2025). Variants referring to the same entity were consolidated into a single standardised form. For example, "Mejia-Ramos J.P." was standardised as "Mejía-Ramos J.P.," while the typographical error "Inglisa" was corrected to "Inglis." This standardisation procedure was implemented using a thesaurus.txt file for VOSviewer and a synonyms.csv file for the R-based workflow; both files contained pairs linking the original forms extracted from Scopus to their corresponding standardised forms. The standardised dataset, together with the thesaurus.txt and synonyms.csv files, was subsequently used to conduct the bibliometric analyses and generate the network visualizations. The results are presented in Section 3.

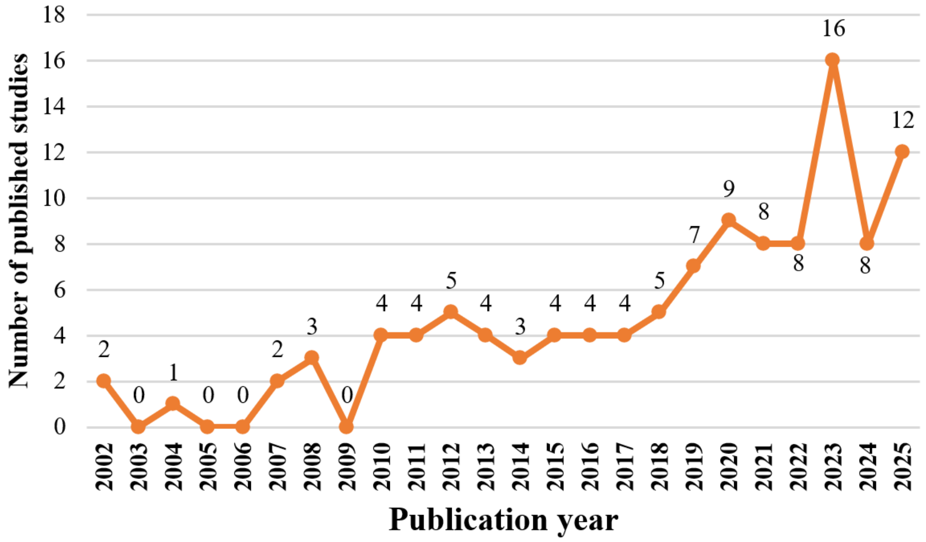
3. Results

3.1. Publication Volume and Temporal Growth Trends of TAP Research in Mathematics Education

To address the first research question, 113 documents on TAP in mathematics education indexed in the Scopus database through the end of 2025 were analysed. These comprised 98 journal articles (86.73%), nine book chapters (7.96%), and six conference papers (5.31%). The annual publication trend is presented in Figure 3. The year 2002 marked the first appearance of research on TAP in mathematics education in the dataset, with two documents. During 2002–2009, publication output remained relatively limited and fluctuated considerably, with only eight documents recorded in total; no documents were identified in 2003, 2005, 2006, or 2009. From 2010 onward, documents appeared more consistently, generally ranging from three to five per year during most of the 2010–2018 period. A clearer upward trend emerged from 2019, with the number of documents increasing from seven in 2019 to nine in 2020, followed by eight documents in both 2021 and 2022. Publication output reached its highest level in 2023, with 16 documents, before declining to eight in 2024 and subsequently increasing to 12 in 2025. Overall, 68 of the 113 documents (60.18%) were published between 2019 and 2025, indicating increased scholarly attention to TAP in mathematics education in recent years, although annual publication output continued to show some fluctuations.

Figure 3

Annual scientific production of research on TAP in mathematics education through 2025



3.2. Most Influential Researchers in TAP Research in Mathematics Education

Table 3 presents the authors with the most prominent influence on research concerning TAP in mathematics education, based on the number of documents (D), total Scopus citations (TC), citations per document (TC/D), and local citations (LC). Mejía-Ramos JP recorded the highest total citation count (TC = 196) across four documents, followed by Conner A (TC = 189; D = 6), Simpson A (TC = 177; D = 2), Inglis M (TC = 166; D = 3), and Pedemonte B (TC = 154; D = 5). In terms of citation influence within the analysed dataset, Conner A received the highest number of local citations (LC = 41), followed by Mejía-Ramos JP (LC = 40), Inglis M (LC = 39), Simpson A (LC = 37), and Pedemonte B (LC = 34). By contrast, Zengin Y had the largest number of documents (D = 9), indicating comparatively high publication productivity, although both TC and LC were lower than those of the most highly cited authors. Overall, the findings indicate that Mejía-Ramos JP, Conner A, Simpson A, Inglis M, and Pedemonte B were among the most influential researchers in this body of literature, although their relative prominence varied across the indicators considered.

Table 3

Authors with the highest total citation counts in research on TAP in mathematics education indexed in Scopus through the end of 2025

Author	*D*	*TC*	*TC/D*	*LC*
MEJÍA-RAMOS JP	4	196	49.0	40
CONNER A	6	189	31.5	41
SIMPSON A	2	177	88.5	37
INGLIS M	3	166	55.3	39
PEDEMONTE B	5	154	30.8	34
STEPHAN M	2	116	58.0	23
KRUMMHEUER G	1	116	116.0	22
RASMUSSEN C	1	112	112.0	23
ZENGIN Y	9	110	12.2	22
HERBST P	3	86	28.7	11
CHAZAN D	3	83	27.7	6
ZACHARIADES T	3	82	27.3	28

Note. D: number of documents; TC : Total Scopus citations; TC/D: Citations per document: LC: Local citations within the analysed dataset.

3.3. Most Influential Publication Sources in TAP Research in Mathematics Education

Table 4 presents the leading publication sources indexed in Scopus up to the end of 2025, highlighting their relative contribution and influence within this research area.

Table 4
Prominent publication sources contributing to research on TAP in mathematics education indexed in Scopus up to 2025

Source	*D*	*TC*	*h-index*	*PYS*	*SJR 2025*
Educational Studies in Mathematics	16	533	13	2007	1.642
Journal of Mathematical Behavior	10	387	9	2002	1.249
Mathematical Thinking and Learning	2	69	2	2014	1.666
ZDM – Mathematics Education	3	66	3	2008	1.582
International Journal of Mathematical Education in Science and Technology	8	63	5	2014	0.729
Journal of Mathematics Teacher Education	3	55	3	2013	1.355
Teachers College Record	1	44	1	2012	0.884
Cognition and Instruction	2	42	2	2012	0.948
Education and Information Technologies	2	35	2	2022	2.131
Educational Research	1	34	1	2004	1.067

Note. D: Number of documents; TC: Total Scopus citations: h-index: h-index within the dataset; PYS: First publication year in the dataset; SJR 2025: SCImago Journal Rank for 2025, accessed on 11 August 2026.

Table 4 presents the most prominent publication sources in research on TAP in mathematics education, ranked by total Scopus citations (TC). *Educational Studies in Mathematics* stands out in terms of both publication output and citation impact, with 16 documents, 533 citations, and an h-index of 13, followed by the *Journal of Mathematical Behavior* with 10 documents, 387 citations, and an h-index of 9. The *International Journal of Mathematical Education in Science and Technology* also shows a relatively high publication output (D = 8), although its total citation count is comparatively lower (TC = 63). In terms of temporal presence, the *Journal of Mathematical Behavior* was the earliest source represented in the dataset (PYS = 2002), followed by *Educational Research* (PYS = 2004) and *Educational Studies in Mathematics* (PYS = 2007). In addition to these dataset-specific indicators, SJR 2025 was included as a contextual measure of the broader scholarly standing of the journals. *Education and Information Technologies* recorded the highest SJR value (2.131), followed by *Mathematical Thinking and Learning* (1.666) and *Educational Studies in Mathematics* (1.642). Overall, *Educational Studies in Mathematics* and the *Journal of Mathematical Behavior* emerge as the most prominent sources in the analysed literature in terms of publication output and citation impact, while PYS provides information on their temporal presence in the field and SJR offers complementary context regarding their broader scholarly standing.

3.4. Dominant and Emerging Thematic Trends in TAP Research in Mathematics Education

Prior to the keyword analysis, the full texts of documents with missing Author Keywords were examined to determine whether author-provided keywords were present in the articles but absent from the Scopus export. This verification resulted in the recovery of keywords for one document, leaving 16 documents without Author Keywords, representing 14.16% of the dataset; Biblioshiny classified the completeness of this field as acceptable. Subsequently, the keyword co-occurrence network was analysed using VOSviewer. Across the 113 documents, a total of 290 keywords were identified. To assess the robustness of the network and determine an appropriate inclusion threshold, a sensitivity analysis was conducted using minimum occurrence thresholds of two, three, and four, while all other network parameters were held constant. The principal thematic structure remained broadly consistent across these thresholds. Accordingly, a minimum occurrence threshold of two was retained to provide broader thematic coverage. Ultimately, 35 keywords met the inclusion criterion and were included in the co-occurrence analysis.

In this analysis, Occurrences denotes the number of documents in which a keyword appears; Link Strength [LS] represents the strength of the co-occurrence relationship between a pair of keywords; and Total Link Strength [TLS] refers to the sum of the strengths of all links between a given keyword and other keywords in the network. Accordingly, higher Occurrences indicate greater keyword prevalence, higher Link Strength indicates stronger co-occurrence between two keywords, and higher TLS reflects greater overall connectivity within the thematic structure of the field (van Eck & Waltman, 2010).

Table 5
Author keywords with the highest total link strength in research on TAP in mathematics education indexed in Scopus through 2025

Keyword	*Occurrences*	*Total Link Strength (TLS)*
Toulmin's argumentation pattern	47	83
Argumentation	29	59
Proof	25	47
Mathematical reasoning	12	27
GeoGebra	7	17
Collective argumentation	10	16
ACODESA method	5	14
Dynamic geometry software (DGS)	5	14
Mathematics teacher education	8	12
Preservice teachers	7	12

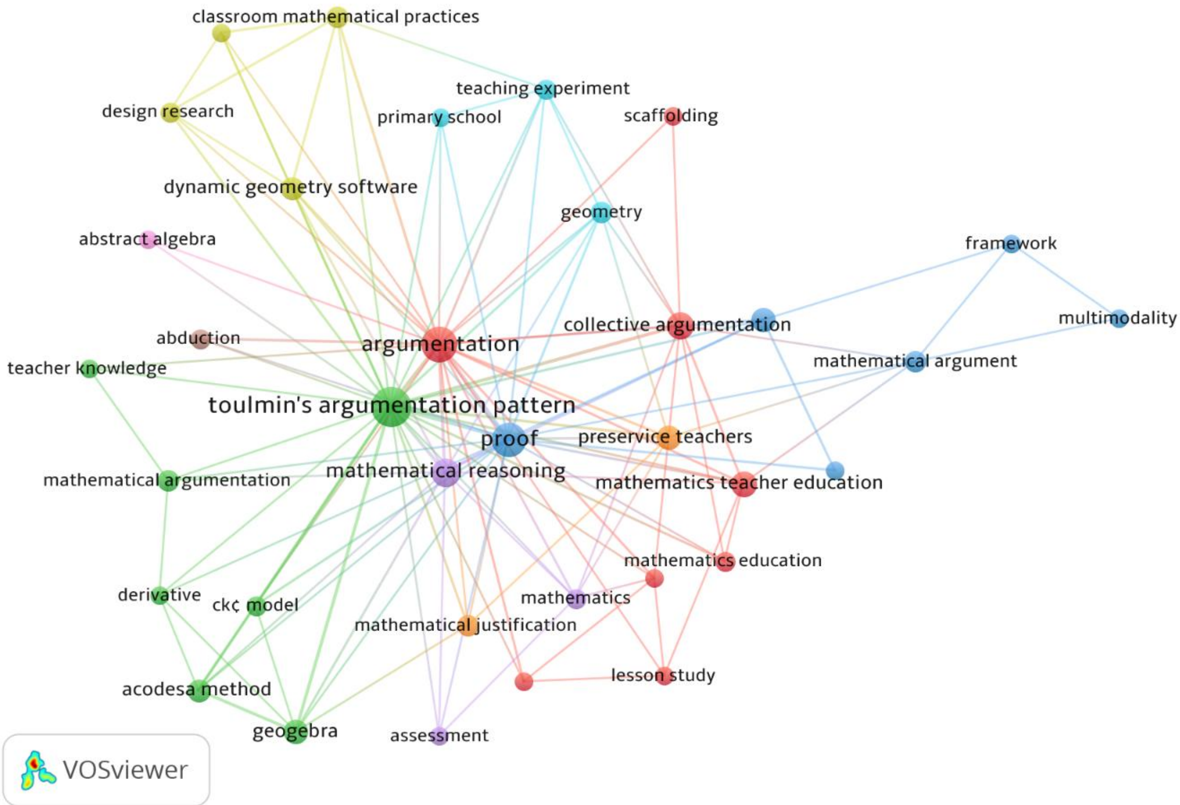
Table 5 presents the keywords with the highest TLS values. *Toulmin's argumentation pattern* was the most prominent keyword, with 47 occurrences and a TLS of 83, followed by *argumentation* (29 occurrences; TLS = 59) and *proof* (25 occurrences; TLS = 47), indicating the central position of argumentation and proof within the thematic structure of the literature. *Mathematical reasoning* also showed substantial connectivity (12 occurrences; TLS = 27). Technology-related keywords included *GeoGebra* (7 occurrences; TLS = 17), *ACODESA method* (5; TLS = 14), and *dynamic geometry software* (5 occurrences; TLS = 14). Teacher-education-related keywords included *mathematics teacher education* (8 occurrences; TLS = 12) and *preservice teachers* (7 occurrences; TLS = 12).

To clarify the thematic structure and relationships among research directions, a keyword co-occurrence network comprising 35 keywords was constructed using VOSviewer (see Figure 4). In the network, node size represents keyword occurrence, links indicate co-occurrence relationships, link thickness reflects the strength of the relationship between two keywords, and colours distinguish thematic clusters. The 35 keywords were grouped into nine clusters, four of which represent the principal thematic directions of the field.

The *Argumentation in teacher education and classroom interaction* cluster (red) comprises *argumentation, collective argumentation, discourse, lesson study, mathematics education, mathematics teacher education, qualitative research*, and *scaffolding*. The *Application of TAP in mathematical argumentation and technology-enhanced environments* cluster (green) includes *ACODESA method, cK¢ model, derivative, GeoGebra, mathematical argumentation, teacher knowledge*, and *Toulmin's argumentation pattern*. The *Structure of mathematical arguments and proof* cluster (blue) comprises *argument, framework, mathematical argument, multimodality, persuasion*, and *proof*. The *Classroom mathematical practices and inquiry-based learning environments* cluster (yellow) includes *classroom mathematical practices, design research, dynamic geometry software*, and *inquiry-based learning*.

Figure 4

Co-occurrence network of author keywords in research on TAP in mathematics education through 2025

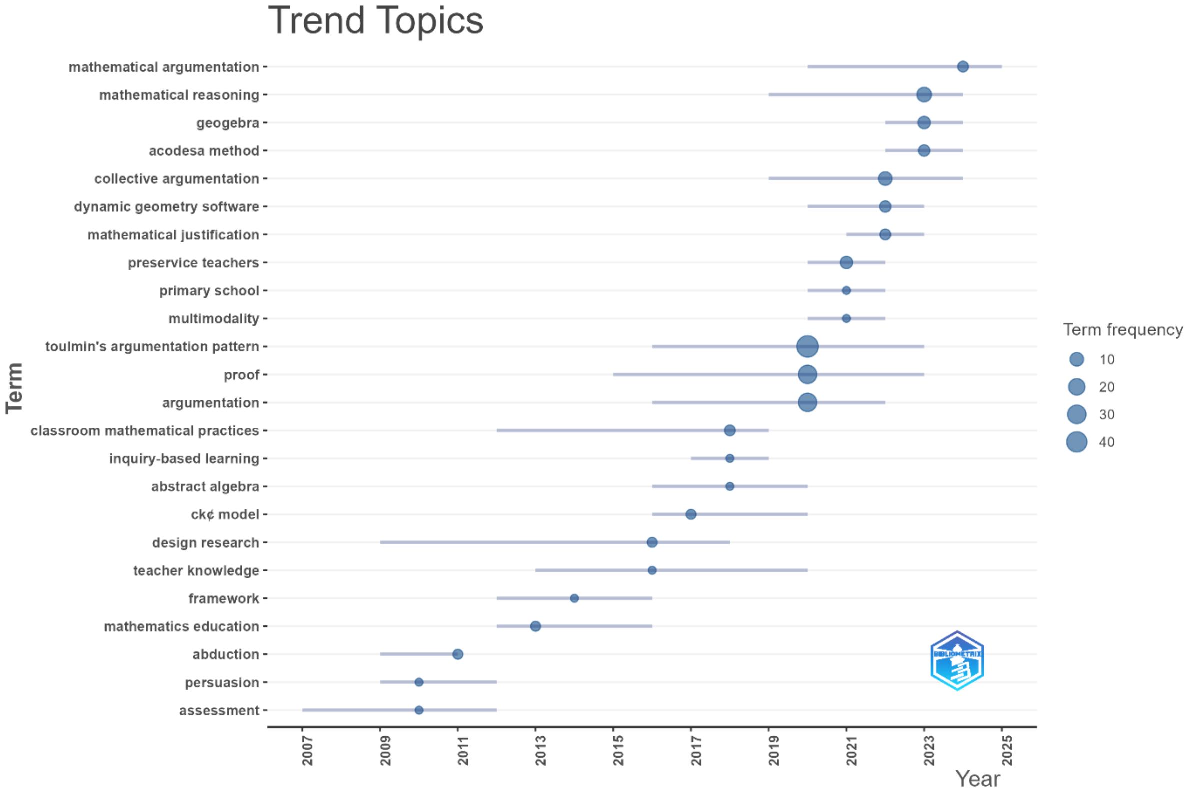


Note. Node size represents keyword occurrence frequency, while link thickness indicates link strength. LS = link strength.

Notably, the two links with the highest link strength in the co-occurrence network both involve *Toulmin's argumentation pattern*. The link between *proof* and *Toulmin's argumentation pattern* has the highest link strength (LS = 13), followed by the link between *argumentation* and *Toulmin's argumentation pattern* (LS = 12). These values indicate that *Toulmin's argumentation pattern* frequently co-occurs with keywords related to proof and argumentation within the analyzed dataset.

Figure 5 illustrates the temporal shift in the thematic focus of research on TAP in mathematics education. Bubble size represents keyword frequency, the position of each bubble on the time axis indicates the median publication year, and the horizontal line reflects the period in which the associated publications are concentrated. In the earlier period, research primarily centred on foundational issues in argumentation and mathematics education (*assessment, persuasion, abduction,* and *mathematics education*). From around 2016, the focus broadened towardss pedagogical contexts and classroom practices (*teacher knowledge, design research, classroom mathematical practices,* and *inquiry-based learning*). Around 2020, mathematical argumentation and proof emerged as prominent themes and maintained a sustained presence over several years (*Toulmin's argumentation pattern, argumentation,* and *proof*). More recently, the thematic focus has expanded further to include collective argumentation, mathematical justification and reasoning, teacher education, and technology-supported learning environments (*collective argumentation, mathematical justification, mathematical reasoning, preservice teachers, GeoGebra, ACODESA method,* and *dynamic geometry software*). Overall, these patterns indicate an increasingly diverse thematic scope of TAP research in both content and context, consistent with the thematic structure identified in the keyword co-occurrence network presented in Figure 4.

Figure 5
Temporal trends in research topics on TAP in mathematics education indexed in Scopus through 2025



4. Discussion

Between 2020 and 2025, 61 publications on TAP in mathematics education were identified, corresponding to an average of 10.2 documents per year and exceeding the 52 publications produced during the entire period before 2020. This pattern indicates a marked increase in scholarly attention to the topic in recent years. The trend can be considered in the broader context of the growing emphasis in mathematics education on the development of thinking and reasoning competencies. Influential frameworks such as the *Common Core State Standards* (Common Core State Standards Initiative, 2010) and Niss's (2003) framework of mathematical competencies had already highlighted the importance of constructing, justifying, and evaluating mathematical arguments. More recently, mathematical reasoning has continued to receive emphasis in assessment frameworks and curricula across different contexts, including PISA (OECD, 2019), Türkiye (Kelly et al., 2020), and Indonesia (Muhammad et al., 2023). At the same time, the expansion of dynamic mathematics environments and educational technologies has provided additional contexts for investigating and analysing mathematical argumentation, within which TAP has continued to be employed as an appropriate analytical framework (Turós et al., 2025). Overall, the increase in publications since 2020 may be viewed in relation to the growing emphasis on mathematical reasoning and the increasing diversity of instructional contexts.

Table 3 shows that the five most influential authors in research on TAP in mathematics education through the end of 2025 were Mejía-Ramos JP, Conner A, Simpson A, Inglis M, and Pedemonte B. Among them, Mejía-Ramos JP, Conner A, and Pedemonte B were affiliated with institutions in the United States, whereas Simpson A and Inglis M were affiliated with institutions in the United Kingdom. Notably, all four publications involving Simpson A and Inglis M in the dataset were co-authored with Mejía-Ramos JP. This pattern indicates the prominent presence of authors with academic links to the United States among the highly influential researchers, consistent with previous bibliometric studies identifying the United States as one of the important centres of research on mathematical argumentation (Kartika et al., 2023). In the more recent period

(2021–2025), the United States continued to account for a substantial number of authors, while Türkiye and Indonesia also recorded increasing numbers of contributors, with 21 and 12 authors, respectively. These findings indicate that research on TAP in mathematics education has been conducted across an increasingly diverse range of national contexts, with Türkiye and Indonesia showing notable levels of participation in recent years (Tristanti & Nusantara, 2021; Zengin & Tapan Broutin, 2025).

The simultaneous consideration of SJR and the h-index can provide complementary information on different aspects of publication sources. SJR reflects a journal's relative standing within the citation network, whereas the h-index captures the cumulative relationship between publication output and citations (Roldan-Valadez et al., 2019). Within the dataset analysed in this study, *Educational Studies in Mathematics* accounted for 16 publications and recorded the highest total citation count, with an SJR 2025 value of 1.642 and an h-index of 13. These findings indicate that the journal has a substantial presence within the body of TAP-related research in mathematics education examined here. Several highly cited publications in the dataset also appeared in this journal; for example, the article by Inglis, M., Mejía-Ramos, J. P., and Simpson, A., entitled "*Modelling mathematical argumentation: The importance of qualification*". The journal's broad coverage of core issues in mathematics education, together with its relatively regular publication schedule of eight issues per year, may partly account for its frequent representation in the dataset. In addition, the *Journal of Mathematical Behavior* contributed 10 publications with a total of 387 citations, indicating that it is also a noteworthy publication venue within the analysed corpus, particularly for studies examining TAP in practice-oriented and classroom-interaction contexts.

Temporal trend analysis suggests that the thematic scope of TAP-related research in mathematics education has gradually broadened from a primary emphasis on the internal structure of arguments toward classroom argumentation practices, collective argumentation, and technology-mediated learning environments. This pattern broadly resonates with developments in the international literature, although existing reviews do not establish a simple chronological shift from structural to social perspectives. Lehmann and Friend (2025), for example, found that Toulmin's model, either explicitly adopted or implicitly reflected through its constituent elements, was the predominant analytical framework in research on K–12 mathematical argumentative writing. Their review also identified social and integrated cognitive–social perspectives alongside the still-dominant cognitive orientation, indicating that mathematical argumentation is increasingly examined in relation to classroom interaction and disciplinary practice rather than solely as an individual cognitive product. This broader analytical orientation is also reflected in work that foregrounds claims, evidence, and reasoning as situated components of student thinking in STEM-rich learning environments (Slavit et al., 2022).

The theoretical flexibility of TAP is further illustrated by studies that combine it with complementary frameworks. Pedemonte and Balacheff (2016), for example, integrated Toulmin's model with the cK¢ model to examine relationships among students' conceptions, argumentation, and proof construction. Their approach demonstrates how Toulmin's framework can move beyond identifying argument components by being combined with models that illuminate the mathematical knowledge and conceptions underlying warrants and backings. A related extension toward the social organization of argumentation is evident in Zhuang and Conner (2022), who coordinated Toulmin's model with Habermas's construct of rationality to investigate how teachers' questioning supported students' participation in collective mathematical argumentation.

Technology-supported environments represent another extension of this analytical trajectory. Urhan and Zengin (2024) combined Toulmin's model with Habermas's construct of rationality to analyse university students' argumentation and proving in a GeoGebra-supported environment involving collaborative learning, debate, and self-reflection. Their findings showed that the semiotic mediation afforded by GeoGebra, together with collaborative and dialogic activity, supported the construction and validation of arguments and enabled students to contribute backing and rebuttals to collective arguments. More recently, Toulmin-based analysis has been extended to human–AI mathematical interaction. Urhan et al. (2024) used Toulmin's model together with Habermas's rationality framework to analyse argumentation between a university student and ChatGPT around calculus concepts. The study showed that ChatGPT could support particular aspects of students' warrants and reasoning, while also producing mathematically problematic and inconsistent responses, highlighting both the analytical potential of TAP for examining AI-mediated reasoning and the need for critical scrutiny of AI-generated mathematical claims. This development parallels a broader movement in mathematics education toward examining AI as a dialogical resource for explanation and reasoning while remaining attentive to the accuracy and reliability of its mathematical outputs (Memiş, 2025).

Taken together, the present findings indicate that the use of TAP in mathematics education has gradually extended beyond the identification of structural components within individual arguments toward analyses that connect argument structure with cognitive processes, classroom interaction, collective reasoning, and technology-mediated learning environments. The capacity of TAP to be coordinated with complementary theoretical perspectives and applied across different forms of mathematical activity is evident in studies combining Toulmin's framework with models of mathematical conceptions and proof, theories of rationality, collaborative argumentation, dynamic mathematics software, and human–AI interaction (Pedemonte & Balacheff, 2016; Urhan et al., 2024; Urhan & Zengin, 2024; Zhuang & Conner, 2022). This development also resonates with broader directions in contemporary mathematics education. Recent studies increasingly foreground dialogic questioning and reasoning in non-routine problem solving, as well as learning environments in which students articulate, compare, reconsider, and revise mathematical ideas through interaction with others (Açıkgöz & Yıldız, 2026; Şahal & Akdaş, 2026). Collaborative mathematics environments similarly provide opportunities for peer explanation, shared problem solving, and collective engagement with mathematical ideas, further highlighting the social dimension of mathematical reasoning (Osei et al., 2026). At the same time, growing attention to dynamic mathematical software and technology-supported learning illustrates how mathematical activity is increasingly mediated by digital tools, feedback, and technologically enriched learning environments (Berhe et al., 2025; Davor et al., 2026). Future TAP research could therefore move beyond documenting the presence or absence of argument components to examine how claims, data, warrants, backing, qualifiers, and rebuttals are constructed, challenged, justified, negotiated, and revised during collective argumentation, within dynamic mathematics environments, and across emerging technology-mediated contexts, including human–AI interaction. Such work could provide a more process-oriented account of how mathematical arguments develop and how social, pedagogical, and technological forms of mediation shape argumentative activity. Nevertheless, because the findings of the present study are derived from keyword co-occurrence networks, the observed patterns should be interpreted as indications of thematic evolution within the literature rather than as evidence that combining TAP with particular technologies, pedagogical arrangements, or complementary theoretical frameworks necessarily leads to improved learning or pedagogical outcomes.

5. Limitations

Despite the findings obtained, this study has several limitations. First, the data were collected exclusively from Scopus and were restricted to English-language publications; consequently, the scope of the literature is dependent on the database's coverage and language criteria. In addition, delays in indexing may affect the inclusion of newly published studies close to the data retrieval date. Second, the dataset of 113 documents is relatively limited for some bibliometric techniques, which are typically applied to larger datasets (Donthu et al., 2021; Rogers et al., 2020; Williams & Bornmann, 2016). However, in narrow and specialised research fields, datasets comprising fewer than 200 documents are still relatively common and can yield useful insights in bibliometric analyses (Fajri et al., 2025; Hanafi et al., 2025). Given the limited volume of international publications on TAP in mathematics education, the dataset of 113 documents was considered appropriate, although the findings should be interpreted cautiously within the scope of the study. Third, keyword standardisation and the selection of analytical thresholds involve researcher judgement and may influence the resulting network structure. Finally, VOSviewer maps should be interpreted primarily as exploratory tools for identifying structures and relationships within the data rather than as confirmatory evidence (Donthu et al., 2021; Kirby, 2023).

6. Conclusion and Implications

This study provides a bibliometric overview of the development of TAP-related research in mathematics education, drawing on 113 English-language publications indexed in Scopus up to the end of 2025. The findings indicate that scholarly interest in TAP has increased in recent years, with publications from 2019 to 2025 accounting for 60.18% of the analysed corpus. Mejía-Ramos, J. P., Conner, A., Simpson, A., Inglis, M., and Pedemonte, B. were among the most prominent authors according to the citation-based indicators used in the study, while Zengin, Y. recorded the highest number of publications. At the source level, *Educational Studies in Mathematics* and *Journal of Mathematical Behavior* showed a substantial presence in the dataset in terms of both publication output and citation impact. Thematically, TAP-related research continues to address core issues in mathematical argumentation and proof, while its scope has gradually broadened to encompass collective argumentation, teacher education, classroom practice, and technology-supported learning environments. Temporal patterns further suggest growing interest in examining TAP within interactive, justificatory, and argument-development processes across emerging learning contexts.

From a practical perspective, the findings suggest that TAP may serve as a useful reference framework for designing learning activities that encourage students to explain, justify, critique, and refine mathematical arguments. Its integration with group discussion, collaborative learning, and digital technologies also points to promising possibilities for classroom implementation. Future bibliometric research could extend the scope to additional databases and languages in order to provide a more comprehensive account of the development of this field.

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Data availability: The data used in this study were obtained from publicly available bibliographic records indexed in Scopus. No new primary data were generated or collected. The data supporting the findings of this study are available from the corresponding author upon reasonable request.

Declaration of interest: The authors declare that they have no conflict of interest.

Ethics declaration: As this study is a review article based exclusively on previously published and publicly available literature, it did not involve human participants, animals, personal data collection, or any form of experimental intervention. Therefore, ethical approval and informed consent were not required for this study.

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